# The Linear Approximation Method to the Modified Hopfield Neural Network Parameters Analysis

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## Abstract

The dynamic of Hopfield network is usually described by the system of differential equations. Our idea is to modify Hopfield network in aim to allow its behavior description by the system of transcendental exponential equations solvable analytically by the Special Trans Function Theory (STFT). Furthermore, the linear approximation method to the system of transcendental exponential equations describing the modified Hopfield network, based upon the STFT, has been discussed in some details.

## 1 Introduction

The Hopfield type neural network is an important one due to its applicability in solving associative memory, pattern recognition and optimization problems [1]-[3]. The segment of the nonlinear active electronic circuit in Figure 1 represents classical Hopfield neural network consisting of $n$ neurons. This circuit contains resistors, capacitors, ideal current sources and amplifiers with activation functions, which are often differentiable monotonically increasing ones [4].

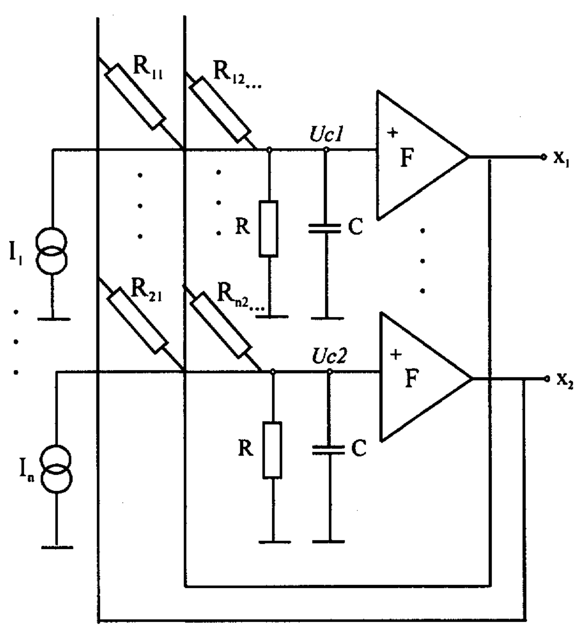


**Fig. 1.** The classical Hopfield network electronic model consisting of $n$ neurons

In the paper capacitors at the amplifiers inputs have been replaced with the inverse polarized diodes with large capacity of approximately $0.5 \div 0.9\mu F$ (Figure 2). By these replacements, the classical Hopfield model analysis come to be moved from the field of linear differential equations to the field of exponential equations solvable easily by the linear approximation method proposed throughout the next sections.

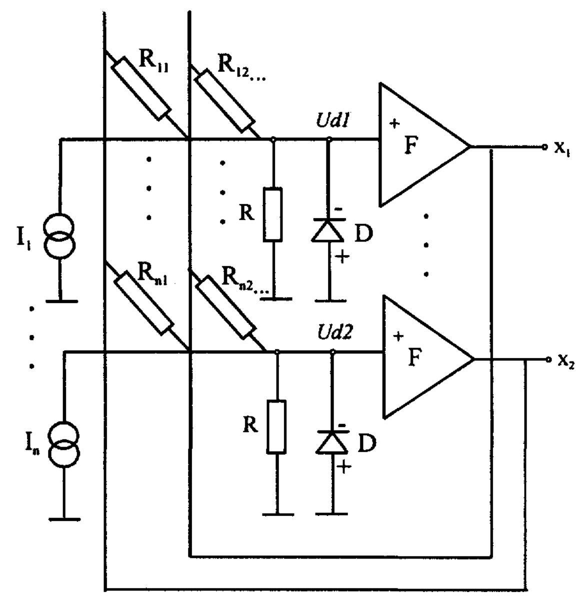


**Fig. 2.** The modified Hopfield network architecture consisting of $n$ neurons

## 2 The differential equations transformation into the exponential ones

The electronic circuit segment of Hopfield network with $n$ neurons is given in Figure 1 and its dynamic can be expressed with the linear differential equation of type:

$$C\frac{du_{C_i}}{dt}+\frac{u_{C_i}}{R}=\sum_{j=1}^{n}\frac{1}{R_{ij}}\left(f\left(u_{C_i}\right)-u_{C_i}\right)+I_i\;;\quad i=\overline{1,n} \quad (1)$$

where $I_i$ is an external input signal (or bias) gained from the ideal current source, $u_{C_i}$ is i-th capacitor voltage, $f\left(u_{C_i}\right)$ is a neuron output voltage, $R$ is the same resistance for each amplifier, while $R_{ij}$ are the resistors representing network weights. By replacing capacitors (Figure 1) with the inverse polarized diodes (Figure 2) with large capacity, at the amplifiers inputs, becomes possible to replace the system of the linear differential equations (1) with a system of exponential equations:

$$i_s\left(1-e^{\frac{-u_{d_i}}{V_T}}\right)+\frac{u_{d_i}}{R}=\sum_{j=1}^{n}\frac{f(u_{d_i})-u_{d_i}}{R_{ij}}+I_i;\quad i=\overline{1,n}\cdot \qquad (2)$$

where $I_i$ is i-th external current or bias, $i_s$ is diode saturation current, $u_{d_i}$ is i-th neuron input signal, that is diode voltage, $V_T$ is the thermal voltage, and $f(u_{d_i})$ is i-th neuron activation function. The system of exponential equations (2) is to be linear approximated by the Special Trans Function Theory (STFT) approach, in the manner being described in the next section.

## 3 The linear approximation method based on STFT

Let us suppose that i-th neuron activation function, that is i-th amplifier transfer function, has the simplest form $f(u_{d_i})=C_{1_i}=const.$ Under this assumption equation (2) can be rewritten as follows:

$$i_s\left(1-e^{\frac{-u_{d_i}}{V_T}}\right)+\frac{u_{d_i}}{R}=\sum_{j=1}^{n}\frac{C_{1_i}-u_{d_i}}{R_{ij}}+I_i;\quad i=\overline{1,n}\cdot \qquad (3)$$

Now, the term $e^{-\frac{u_{d_i}}{V_T}}$ can be approximated with:

$$e^{\frac{-u_{d_i}}{V_T}}=1-\frac{u_{d_i}}{V_T}+\delta_i;\quad i=\overline{1,n} \qquad (4)$$

where $\delta_i$ is a linear approximation error. By taking into account this linear approximation, equation (3) takes the form:

$$i_s\left(\frac{-u_{d_i}}{V_T}-\delta_i\right)+\frac{u_{d_i}}{R}=\sum_{j=1}^{n}\frac{C_{1_i}-u_{d_i}}{R_{ij}}+I_i;\quad i=\overline{1,n}\cdot \qquad (5)$$

It is to be pointed out that equation (4) can be solved analytically by the Special Trans Function Theory [5]-[7]. Namely, for given $\delta_i$ and $V_T$, we can obtain value of $u_{d_i}$ in analytical closed-form:

$$u_{d_i}=V_T\cdot trans(\delta_i,V_T);\quad i=\overline{1,n} \qquad (6)$$

where $trans(\delta_i,V_T)$ is new trans function defined as follows:

$$trans(\delta_i,V_T)=\lim_{x\to\infty}\left[\ln\left(\frac{\sum_{k=0}^{[x+1]}(-1)^k e^{-k\xi}\frac{(x+1-k)^k}{k!}}{\sum_{k=0}^{[x]}(-1)^k e^{-k\xi}\frac{(x-k)^k}{k!}}\right)\right];$$

$$\xi=1+\delta_i;\, i=\overline{1,n} \qquad (7)$$

The equation (7) is obtained by classical STFT approach [8]. By using in this manner obtained $u_{d_i}$, we can now determine the new value of $\delta_i$, that is ${}^{(1)}\delta_i$, as remain part of MacLaurin series:

$${}^{(1)}\delta_i=\frac{(u_{d_i}/V_T)^2}{2!}-\frac{(u_{d_i}/V_T)^3}{3!}+\frac{(u_{d_i}/V_T)^4}{4!}-\frac{(u_{d_i}/V_T)^5}{5!}+\ldots \qquad (8)$$

By means of this new value for $\delta_i$, i.e. ${}^{(1)}\delta_i$, the new one for $u_{d_i}$, that is for ${}^{(1)}u_{d_i}$, can be calculated on the base of transformed equation (5), which after some elementary transformations takes the form:

$${}^{(1)}u_{d_i}=\frac{\sum_{j=1}^{n}\frac{C_{1_i}}{R_{ij}}+I_i+i_s\,{}^{(1)}\delta_i}{\frac{i_s}{V_T}+\frac{1}{R}+\sum_{j=1}^{n}\frac{1}{R_{ij}}};\, i=\overline{1,n} \qquad (9)$$

The above procedure is to be continued until not the inequalities $\left|{}^{(1)}\delta_i-\delta_i\right|\le\varepsilon$ and $\left|{}^{(1)}u_{d_i}-u_{d_i}\right|\le\zeta$, for enough small real $\varepsilon$ and $\zeta$, be satisfied.

### 3.1 The numerical example

Some numerical results related to previously described method to the $u_{d_i}$ estimation, for given values of $R, R_1, i_s, V_T, C_1$ and $I$, in the case of one neuron, are presented in Table 1.

The same method could be applied to two or more neurons forming the network. The iterative procedures should be performed independently, for each neuron, until all $\delta_i$ and $u_{d_i}$ $(i=\overline{1,n})$ be obtained with the appropriate accuracy.

**Table 1.** The numerical results obtained through 15 iterations, for
$R = 0.7M\Omega;\ R_I = 0.5M\Omega;\ i_s = 1\mu A;\ I = 1nA;\ C_I = 0.5V$ and $V_T = 25mV$

| It. | $\delta$ | ${}^{(l)}\delta$ | $\left\lvert {}^{(l)}\delta - \delta \right\rvert$ |
|---|---|---|---|
| 1. | 0.300000 | 0.299393 | 0.606883 E-03 |
| 2. | 0.299393 | 0.298790 | 0.602752 E-03 |
| 3. | 0.298790 | 0.298191 | 0.598671 E-03 |
| 4. | 0.298191 | 0.297597 | 0.594636 E-03 |
| 5. | 0.297597 | 0.297006 | 0.590649 E-03 |
| 6. | 0.297006 | 0.296419 | 0.586707 E-03 |
| 7. | 0.296419 | 0.295836 | 0.582809 E-03 |
| 8. | 0.295836 | 0.295257 | 0.578957 E-03 |
| 9. | 0.295257 | 0.294682 | 0.575148 E-03 |
| 10. | 0.294682 | 0.294111 | 0.571383 E-03 |
| 11. | 0.294111 | 0.293543 | 0.567660 E-03 |
| 12. | 0.293543 | 0.292979 | 0.563978 E-03 |
| 13. | 0.292979 | 0.292419 | 0.560337 E-03 |
| 14. | 0.292419 | 0.291862 | 0.556736 E-03 |
| 15. | 0.291862 | 0.291309 | 0.553175 E-03 |
| It. | $u_{d_i}$ | ${}^{(l)}u_{d_i}$ | $\left\lvert {}^{(l)}u_{d_i} - u_{d_i} \right\rvert$ |
| 1. | 0.022222 | 0.019729 | 2.492305 E-03 |
| 2. | 0.022196 | 0.019718 | 2.478252 E-03 |
| 3. | 0.022170 | 0.019706 | 2.464277 E-03 |
| 4. | 0.022145 | 0.019694 | 2.450378 E-03 |
| 5. | 0.022120 | 0.019683 | 2.436556 E-03 |
| 6. | 0.022094 | 0.019672 | 2.422808 E-03 |
| 7. | 0.022069 | 0.019660 | 2.409135 E-03 |
| 8. | 0.022044 | 0.019649 | 2.395536 E-03 |
| 9. | 0.022020 | 0.019638 | 2.382010 E-03 |
| 10. | 0.021995 | 0.019627 | 2.368556 E-03 |
| 11. | 0.021971 | 0.019616 | 2.355173 E-03 |
| 12. | 0.021947 | 0.019605 | 2.341861 E-03 |
| 13. | 0.021922 | 0.019594 | 2.328619 E-03 |
| 14. | 0.021898 | 0.019583 | 2.315447 E-03 |
| 15. | 0.021875 | 0.019572 | 2.302343 E-03 |

## 4 The capacitor and the diode equalization

The basic question related to the proposed Hopfield neural network modification is undoubtedly: under which condition(s) the capacitor can be replaced with an inverse polarized diode? The equalization can be realized under the assumption that the current $i(t)$ (Figure 3) takes the form:

$$i(t) \approx \gamma_i e^{\beta_i t} \tag{10}$$

where

$$\gamma_i = \frac{\alpha_i i_s}{1-\alpha_i e^{-\beta_i t}};\ \alpha_i = \frac{i(0)}{i(0)+i_s} \text{ and } \beta_i = \frac{i_s}{CV_T}.$$

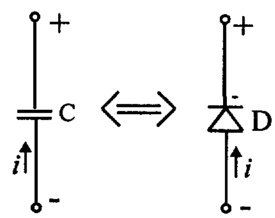


**Fig. 3.** The capacitor and the inverse polarized diode equalization

The expression (10) for current $i(t)$ is obtained through following steps:

(a) by equalizing capacity (C) voltage $u_C$ with an inverse polarized diode (D) voltage $u_d$:

$$u_C = \frac{1}{C}\int_0^t i(t)dt \Leftrightarrow u_d = -V_T\, ln\left(\frac{i(t)+i_s}{i_s}\right), \tag{11}$$

that is

$$\frac{1}{C}\int_0^t i(t)dt = -V_T\, ln\left(\frac{i(t)+i_s}{i_s}\right); \tag{12}$$

(b) by differentiation the above equalization, we get:

$$\frac{i(t)}{C} = -V_T \frac{i'(t)}{i(t)+i_s}; \tag{13}$$

(c) now, we are performing some elementary transformations (formulae (14)-(17)):

$$\frac{1}{CV_T} = -\frac{i'(t)}{i(t)}\cdot\frac{1}{(i(t)+i_s)} = -\frac{i'(t)}{i(t)(i(t)+i_s)}; \tag{14}$$

$$\frac{1}{CV_T} = -\frac{i'(t)}{i_s}\left(\frac{1}{i(t)} - \frac{1}{i(t)+i_s}\right); \tag{15}$$

$$\frac{i_s}{CV_T} = -\frac{i'(t)}{i(t)} + \frac{i'(t)}{i(t)+i_s} \Big/ \int_0^t; \tag{16}$$

$$\frac{i_s}{CV_T}t = -ln\frac{i(t)}{i(0)} + ln\left(\frac{i(t)+i_s}{i(0)+i_s}\right) = ln\frac{(i(t)+i_s)i(0)}{(i(0)+i_s)i(t)} \tag{17}$$

d) by using replacements $\alpha_i = \frac{i(0)}{i(0)+i_s}$, as well as $\beta_i = \frac{i_s}{CV_T}$, we finally obtain expression for $i(t)$ in the following form:

$$i(t) = \frac{\alpha_i i_s e^{-\beta_i t}}{1-\alpha_i e^{-\beta t}} \approx \gamma_i e^{-\beta_i t} \tag{18}$$

which is identically the same as (10). By supplying the current in the circuit representing Hopfield modified structure, changing in time in accordance with (10), the capacitor and inverse polarized diode equalization can be performed. In other words, the condition of the capacitor and the diode equalization is provided in this manner rather theoretically. This equalization is of upmost importance in stability and network energy function analysis. It is to be noted that the stability and energy function analyses of the modified Hopfield-type neural network are to be the subjects of further more rigorous investigation.

## 5 Conclusions

The idea of Hopfield neural network electronic model modification by the capacitor replacement with the inverse polarized diode with great PN junction capacity has been proposed in the paper. It has been done in aim to enable Hopfield network architecture analysis in the domain of exponential equations instead of in the domain of differential ones. Besides, the exponential equations, describing modified Hopfield architecture, have been linear approximated and solved analytically by STFT approach. The obtained numerical results confirm in a way the validity and applicability of proposed method.

The equalization between the capacitor and the diode at the neuron input has been examined in the paper, as well. It is shown that this equalization can be established under the condition that the current in the modified Hopfield structure has the appropriate continuous-time form.

The further investigation should be oriented toward modified Hopfield architecture stability and energy function analysis, which are of prior importance in improving system performance and in solving some difficult problems, like some of the associative memory, the pattern recognition and the optimization problems are.

## References

[1] Juang J-C. (1999), Stability Analysis of Hopfield-Type Neural Networks, *IEEE Transactions on Neural Networks*, vol. 10, no. 6, pp 1336-1374

[2] Quiao H., et al. (1999), Nonlinear Measures: A New Approach to Exponential Stability Analysis for Hopfield-Type Neural Networks, *IEEE Transactions on Neural Networks*, vol. 12, no. 2, pp 360-369

[3] Kakeya H., Okabe Y. (2000), Fast combinatorial optimization with parallel digital computers, *IEEE Transactions on Neural Networks*, vol. 11, no. 6, pp 1323-1331

[4] Hassoun M. H. (1995), *Fundamentals of Artificial Neural Networks*, The MIT Press, London

[5] Bauk S., Perovich S. M. (2003) STFT iterative procedure for estimating probability of clients arrivals in case of M/M/1 queue, *SYM-OP-IS*, S&M, pp 735-737

[6] Bauk S., Perovich S. M. (2003), Procjena vjerovatnoće nastupanja otkaza primjenom teorije specijalnih trans funkcija, *NSS-OMO*, S&M, pp 60-63

[7] Bauk S., Perovich S. (2004) Considering the special trans function theory approach to the probability estimation (submitted to the *Statistics and Probability Letters*)

[8] Perovich S. (2004) Research monograph – The Special Trans Function Theory, University of Montenegro