The Analytical Analysis of Hopfield Neuron Parameters by the Application of Special Trans Function Theory

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Abstract

The subject of the theoretical analysis presented in the paper is Hopfield neuron electronic model modification based upon capacitor replacement with an inverse polarized diode. The modified neuron parameters have been analytically analyzed by application of the Special Trans Function Theory (STFT). The obtained results are presented numerically and graphically.

1 Introduction

Well known Hopfield network can be thought of as a single-layer neural network of continuous nonlinear units mutually completely connected with feedback [1]. The nonlinear active electronic circuit shown in Fig. 1 represents this network active unit, that is one neuron. As obvious this circuit consists of resistors, capacitor, an ideal current source and a nonlinear amplifier. Our idea is to replace the capacitor (C) with an inverse polarized diode (D), as shown in Fig. 2. Namely, in this manner, our intention is to shift in a way Hopfield neuron parameters analysis from the domain of linear differential equation to the domain of exponential transcendental equation solvable analytically by the application of Special Trans Function Theory (STFT) [2]-[13].

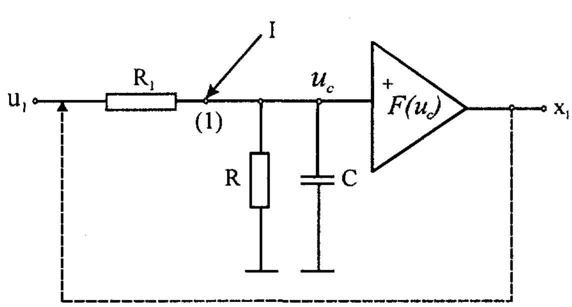

Fig. 1. Hopfield neuron model

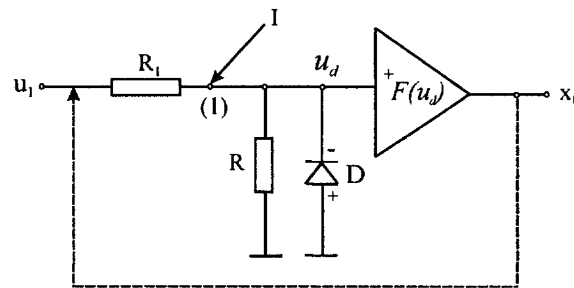

Fig. 2. Modified Hopfield neuron model

2 On Hopfield neuron model transformation

The electronic circuit segment of Hopfield network representing one neuron is given in Fig. 1. The equation describing the evolution of its output state can be obtained by applying Kirchoff's current law to the node (1) in the form:

$$C\frac{du_c}{dt}+\frac{u_c}{R}=\frac{u_I}{R_I}+I \quad (1)$$

where I is an external input signal (or bias) gained from an ideal current source, u_c is capacitor voltage and $F(u_c)$ is an activation function (amplifier transfer function), which is usually of type: $F(u_c)=\dfrac{1}{1+e^{-u_c}}$.

By replacing capacitor (Fig. 1) with an inverse polarized diode (Fig. 2) with large capacity of approximately $0.9\mu F$, we are in position to replace differential equation (1) with an exponential equation of type:

$$i_s\left(1-e^{\frac{-u_d}{V_T}}\right)+\frac{u_d}{R}=\frac{u_I}{R_I}+I \quad (2)$$

where i_s is the saturation current; u_d is the neuron input signal, i.e. diode voltage; V_T is the thermal voltage; I is an external current or bias.

The multiplication of equation (2) by the term $\frac{R}{V_T}$ gives the following equation:

$$\frac{Ri_s}{V_T}-\frac{Ri_s}{V_T}\cdot e^{-\frac{u_d}{V_T}}+\frac{u_d}{V_T}=\frac{R}{V_T}\cdot\frac{u_1}{R_1}+\frac{RI}{V_T} \quad (3)$$

By introducing several transformations of the form:

$$y=\frac{u_d}{V_T};\alpha=-\frac{Ri_s}{V_T};\Omega=\frac{R}{V_T}\cdot\frac{u_1}{R_1};\beta=\frac{RI}{V_T}$$

we get transcendental equation of type:

$$\alpha e^{-y}-\alpha+y=\Omega+\beta . \quad (4)$$

Finally, by the replacement: $\alpha_1=\alpha+\Omega$, the transcendental equation:

$$\alpha e^{-y}+y=\alpha_1+\beta \quad (5)$$

appears.

This equation is directly solvable by application of the Special Trans Function Theory (STFT). Thus, by solving (5) in analytical closed form becomes possible to analyze analytically diode voltage u_d with respect to values u_1, R_1, R and (or) I. Besides, by STFT application it becomes possible to obtain analytically *gradients:* $\frac{\Delta u_d}{\Delta u_1};\frac{\Delta u_d}{\Delta R_1};\frac{\Delta u_d}{\Delta R}$ and (or) $\frac{\Delta u_d}{\Delta I}$.

3 Genesis of the analytical closed form solution to the equation (5)

According to the Special Trans Function Theory [2]-[13], in aim to solve equation (5) analytically in closed-form, first of all, an appropriate equation for identification (EQID), to (5), is to be found. The theoretical method to the genesis of EQID is the intuitive or analogy. Thus, in this case, EQID is linear differential equation and takes the following form:

$$\varphi'(z)+\alpha\varphi(z-1)-\alpha_2\varphi(z)=0 \quad (6)$$

where $\alpha_2=\alpha_1+\beta$. The asymptotic solution of this equation has the form

$$\varphi_{as}=e^{yz} \quad (7)$$

and it is essential to the EQID genesis. Consequently, by the replacement of this asymptotic solution into the equation (6), the equation (5) is to be obtained. On the other hand, equation (6) is solvable analytically by Laplace transform. Thus, after applying Laplace transform equation (6) takes the form

$$\Phi(s)\cdot\left(s+\alpha e^{-s}-\alpha_2\right)=\varphi(0) \quad (8)$$

where

$$\Phi(s)=L(\varphi(z)) \quad (9)$$

or

$$\Phi(s)=\frac{\varphi(0)}{s-\alpha_2+\alpha e^{-s}} . \quad (10)$$

Finally,

$$\Phi(s+\alpha_2)=\frac{\varphi(0)}{s+\alpha e^{-\alpha_2}\cdot e^{-s}}=\frac{\varphi(0)}{s\left(1+\frac{\alpha e^{-\alpha_2}}{s}e^{-s}\right)}$$

or

$$\frac{\varphi(0)}{s}\sum_{n=0}^{\infty}(-1)^n\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{e^{-ns}}{s^n} \quad (11)$$

under the condition $\frac{\alpha e^{-\alpha_2}}{s}e^{-s}\ll 1$.

Now, inverting term by term, in original *z domain,* we obtain

$$\varphi(z)e^{-\alpha_2 z}=\varphi(0)\sum_{n=0}^{[z]}(-1)^n\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z-n)^n}{n!} \quad (12)$$

and finally,

$$\varphi(z)=\varphi(0)e^{\alpha_2 z}\sum_{n=0}^{[z]}(-1)^n\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z-n)^n}{n!} . \quad (13)$$

According to the unique solution principle from equations (7) and (13) follows:

$$\lim_{z\to\infty}\left(\frac{\varphi(z)}{\varphi_{as}(z)}\right)=1 \quad (14)$$

or

$$\lim_{z\to\infty}\frac{\varphi(z+1)}{\varphi(z)}=\frac{\varphi_{0as}e^{y(z+1)}}{\varphi_{0as}e^{yz}}=e^y . \quad (15)$$

From the above equalization we have:

$$y=\lim_{z\to\infty}\left[\ln\left(\frac{\varphi(z+1)}{\varphi(z)}\right)\right]=\lim_{z\to\infty}\left[\ln\left(\frac{e^{\alpha_2(z+1)}}{e^{\alpha_2 z}}\cdot\frac{\varphi_0(z+1)}{\varphi_0(z)}\right)\right] \quad (16)$$

so, the final expression for y becomes:

$$y=\alpha_2+\lim_{z\to\infty}\left[\ln\left(\frac{\varphi_0(z+1)}{\varphi_0(z)}\right)\right]=\alpha_2+trans_N(\alpha,\Omega,\beta) \quad (17)$$

where

$$\varphi_0(z)=\sum_{n=0}^{[z]}(-1)^n\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z-n)^n}{n!} \tag{18}$$

and

$$\varphi_0(z+1)=\sum_{n=0}^{[z+1]}(-1)^n\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z+1-n)^n}{n!}. \tag{19}$$

The error function is defined as:

$$G=\alpha e^{-y}+y-\alpha_1-\beta \tag{20}$$

for y derived by (17), and it must satisfy the inequality $G\le g_0$, where g_0 is enough small real positive number. Therefore, the diode voltage u_d, as neuron input signal, could be obtained by using special trans function, instead of linear differential equation (1) (applicable to the classical Hopfield neuron model) in the following manner:

$$u_d=V_T\left(\alpha_2+trans_N(\alpha,\alpha_2)\right) \tag{21}$$

where $trans_N(\alpha,\alpha_2)$ is a new neuron trans function defined as:

$$trans_N(\alpha,\alpha_2)=\lim_{z\to\infty}\left[\ln\left(\frac{\sum_{n=0}^{[z+1]}(-1)^n\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z+1-n)^n}{n!}}{\sum_{n=0}^{[z]}(-1)^n\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z-n)^n}{n!}}\right)\right].$$

4 The neuron input signal derivation function

The partial derivation function of the neuron input signal u_d upon voltage u_1 value, can be defined on the base of the equation (21). Let us note, that α_2 is a function of voltage u_1, so:

$$\frac{\Delta u_d}{\Delta u_1}=V_T\left(\frac{\Delta\alpha_2}{\Delta u_1}+\frac{\Delta trans_N(\alpha,\alpha_2)}{\Delta\alpha_2}\cdot\frac{\Delta\alpha_2}{\Delta u_1}\right) \tag{22}$$

since, $\frac{\Delta\alpha_2}{\Delta u_1}=\frac{R}{V_T R_1}$ follows that:

$$\frac{\Delta trans_N(\alpha,\alpha_2)}{\Delta\alpha_2}\cdot\frac{\Delta\alpha_2}{\Delta u_1}=\lim_{z\to\infty}\left(\frac{\sum_{n=1}^{[z+1]}(-1)^{n+1}\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z+1-n)^n}{(n-1)!}}{\sum_{n=0}^{[z+1]}(-1)^n\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z+1-n)^n}{n!}}-\frac{\sum_{n=1}^{[z]}(-1)^{n+1}\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z-n)^n}{(n-1)!}}{\sum_{n=0}^{[z]}(-1)^n\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z-n)^n}{n!}}\right)\cdot\frac{R}{R_1} \tag{23}$$

and finally, it is observed that

$$\frac{\Delta u_d}{\Delta u_1}=V_T\cdot\frac{\Delta\alpha_2}{\Delta u_1}\left(1+\frac{\Delta trans_N(\alpha,\alpha_2)}{\Delta\alpha_2}\right)=$$
$$=\left[1+\lim_{z\to\infty}\left(\frac{\sum_{n=1}^{[z+1]}(-1)^{n+1}\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z+1-n)^n}{(n-1)!}}{\sum_{n=0}^{[z+1]}(-1)^n\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z+1-n)^n}{n!}}-\frac{\sum_{n=1}^{[z]}(-1)^{n+1}\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z-n)^n}{(n-1)!}}{\sum_{n=0}^{[z]}(-1)^n\left(\alpha e^{-\alpha_2}\right)^n\cdot\frac{(z-n)^n}{n!}}\right)\right]\cdot\frac{R}{R_1}. \tag{24}$$

Consequently, it is possible to track changes of u_d analytically with respect to the voltage u_1, changing for small arbitrary chosen positive values. The appropriate numerical results and graphical presentations are given in the next section.

5 The numerical results analysis

In this section some simulation results obtained by previously performed theoretical analysis to the equation (5) are presented. Thus, in the Table 1 are given some numerical results of y, that is, of diode voltage u_d and error $|G|$ for different values of integer $[z]$ controlling number of the solution accurate digits. The other values being involved in proposed Hopfield neuron model analysis are given:

$$R=0.1M\Omega;\ R_1=0.5M\Omega;\ i_s=1\mu A;$$
$$V_T=25mV;\ u_1=0.5V \text{ and } I=1nA.$$

Table 1. Values for y, u_d and $|G|$ obtained by STFT for various $[z]$

$[z]$	y	$u_d[mV]$
5	1.20502516381988	30.125629095497
10	1.20398526249551	30.099631562388
15	1.20398516473426	30.099629118356

20	1.20398516603802	30.099629150950
25	1.20398516603764	30.099629150941
$[z]$	$\lvert G \rvert$	
5	2.28733096933276 E-03	
10	2.12205883757394 E-07	
15	2.86742806929197 E-09	
20	8.27563712002544 E-13	
25	2.89004931097736 E-15	

Some other results for u_d and error $|G|$, obtained for various values of voltage u_I and resistance R_I, while parameters R, i_S, V_T and *I take the same values as in* the previous example, are shown in Table 2.

The essential advantage of the STFT approach is the possibility of the partial derivation of the diode voltage u_d. Namely, by using a fixed value of u_I, we are in position to calculate $\Delta u_d / \Delta u_1$ on the basis of equation (24). Some derivation $\Delta u_d / \Delta u_1$ results, for u_I taking discrete values from the interval ($0.1 \div 0.5V$), are presented in Figure 3.

Table 2. The values of u_d and $|G|$ for different u_I and R_I ($[z] = 25$)

$R_I = 0.5[M\Omega]$		
$u_I[V]$	$u_d[mV]$	$\lvert G \rvert$
0.1	4.29953528	2.11292764263141 E-06
0.2	9.22965817	3.86432836806117 E-08
0.3	14.99410535	7.57895527586649 E-10
0.4	21.84172434	5.53645670975378 E-12
0.5	30.09962915	2.89004931097736 E-15
$u_I = 0.5[V]$		
$R_I[M\Omega]$	$u_d[mV]$	$\lvert G \rvert$
0.50	30.0996291	2.89004931097736 E-15
0.55	26.1471403	8.21911982917811 E-15
0.60	23.1101060	2.19401927181728 E-12
0.65	20.7055690	6.46926609504384 E-12
0.70	18.7552700	5.86914995770460 E-11

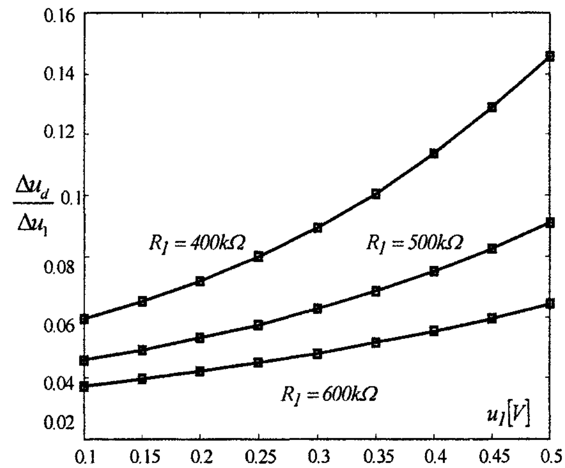

Fig. 3. The partial derivation $|\Delta u_d / \Delta u_1|$ graphical presentation (obtained by equation (24))

6 Conclusions

The paper has proposed an original approach to the modified Hopfield neuron electronic model parameters analytical analysis by the application of recently developed Special Trans Function Theory (STFT). The obtained numerical results confirm applicability and validity of the proposed method. Namely, the architecture of Hopfield electronic neuron model has been changed by replacing capacitor at the amplifier input (representing neuron body) with an inverse polarized diode whose capacity is to be similar to biological neuron membrane capacity (order of μF). This modification comes to remove Hopfield neuron parameters analysis from the field of the linear differential equation to the field of the exponential transcendental equation solvable analytically by STFT approach.

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