SATISFICING

by
Roy Radner

Professor of Economics and of Statistics[1]
University of California, Berkeley, California, USA

1. INTRODUCTION AND MOTIVATION

As decision theorists have succeeded in extending their analyses into new domains, and have aspired to new levels of both realism and rigor, they have attempted to apply the rationality postulate to more and more complicated decision problems. In particular, decision theorists have become more concerned with the complexities associated with time, uncertainty, and interpersonal conflict and cooperation; and advances in mathematical theories of optimization, statistical decision-making, and games have provided new concepts and tools for the study of rational behavior in the face of such complexities.

Nevertheless, the very success and expansion of these theories have brought into sharper focus a deep problem for the widespread application of the rationality postulate in decision theory. It is now clear that specialists are far from finding "optimal solutions" to such restricted problems as (1) the management of a network of warehouses under general conditions of uncertain demand, (2) winning a game of chess, or (3) administering a department of mathematics. It is probably not good positive theory to take very seriously an assumption that anyone behaves according to a sequential strategy that maximizes an expected lifetime (or infinite horizon) utility, nor is it good advice to a manager to recommend adoption of the solution of an optimization problem that there is no prospect of solving in the next hundred years.

In other words, decision theory is facing more and more clearly the problem

[1] This paper is based on research supported by the National Science Foundation, USA.

of <u>the limits of rationality</u>. I am not speaking here simply of what is often described as the cost of information, but rather of the limited capacities of humans (and machines) for imagination and computation. These limits create theoretical problems on at least two levels. First, there is the profound logical or philosophical problem of defining what one means by "rationality" in the presence of such limits;[2] I shall not discuss this problem here. Second, there is the problem of describing, in terms amenable to theoretical analysis, the different ways humans do behave in complex decision-making situations, and of deducing the consequences of different modes of behavior.

If we are not to discard entirely the rationality postulate in economic theory, then we must elaborate more sophisticated and empirically relevant concepts of rational behavior, which nevertheless retain the important insights provided by the notion of "economic man." Simon has used the term <u>bounded rationality</u> to describe such behavior.[3] I shall not attempt here to give a precise definition of bounded rationality. However, three aspects of bounded rationality do seem important for decision theory: (1) existence of goals, (2) search for improvement, and (3) long-run success.

It is no doubt useful to explain much of economic behavior in terms of "goals" or "motives," and normative economics would appear to be meaningless without reference to goals. On the other hand, an individual economic agent may have "conflicting" goals, and it may be bad psychology in many instances to assume that these conflicts are resolved in terms of a single transitive preference ordering. Such conflicts may be "resolved" in a dynamic way by various mechanisms for switching attention and effort, with results that do not appear to be transitive. (There are, perhaps, useful analogies between individuals with conflicting goals and groups of individuals with conflicting interests.) Also,

---

[2]See, for example, Savage, 1954, pp. 8-17, 59, 83, and Marschak and Radner, 1972, pp. 314-317.

[3]Simon's description is somewhat more general. "Theories that incorporate constraints on the information-processing capacities of the actor may be called <u>theories of bounded rationality</u>." (See H. A. Simon, Ch. 8 of McGuire and Radner, 1972; see, also, Simon, 1959.)

the set of goals may be endogenous, so that, through time, some goals may be dropped and others added to the list.

Even if the theorist draws back from assuming that economic agents behave according to optimal lifetime strategies, it is no doubt useful to postulate that they search for improvements, at least from time to time, and that they take advantage of perceived improvements. How, and under what circumstances, agents search for improvements, and how these improvements are perceived, is, of course, an important subject of study. If repeated improvements can be made in the solution of the same problem, then we have a situation of "expanding rationality." On the other hand, an environment that changes at unpredictable times and in unpredictable directions may make past improvements obsolete, so that the individual is engaged in a race between improvement and obsolescence.

A strategy of search may itself be the object of an improvement effort (as in the planning of research and development), but this leads to a "regression" in the model of decision-making; one eventually reaches a level of behavior at which it is no longer fruitful to assume that the search for improvement is itself being conducted "optimally."

The notion of "adjustment," as it has commonly been used in economic theory, is in the spirit of bounded rationality in the following sense. At a given date the economic agent adopts a particular action (or strategy) that is optimal with respect to the agent's formulation of the decision problem and the agent's "expectations." At the next date, the agent receives new information, which causes him to revise his expectations in <u>a way that was not anticipated at the previous date</u>, or even causes him to revise his formulation of the decision problem. This revision of expectations or of problem formulation is to be distinguished from the behavior of a Bayesian statistician with an optimal sequential decision rule, who periodically revises his <u>a posteriori</u> probability distribution on the states of the environment in response to new information, according to a well-defined and completely anticipated (optimal) transformation.

In a similar spirit, a realistic treatment of the search for improvement in a theory of bounded rationality would not follow the present lines of development of the theory of optimal search.[4] Optimal search theory began with a few interesting theorems showing that for some simple search problems the optimal policies could be described in terms of "aspiration levels" and "satisficing." To take a well-known example, suppose that one is searching for larger values in a sequence of independent and identically distributed random variables (with known probability distribution), but there is a constant cost per observation. If one's objective is to maximize the expected value of the difference between the largest value observed and the total cost of observation, then the optimal sequential stopping rule is characterized by an "aspiration level," i.e., there exists a number, the aspiration level, such that one stops searching as soon as one observes a value that is greater than or equal to the aspiration level. However, there are fairly simple (and plausible) examples of search problems in which the optimal policy cannot be characterized by an aspiration level, or even by a rule that determines the aspiration level at each date as a function of the past history of observations. Rather than attempt to characterize optimal search in a greater variety of more and more complicated problems, the theorist following the approach of bounded rationality would observe that aspiration-level and satisficing behavior is common, even in complicated problems, and would endeavor to understand the implications of such behavior in a variety of situations.

In this lecture I shall explore the consequences of satisficing in the context of a simple model of the allocation of an agent's effort to the search for improvement in one or more activities. For any fixed allocation of effort, the performance of each activity is assumed to be a random walk, or more generally, a semimartingale. The expected rate of change per unit time for each activity depends on the effort allocated to it. This expected rate of change is positive if all of the agent's effort is allocated to the activity, and negative if none is. A behavior is a rule that determines, at each date, the current allocation of effort among the activities as a function of the past history of performance

---

[4]See, for example, MacQueen, 1964, and Rothschild, 1973.

up to that date.

In such a model, performance of the several activities will typically not approach a steady state, even in a stochastic sense, except for very special values of the parameters. In these notes, I examine "long-run success" (i.e., asymptotic performance) with respect to two criteria: (1) the probability of survival, i.e., the probability that performance on one or more activities never falls below certain prescribed levels, and (2) the long-run average rate of growth per unit time.

2. SINGLE OBJECTIVE

2.1. General Formulation of a Satisficing Process

I start with a general formulation of a process of intermittent search for improvement with respect to a single objective. Consider a basic probability space, (X, F, P), where F is a sigma-field of subsets of X, and P is a probability measure on F. Let (F_t), t = 0,1,2,..., be an increasing sequence of subfields of F; F_t is to be interpreted as the set of observable events through date t. Let $\{U(t)\}$ be a corresponding sequence of integer-valued random variables on X, such that U(t) is F_t-measurable; U(t) will be called the performance at t, relative to a given single objective. Finally, let (T_n), n = 0,1,2,..., be a nondecreasing sequence of random times, possibly taking on the value plus infinity, such that $T_n < T_{n+1}$ if T_n is finite; for n odd, T_n is to be interpreted as a date at which a period of search for improvement begins, and T_{n+1} as the date at which that period ends. (A random time T is an integer-valued random variable, possibly equal to plus infinity, such that the event (T = t) is F_t-measurable.) Take $T_0 = 0$.

An interval $(T_n \le t < T_{n+1})$ will be called a search period if n is odd and a rest period if n is even. To capture the idea of intermittent search for improvement I assume: for $T_n \le t < T_{n+1}$,

$$(2.1)\qquad \begin{aligned} E[U(t+1) \mid F_t] &\ge U(t), \quad \text{if } n \text{ is odd,} \\ E[U(t+1) \mid F_t] &\le U(t), \quad \text{if } n \text{ is even.} \end{aligned}$$

In other words, $U(t)$ is a submartingale during the search periods, and a supermartingale during the rest periods.

To capture the idea of "satisficing," let $\{S(t)\}$ be a sequence of random variables such that $S(t)$ is F_t-measurable; $S(t)$ is to be interpreted as the "satisfactory level of performance" at date t. The random times T_n are determined by: for n even,

$$(2.2) \quad \begin{aligned} &T_{n+1} \text{ is the first } t > T_n \text{ such that } U(t) < S(t), \\ &T_{n+2} \text{ is the first } t > T_{n+1} \text{ such that } U(t) \geq S(t); \end{aligned}$$

this is qualified by the convention that, for any n, if T_n is infinite, then so is T_m for every $m > n$.

In the next sections, more specific assumptions will be made about the processes $U(t)$ and $S(t)$.

2.2. A Favorable Satisficing Process

Let $Z(t)$ be the successive increments of the process $U(t)$; thus $Z(t+1) = U(t+1) - U(t)$. Let ξ, η, and β be given positive numbers. For $T_n \leq t < T_{n+1}$, assume:

$$(2.3) \quad \begin{aligned} &\text{(i) for n even (rest),} \\ &\qquad E[Z(t+1) \mid F_t] \leq -\xi , \\ &\qquad S(t) = U(T_n) - \beta + 1; \\ &\text{(ii) for n odd (search),} \\ &\qquad E[Z(t+1) \mid F_t] \geq \eta , \\ &\qquad S(t) = U(T_{n-1}) . \end{aligned}$$

Thus, if a search period ends with $U(T_n) = u$, then the next search period begins as soon as $U(t)$ reaches or falls below $(u - \beta)$, and ends thereafter as soon as $U(t)$ reaches or exceeds u again. During such a search period, u may be called the "aspiration level." For technical reasons, assume further that there is a number b such that

$$(2.3; \text{iii}) \quad |Z(t)| \leq b, \text{ for all } t.$$

Using an inequality of Freedman, 1973, one can prove:

Proposition 1. The random times T_n have finite expectations; indeed, there are numbers μ_0 and μ_1 such that, for all n,

$$(2.4) \qquad E[T_{n+1} - T_n \mid F_{T_n}] \leq \begin{cases} \mu_0 \text{, if n is even,} \\ \mu_1 \text{, if n is odd.} \end{cases}$$

For any nonnegative integer k, let $V_k = U(T_{2k})$. The V_k are the performance levels at which successive search periods end, and each V_k is the aspiration level for the next succeeding search period. It is clear that the V_k form a non-decreasing sequence. If, during search, performance can (with positive probability) increase by more than one unit at a time, then V_k will actually increase from time to time. I shall say that the process is strictly favorable if there is a (strictly) positive number υ such that, for every k,

$$(2.5) \qquad E[V_{k+1} \mid F_{2k}] \geq V_k + \upsilon .$$

Again using Freedman, 1973, one can prove:

Proposition 2. If the process is strictly favorable, then

$$\liminf_{k\to\infty} \frac{V_k}{k} \geq \upsilon \text{, almost surely.}$$

2.3. Random-Walk Search and Rest

In the model of Section 2.2, assume further that, during rest the increments $Z(t+1)$ are independent and identically distributed, with mean $-\xi$, and during search they are also independent and identically distributed, with mean η . In other words, during rest the performance process is a random walk with negative drift, and during search it is a random walk with positive drift. To minimize technical complications, assume further that these random walks are integer-valued and aperiodic.

Let $a(t) = 1$ during search, and 0 during rest. The process

$\{a(t-1), U(t), S(t)\}$ is a Markov chain with countably many states and a single class. Let $D(t) = U(t) - S(t)$. The process $\{a(t-1), D(t)\}$ is also Markovian, with a single class.

Proposition 3. The process $\{a(t-1), D(t)\}$ is positive recurrent. Let $\bar{a}$ denote the long-run frequency with which $a(t) = 1$; and let $\bar{\zeta} = \bar{a}\eta - (1-\bar{a})\xi$; then, almost surely,

$$\lim_{t\to\infty} \frac{U(t)}{t} = \bar{\zeta} . \tag{2.6}$$

If $\bar{\zeta} > 0$, then the process is strictly favorable, in the sense of Proposition 2. In the present case, the sequence (V_k) is a random walk. However, the sequence $U(t)$ is not a random walk, nor even a submartingale. Nevertheless, one can prove for $\{U(t)\}$ the following result.

Proposition 4. If the process is strictly favorable ($\bar{\zeta} > 0$), then there exist positive numbers H and K such that, if $U(0) \equiv u > \beta + b$, then

$$\text{Prob}\ \{U(t) \le 0 \text{ for some } t | F_0\} \le He^{-Ku} .$$

If $\bar{\zeta} = 0$, then the above probability is 1.

Let us say that the process survives if the performance $U(t)$ remains positive for all t. Taken together, Propositions 3 and 4 assert that, for a strictly favorable process, with random-walk rest and search, in the long-run performance increases at a positive average rate per unit time, and the probability of survival approaches unity exponentially as a function of the initial performance level, $U(0)$. This implies further that, if the process has "survived" for a long time, then the performance level is probably very high, and therefore the conditional probability of subsequent survival is close to unity. If the process is not strictly favorable, then the probability of survival is zero.

3. "PUTTING OUT FIRES"

A manager usually supervises more than one activity. For any given level of search effort per unit time, the opportunity cost of searching for improvement in one activity is the neglect of others. Consider a stochastic process $\{U(t), F_t\}$, as in the first paragraph of Section 2.1, but let $U(t)$ be a vector with coordinates $U_i(t)$, $i = 1,\ldots,I$, where $U_i(t)$ is a measure of performance of activity i at date t. An allocation behavior is a sequence, $\{a(t)\}$, where $a(t)$ is an F_t-measurable random vector with coordinates $a_i(t)$, $i = 1,\ldots,I$, such that, for any date t, exactly one coordinate of $a(t)$ is 1, and the other coordinates are 0. If $a_i(t) = 1$, this is interpreted as a search for improvement in activity i at date t.

Concerning the process $U(t)$, I shall make assumptions analogous to those of Section 2.3. As before, let

$$Z(t+1) = U(t+1) - U(t). \tag{3.1}$$

For the conditional distribution of $Z(t+1)$, given F_t, assume:

(3.2a) The distribution of $Z(t+1)$ depends only on $a(t)$.

(3.2b) $EZ_i(t+1) = a_i(t)\eta_i - [1 - a_i(t)]\xi_i$, where ξ_i and η_i are given positive parameters.

(3.2c) $\text{Var } Z_i(t+1) = s_i(a_i[t])$, where $s_i(0)$ and $s_i(1)$ are given positive parameters.

(3.2d) The coordinates of $Z(t+1)$ are mutually independent.

To minimize technical complications, I also assume:

(3.2e) The coordinates of $Z(t+1)$ are integer-valued, uniformly bounded by b, and aperiodic.[5]

A common managerial behavior is to pay attention only to those activities that are giving the most trouble; this is colloquially called "putting out fires." Formally, let

$$M(t) = \min_i U_i(t) , \tag{3.3}$$

and define putting out fires by

---

[5]A random variable is aperiodic if 1 is the greatest common divisor of its support.

(3.4)
(a) if $U_i(t) > M(t)$, then $a_i(t) = 0$;

(b) if $U_i(t) = M(t)$ and $a_i(t - 1) = 1$, then $a_i(t) = 1$;

(c) if neither (a) nor (b) holds, then $a_i(t) = 1$ for i = the smallest j such that $U_j(t) = M(t)$.

To compare putting out fires with the satisficing model of Section 2, roughly speaking, the satisfactory level of performance of any activity is here defined to be equal to $M(t) + 1$.

To describe the properties of the performance process under putting out fires, I first define

$$\bar{\zeta} = (1 - \sum_i \frac{\xi_i}{\eta_i + \xi_i})/(\sum_i \frac{1}{\eta_i + \xi_i}) . \tag{3.5}$$

$$\bar{a}_i = \frac{\bar{\zeta} + \xi_i}{\eta_i + \xi_i} , \quad i = 1,\ldots,I. \tag{3.6}$$

If the limit, as t increases, of $U_i(t)/t$ exists, I shall call this limit the <u>rate of growth of activity i</u>. If $M(t) > 0$ for all t, I shall say that the performance process <u>survives</u>. Define $W(t) = U(t) - M(t)$.

<u>Proposition 3.1</u>. Under putting-out-fires behavior, if $\bar{\zeta} > 0$, then the Markov chain $\{a(t - 1), W(t)\}$ is ergodic, and for each activity i,

(a) the long-run frequency with which $a_i(t) = 1$ is almost surely equal to $\bar{a}_i$;

(b) the rate of growth of $U_i(t)$ is almost surely $\bar{\zeta}$ (the same for all activities);

furthermore, if $M(0) > 0$; and if, for every i, $\text{Prob}\{Z_i(t + 1) \geq 0 | a_i(t) = 0\} > 0$, then

(c) the probability of survival is positive.

In the context of the model defined by (3.2a)-(3.2e) one could explore other allocation behaviors, but the limitation of space does not permit that here. I mention, however, that a necessary and sufficient condition that there exist <u>any</u>

allocation behavior with positive probability of survival is $\bar{\zeta} > 0$. In other words, survival is possible with positive probability if and only if it is possible with putting out fires.

In the special case of two activities (I = 2) the conclusions (a) and (b) of Proposition 3.1 are true also if $\bar{\zeta} \leq 0$.

For proofs of the facts mentioned in this section and for an analysis of other allocation behaviors see Radner and Rothschild (1974).

4. BIBLIOGRAPHIC NOTE

The material of this lecture is adapted from Radner, 1973 and 1974, and Radner and Rothschild, 1974. Satisficing plays an important role in the models of stochastic equilibrium and evolution in Winter, 1971, Nelson and Winter, 1972, and Nelson, Winter and Schuette, 1973. Stochastic search for improvement is a key element of a decentralized resource allocation process that converges to Pareto optimal allocations in the presence of nonconvexities, as described in Hurwicz, Radner, and Reiter, 1973. Related stochastic adjustment processes for reaching the core of a game are described in Green, 1970, and in Neuefeind, 1971.

REFERENCES

Freedman, D., "Another Note on the Borel-Cantelli Lemma and the Strong Law," Ann. Probability 1 (1973), no. 6, 910-925.

Green, J., "Some Aspects of the Use of the Core as a Solution Concept in Economic Theory," unpublished thesis, Rochester, 1970; also, Econometrica, forthcoming.

Hurwicz, L., R. Radner, and S. Reiter, "A Stochastic Decentralized Resource Allocation Process," Technical Report No. 20, Center for Research in Management Science, University of California, Berkeley, 1973; also, Econometrica, forthcoming.

MacQueen, J. B., "Optimal Policies for a Class of Search and Evaluation Problems" Management Sci. 10 (1964), no. 4, 746-759.

Marschak, J. and R. Radner, Economic Theory of Teams, Yale University Press, New Haven, 1972.

McGuire, C. B. and R. Radner (eds.), Decision and Organization, North-Holland Publishing Company, Amsterdam, 1972.

Nelson, R. R. and S. G. Winter, "Toward an Evolutionary Theory of Economic Capabilities," Discussion Paper No. 44, Institute of Public Policy Studies, University of Michigan at Ann Arbor, 1972.

Nelson, R. R., S. G. Winter, and H. L. Schuette, "Technical Change in an Evolutionary Model," Discussion Paper No. 45, Institute of Public Policy Studies, University of Michigan at Ann Arbor, 1973.

Neuefeind, W., "A Tatonnement Process for N-Person Games," CORE Discussion Paper No. 7136, 1971.

Radner, R., "Aspiration, Bounded Rationality, and Control," Presidential Address, Oslo Meeting of the Econometric Society, August 1973 (unpublished manuscript).

Radner, R., "A Behavioral Model of Cost Reduction," Technical Report No. OW-2, Center for Research in Management Science, University of California, Berkeley, 1974; to appear in the Bell J. Econ. and Management Sci., Spring, 1975.

Radner, R. and M. Rothschild, "Notes on the Allocation of Effort," Technical Report No. OW-1, Center for Research in Management Science, University of California, Berkeley, 1974.

Rothschild, M., "Searching for the Lowest Price when the Distribution of Prices is Unknown," J. Political Economy 82 (1974), 689-712.

Savage, L. J., The Foundations of Statistics, Wiley, New York, 1954.

Simon, H. A., "Theories of Decision-Making in Economics and Behavioral Science," Amer. Econ. Rev. 49 (1959), no. 3, 253-283.

Simon, H. A., "Theories of Bounded Rationality," Ch. 8 in C. B. McGuire and R. Radner (eds.), Decision and Organization, North-Holland Publishing Company, Amsterdam, 1972.

Winter, S. G., "Satisficing, Selection, and the Innovating Remnant," Quart. J. Econ. 85 (1971), 237-261.