Ant-based distributed optimization for supply chain management[1]

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Abstract

Multi-agent systems are the best approach for an efficient supply chain management. However, the control of each sub-system in a supply-chain is a complex optimization problem and therefore the agents have to include powerful optimization resources along with the communication capacities. This paper presents a new methodology for supply-chain management, the distributed optimization based on ant colony optimization, where the concepts of multi-agent systems and metaheuristics are merged. A simulation example, with the logistic and the distribution sub-systems of a supply-chain, shows how the distributed optimization outperforms a centralized approach.

1 Introduction

In order to improve competitiveness and profitability, most of the companies today are organized as *supply-chains*: a world-wide network of external partners (suppliers, warehouses and distribution centers) through which raw materials are acquired, transformed into products and delivered to costumers [1]. The company's job is no longer to produce the goods, but to manage all the different partners in a coordinated manner such that in the end the costumer receives a quality product on a certain desired date.

The different partners in a supply-chain operate under different sets of constraints and objectives. However, the systems are highly interdependent and the optimization of objectives such as on-time deliveries or costs of one partner will influence the performance of the remaining partners. The supply-chain is a pure distributed system with several parallel and independent optimization problem and the coherence between the different decision making centers can be accomplished by a multi-agent based framework, based on explicit communication between constituent agents to control multiple systems [1, 5].

This paper introduces an innovative management methodology based on the description of the supply chain as a distributed optimization problem. The optimization problems are solved by the ant colonies metaheuristic that can also be used as a multi-agent framework.

2 Description of a supply-chain

A typical supply chain has at least two partners: the *logistic* system, that collects the orders from the customers, purchases the components from external suppliers and schedules the components gathered in cross-docking centers, e.g. airports, see [5]); and the *distribution* system, an external company that collects the components at the cross-docking centers and delivers them to the clients as orders. The task of each system can be modeled as an optimization problem.

2.1 Logistic process

The logistic system receives every day new orders requested by different clients, where an order o_j is a set of different types of components in certain quantities, with a certain due date d_j. The different components and their quantities are purchased from external suppliers, that deliver the components to the cross-docking centers after a certain period of time. The logistic process task is a scheduling problem that consists of observing the list of n orders and the list of components, and decides which orders are released at date r_j.

The difference between the release date and the due date is called the *lateness* $L_j = r_j - d_j$. The objective is to match the release date with the due date, i.e. to have for all orders $L_j = 0$. This decision step is done once per day. Two disturbances may influence the system: the fact that suppliers service may not be respected; and the fact that some clients ask for desired delivery dates not compatible to supplier services. The optimization

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objective is to minimize the cost function given by

$$f_L = \frac{\sum_{j\in O} |L_j| + \#\{j \notin O\} + \sum_{j\notin O} L_j}{\#\{j \in O : L_j = 0\}} \quad (1)$$

where $\sum_{j\in O} |L_j|$ accounts for the minimization of the lateness of the set of released orders O; $\#\{j \notin O\}$ refers to the minimization of the number of orders not released; $\sum_{j\notin O} L_j$ describes the minimization of the expected lateness of those orders that remain in the system and are delayed; and finally $\#\{j \in O, L_j = 0\}$ accounts for the maximization of the number of orders delivered at the correct date. This problem can be formally described by a disjunctive graph $G = \{V, A\}$, where the vertices V represent the n orders waiting to be released.

2.2 Distribution process

After the scheduling method has decided which orders will be delivered, a distribution company will pick-up the assigned components and deliver them to the different clients. There is a direct correspondence between clients and orders, but clients are described in this case by their geographical location.

In general, a distribution problem consists of determining how many trucks are necessary to transport the orders and which sequence should be followed in order to minimize the transportation costs. We consider here two constraints: the maximum load capacity Q and the maximum travel distance R of each truck. This distribution problem can be modeled as a *Vehicle Routing Problem* (VRP)[2]. In this case, the cost function to be minimized is the distance traveled by all the vehicles

$$f_D = \sum_{i=0}^{m}\sum_{j=0}^{m}\sum_{l=1}^{v} d_{ij} x_{ijl} \quad (2)$$

where $x_{ijl} = \{0, 1\}$ indicates if the vehicle l traveled the distance $d_{i,j}$ from client i to j: if yes, $x_{ijl} = 1$; if not $x_{ijl} = 0$. The problem can be represented by a disjunctive graph $G = \{V, A\}$, where the vertices V represent the location of the clients and the arcs A are associated with the traveling distance d_{ij} between the vertices.

3 Supply-chain management through distributed optimization

Two different strategies can be adopted for the supply chain management: the *centralized* or the *distributed* optimization. In the first case, the logistic partner is the dominant partner and defines the supply chain solution according only to the logistic objectives. The distribution partner can only optimize the static solution provided by the central system. In the distributed approach, both partners are equally important and the final solution is found after the systems agree about the solution that is better for both systems. This is achieved through a distributed optimization description of the supply chain management problem.

Consider that the supply-chain is a system $S = S_L \circ S_D$ consisting of an aggregation $\circ$ of the logistic system S_L and the distribution system S_D. Let $f = f_L \circ f_D$ be the cost function of the system S, where f_L and f_D are the expressions proposed in (1) and (2), respectively. Distributed optimization is a methodology where the two optimization processes are running in parallel and each of the processes is using the intermediate results of the other process. This can be defined as

$$\min f(t) = \min[f_L(t - l_1)] \circ \min[f_D(t - l_2)] \quad (3)$$

where t describes the actual optimization iteration and $t - l_1$ and $t - l_2$ describe previous optimization iterations. If $l_1 \neq l_2$, the distributed optimization is said to be *asynchronous*, which means that at every optimization iteration, the optimization method accesses information from previous and different iterations. The asynchronous method has been the most used method [6], in order to avoid convergence problems of the $\min[f_i]$ optimization methods, for example in cases where the computational effort of one iteration is different from method to method.

Next section shows how this framework is easily implemented using the Ant Colony Optimization methodology.

4 Ant Colony optimization

The Ant Colony Optimization (ACO) methodology [3] is an optimization method suited to find minimum cost paths in optimization problems described by graphs. Consider a problem with n nodes and a colony of g ants. Initially, the g ants are randomly placed in g different nodes. The probability that an ant k in node i chooses node j as the next node to visit is given by

$$p_{ij}^k(t) = \begin{cases} \frac{\tau_{ij}{}^{\alpha} \cdot \eta_{ij}{}^{\beta}}{\sum_{r\notin\Gamma}^{n} \tau_{ir}{}^{\alpha} \cdot \eta_{ir}{}^{\beta}} & \text{if } j \notin \Gamma \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where τ_{ij} and η_{ij} are the entries of the pheromone concentration matrix τ and heuristic function matrix η respectively, for the path (i, j). The pheromone matrix values are limited to $[\tau_{min}, \tau_{max}]$, with $\tau_{min} = 0$ and $\tau_{max} = 1$. Γ is the *tabu list*, which acts as the memory of the ants and contains all the trails that the ants have already passed and cannot be chosen again. The parameters α and β measure the relative importance of trail pheromone and heuristic knowledge, respectively.

After a complete tour, when all the g ants have visited all the n nodes, the pheromone concentration in the trails is updated by

$$\tau_{ij}(t+1) = \tau_{ij}(t) \times (1-\rho) + \Delta\tau_{ij}^{q} \qquad (5)$$

where $\rho \in [0,1]$ expresses the pheromone evaporation phenomenon and $\Delta\tau_{ij}^{q}$ are pheromones deposited on the trails (i,j) followed by ant q that found the best solution $f^{q}(s)$ for this tour:

$$\Delta\tau_{ij}^{q} = \begin{cases} \frac{1}{f^{q}(s)} & \text{if arc } (i,j) \text{ is used by the ant } q \\ 0 & \text{otherwise} \end{cases} \qquad (6)$$

The algorithm runs N times.

4.1 Implementation in the logistic process

In the scheduling problem of the logistic system, the orders waiting to be delivered are the nodes of the graph, and the role of the ants is to find the minimum cost path connecting the orders that should be delivered. We consider that each ant is traveling with a bag with the available stocks and is distributing the stocks between the orders that it is visiting. It only visits orders whose components it is able to deliver. In this way, the ACO only builds feasible solutions. When the stocks' bag is empty or the remaining components are not enough to deliver any missing order, the search for this ant is finished. In this case, the number of visited nodes may not be the same from one ant to another, while for the VRP the number of nodes to visit is fixed and equal to the number of clients to visit [2].

The heuristic function η is the order's lateness, as proposed in [4]: if an order has already a positive lateness, the ant will feel a stronger attraction to visit it, because the order is already delayed. We define the heuristic function as an exponential function in the interval $[0,1]$ where the value 0 is for the order that has the minimum lateness L_{min} and 1 is for the most delayed order L_{max} [4]. The objective is that the orders already delayed attract ants much more than the orders not yet delayed:

$$\eta_j = \frac{e^{\frac{L_j - L_{min}}{L_{max} - L_{min}}} - 1}{e-1} \qquad (7)$$

Notice that in this case the heuristic information is only order dependent, therefore $\eta_j = \eta_{ij}$. The pheromone trails τ_{ij} are also restricted to the interval $[0,1]$, therefore $\alpha < \beta$ will indicate a higher relative weight of the pheromones trail. The Tabu list is the list of orders already delivered by the ant and also the orders which is not possible to visit, due to lack of stocks. The objective function to minimize by each ant k is f_L^k defined in (1).

4.2 Implementation in the distribution process

To solve the VRP, the ACO algorithm constructs solutions by successively choosing clients to visit until all the orders have been delivered. The nodes are the locations of the clients and there is an extra node specifying the localization of the docking center. The heuristic information used in this case is the *saving function*, proposed in [2]:

$$w_{ij} = d_{i0} + d_{j0} - 2 \times d_{ij} + 2 \times |d_{i0} + d_{j0}| \qquad (8)$$

where d_{ij} is the distance between clients i and j, and $d_{0i} = d_{i0}$ is the distance between client i and the docking center 0. The heuristic matrix η is a normalized version of this heuristic:

$$\eta_{ij} = \frac{w_{ij} - \min[w]}{\max[w] - \min[w]}. \qquad (9)$$

Whenever the choice of a location will lead to infeasible solutions for reasons of vehicle capacity Q or total route length R, the cross-docking center is chosen as a final location to close the tour and a new tour with a new vehicle is started. On the next iteration, the algorithm will start from node 0 again and will repeat this procedure until all the clients are visited and all the orders are delivered. The objective function to be minimized is the one defined in (2).

4.3 Distributed optimization

In the logistic sub-system, the solution's search space is defined by the n orders that can be delivered today. The ACO algorithm uses $n \times n$ matrices τ_L and η_L to search for the optimal solution of f_L. In the distribution sub-system, the solution's search space is $O \cup 0$, i.e., it is equivalent to the search space of the logistic center plus the cross-docking center 0. The ACO algorithm uses $(n+1) \times (n+1)$ matrices τ_{D+0} and η_{D+0}.

The optimization problem $f = f_L o f_D$ is solved in an asynchronous way. The $\circ$ operator represents the composition of the individual pheromone matrices τ_L and τ_D. Note that they both represent a path connecting the clients, although based on different features: lateness and distance.

5 Simulation results

In this section, we compare the supply-chain performance using the centralized and distributed optimization approaches. We consider a simulation environment running one-day optimization problems during one fictive month, where each day a certain stochastic number of new orders enter the logistic system. The clients location follow a random distribution around the cross-docking center.

Table 1. Solutions for the one day problem

Optimization	f_L	f_D
Centralized	0.36	362
Distributed	0.36	355

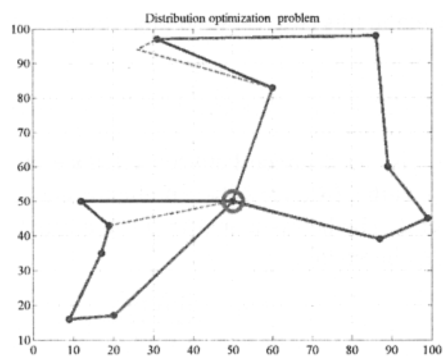

Fig. 1. Distribution problem solution: centralized (-) and distributed(..).

Table 1 shows the results of both logistic (f_L) and distribution (f_D) systems for the one day-problem. In both cases, there are 13 orders to be delivered: 11 orders are distributed at different clients using 2 vehicles, while 2 orders remained in the logistic system. However, when using the distributed optimization approach, the distribution system switched one of the orders that remained in the system by one that was delivered. In logistic terms the result is the same, but in routing terms, it is better, because the traveled distance is smaller. Figure 1 describes this result in detail by representing the cartesian coordinates of the clients' location and the routing solutions for both approaches.

The best cumulative results for the one month problem are presented in Table 2, as well as the mean and variance of the results for 10 different trials. The table presents also the *t-test* probabilities p_t, that indicate if there is a statistical difference between the results. It is clear that the results in terms of logistic optimization are very similar using both approaches, although the best result is obtained using a centralized approach, with a smaller mean result and a narrower variance. However, in terms of the distribution system, it is clear that the system performs better when using the distributed optimization, with lower traveled mean distances and narrower variance. This is confirmed by the t-test (considering a significance level of 0.05) that shows that the results for both approaches in the logistic system case can be considered the same, while for the distribution problem, it shows that the results are statistically different.

Table 2. Solutions for the one-month problem; *t-test* probabilities.

Optimization	f_L	f_D
Centralized	6.74	1511
	(6.86, 0.33)	(1586.8, 81.8)
Distributed	6.91	1422
	(7.28, 0.35)	(1463.2, 48.3)
t-test p_t	0.42	< 0.05

6 Conclusions

This paper introduces a new supply chain management technique, based on the distributed optimization paradigm solved by ant colony optimization. The results show that for a logistic-distribution partners supply chain, the distribution systems performance can be improved without compromising the logistic systems results, i.e. the global systems performance improved just through the exchange of information between the supply-chain partners.

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