Combining Topological and Cardinal Directional Relation Information in Qualitative Spatial Reasoning

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Abstract

Combining different knowledge representation languages is one of the main topics in Qualitative Spatial Reasoning (QSR). In this paper, we combine well known RCC8 calculus (RCC8) and cardinal direction calculus (CDC) based on regions and give the interaction tables for the two calculi. The interaction tables can be used as a tool in solving constraint satisfaction problems (CSP) and consistency checking procedure of QSR for combined spatial knowledge.

1 Introduction

Qualitative spatial reasoning is very useful in improving the reasoning efficiency in answering spatial queries, and can avoid time-consuming quantitative geometry computation. But reasoning with only one aspect of spatial knowledge is not realistic, combining and integrating different kinds of knowledge is an emerging and challenging issue in QSR. [1] has dealt with the combination of topological knowledge and metric size knowledge in QSR, and [2] has combined the cardinal direction knowledge and the relative orientation knowledge.

We first introduce the two formalisms of topological and cardinal directional relations, respectively. The region considered in this paper is non-empty, connected point-set homeomorphic to a unit disk in $\Re^2$.

1.1 RCC8 Calculus (RCC8)

Topology is perhaps the most fundamental aspect of space. Topological relationships are invariant under topological transformations, such as translation, scaling, and rotation. Examples are terms like *neighbor* and *disjoint* [3]. The RCC-8 (Figure 1) is a set of eight jointly exhaustive and pairwise disjoint (JEPD) relations, called base relations, denoted as DC, EC, PO, EQ, TPP, NTPP, TPPi, NTPPi, with the meaning of DisConnected, Extensionally Connected, Partial Overlap, Equal, Tangential Proper Part, Non-Tangential Proper Part, and their converses. Exactly one of these relations holds between any two spatial regions. A reasoning system has been derived according to the property that a situation involving a number of topological relations is possible if and only if the set of model-constraints associated with all of the relations does not entail any of the entailment constraint formulae. A composition table has been obtained by a spatial reasoner using the above technique. For details about the reasoner and composition table, please refer to [4].

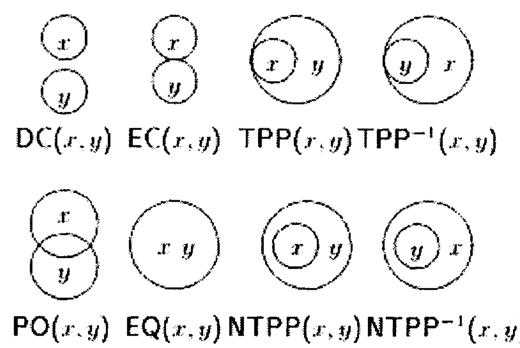

Figure 1. Two-dimensional examples for the eight basic relations of RCC-8

An alternative approach to representing and reasoning about topological relations has been put forward [5], which is based on point-set topology.

1.2 Cardinal Direction Calculus (CDC)

Direction-also called orientation-relationships are important and common-sense linguistic and qualitative properties used in everyday situations and qualitative spatial reasoning.

[6] introduced a direction-relation model for extended spatial objects that considers the influence of the objects' shapes. It uses the projection-based direction partitions and an extrinsic reference system, and considers the exact representation of the target object with respect to the reference frame. The reference frame with a polygon as reference object has nine direction tiles: north (N_A),

northeast (NE_A), east (E_A), southeast (SE_A), south (S_A), southwest (SW_A), west (W_A), northwest (NW_A), and same (O_A). The cardinal direction from the reference object to a target is described by recording those tiles into which at least one part of the target object falls (Figure 2).

$$dir_{RR}(A,B)=\begin{bmatrix} NW_A \cap B & N_A \cap B & NE_A \cap B \\ W_A \cap B & O_A \cap B & E_A \cap B \\ SW_A \cap B & S_A \cap B & SE_A \cap B \end{bmatrix}$$

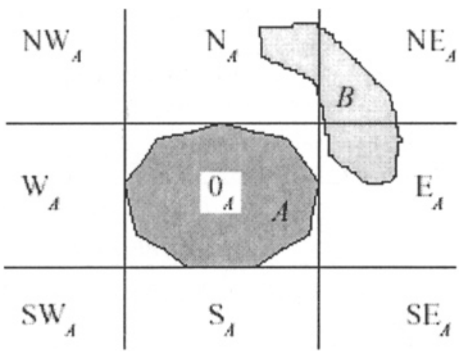

Figure 2. Capturing the cardinal direction relation between two polygons, A and B, through the projection-based partitions around A as the reference object.

For cardinal directions between two polygons, a 3×3 matrix captures the neighborhood of the partition around the reference object and registers the intersections between the target and the tiles around the reference object (Equation 1). The elements in the direction-relation matrix have the same topological organization as the partitions around the reference object. We consider the emptiness and non-emptiness of the nine intersections between the nine tiles formed around the reference object and the exact representation of the target object to describe coarse cardinal directions.

2.Reasoning about combined knowledge of RCC8 and CDC relations

Our main goal is to combine the topological and cardinal direction relation knowledge, which is motivated and inspired by the work of [2]. They try to combine the cardinal direction relations and relative orientation relations based on 2D points. We think our work is more practical, because in the real world we are often faced with regions, not points. And our work is different from theirs in the following aspects:

(1) we investigate two kinds of binary region-based relations; they focus on combining binary and ternary relations, which are all point-based.

(2) We derive additional interaction rules between RCC8 and CDC.

(3) In our procedure, we use two queues instead of one queue used by their work, which can be computed parallel.

Reasoning within the combined knowledge of RCC8 and CDC can be divided into two parts: internal reasoning within the RCC8 and CDC and interaction reasoning between them.

To facilitate the representation of the interaction rules, we denote a basic cardinal direction relation by a set *SB* including at most nine elements, i.e. the nine single-tile [7] cardinal direction relations. For example, *B:S:SE:SN* can be denoted by {*B,S,SE,SN*}. The general cardinal direction relation can be regarded as a set *GB*, whose element is the set *SB*. So we have the relation: $SB \in GB$. The universal relation is the set *BIN*={*B, N, NE, E, SE, S, SW, W, NW*}, and the universe, i.e. the set of all possible cardinal relations, is denoted by *U*.

For two arbitrary regions X and Y, we use *RCC8*(X, Y) to denote the topological relation of X to Y, and we stipulate that *GB*(X, Y) represents the general cardinal direction relation of the primary region X to the reference region Y, and that *SB*(X, Y) represents any basic cardinal relation that belongs to *GB*(X, Y).

2.1 Reasoning within RCC8 and the interaction from RCC8 to CDC (RCC8-To-CDC)

For the internal reasoning within the RCC8, the composition table for pairs of atoms can be found in [8] and [9].

The interaction from the atomic relations of RCC8 to CDC relations has been described in Table 1, where the left column is the eight atomic RCC8 relations (see Fig. 1) and the right column is the cardinal direction relations induced by the left column RCC8 relations. We now give interpretation for this table.

For arbitrary regions X and Y, the table is divided by the value of *RCC8*(X, Y) into eight case as follows:

(1) DC(X, Y): the induced cardinal direction relation *GB*(X, Y) is the universe, i.e. *U*.

(2) EC(X, Y): every basic cardinal direction relation *SB*(X, Y) that belongs to *GB*(X, Y) must includes the relation *B*.

(3) PO(X, Y): every basic cardinal direction relation *SB*(X, Y) that belongs to *GB*(X, Y) must include the relation *B* and an atomic relation that belongs to *BIN*.

(4) TPP(X, Y): the induced cardinal direction relation *GB*(X, Y) includes only one element *B*.

(5) TPPi(X, Y): every basic cardinal direction relation *SB*(X, Y) that belongs to *GB*(X, Y) must include the relation *B* and an atomic relation that belongs to *BIN*.

(6) NTPP: the induced cardinal direction relation *GB*(X, Y) includes only one element *B*.
(7) NTPPi(X, Y): the induced cardinal direction relation *GB*(X, Y) includes only one element *BIN*, i.e. the tiles formed by reference object Y are all occupied by the primary object X.
(8) EQ(X, Y): it is obvious that the induced cardinal direction relation *GB*(X, Y) can only include the relation *B*.
We use *R-to-C* to denote the operation that captures the above interaction between RCC8 knowledge and CDC knowledge, in the direction RCC8-To-CDC, by inferring CDC knowledge from given RCC8 knowledge.
The induced cardinal direction relation by a general RCC8 relation (i.e. the disjunction of atomic RCC8 relations), say *Q* (X, Y), is the union of cardinal direction relations induced by all atomic RCC8 relations that belong to the general RCC8 relation; namely:

$$GB(X,Y) = \bigcup_{r \in Q} R-to-C(r(X,Y)).$$

Table 1. The interaction table from the basic relations of CDF to RCC8 relations

Atomic RCC8 relation	Induced cardinal direction relation
DC	U
EC	$\forall SB \in GB : B \in SB$
PO	$\forall SB \in GB$, $\exists R \in BIN : R \in SB \bigcap B \in SB$
TPP	$\{\{B\}\}$
TPPi	$\forall SB \in GB$, $\exists R \in BIN : R \in SB \bigcap B \in SB$
NTPP	$\{\{B\}\}$
NTPPi	$\{BIN\}$
EQ	$\{\{B\}\}$

2.2 Reasoning within CDC and the interaction from CDC to RCC8 (CDC-To-RCC8)

The converse operation and the composition operation have been investigated in [10] and [7] respectively.
The RCC8 knowledge induced from the basic cardinal direction relations, denoted by *SB*, is presented in the table 2, which describes the interaction between RCC8 knowledge and CDC knowledge, in the direction CDC -To- RCC8, in four cases:
(1)$SB=\{B\}$: the induced RCC8A relation is the disjunction of *DC*, *EC*, *PO*, *TPP*, *NTPP*, *EQ* and *TPPi*.
(2) $\exists R \in BIN : R \in SB \bigcap B \in SB$: the induced RCC8A relation is the disjunction of *DC*, *EC*, *PO TPPi* and *NTPPi*.
(3) Universal relation: the induced RCC8A relation is the disjunction of *DC*, *EC*, *PO*, *TPPi* and *NTPPi*.
$B \notin SB$: the induced RCC8A relation is only *DC*.
We use *C-to-R* to denote the operation that captures the above interaction between RCC8 knowledge and CDC knowledge, in the direction CDC -To- RCC8, by inferring RCC8 knowledge from given CDC knowledge.
The induced RCC8 relation by a general CDC relation (i.e. the disjunction of basic cardinal direction relations), say *Q* (X, Y), is the union of RCC8 relations induced by all basic cardinal direction relations that belong to the general CDC relation; namely:

$$RCC8(X,Y) = \bigcup_{r \in Q} C-to-R(r(X,Y)).$$

Table 2. The interaction table from the basic relations of CDF to RCC8 relations

Basic cardinal direction relation (SB)	*RCC8 relation*
{B}	$DC \vee EC \vee PO \vee TPP \vee NTPP \vee EQ \vee TPP^{-1}$
$\exists R \in BIN : R \in SB \bigcap B \in S$ or *BIN*	$DC \vee EC \vee PO \vee TPP^{-1} \vee NTPP^{-1}$
$B \notin SB$	DC

3 Summary

We have presented the combination of two calculi of spatial relations well-known in Qualitative Spatial Reasoning (QSR): RCC8 calculus and R. Goyal and M. Egenhofer's cardinal direction calculus. In this paper, the interaction between the two kinds of knowledge has been handled, and we have also given two interaction tables. The work in this paper can be applied to the research field of Geographic Information System (GIS), image understanding and computer vision, etc. The fuzzy spatial reasoning with fuzzy knowledge of topological and directional relations can be of very interest in the future.

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