



Gravity Calibration Baseline Between Jeddah and Taif in the Kingdom of Saudi Arabia

Gokhan Arslan, Mehmet Emin Ayhan,
Bandar Saleh Abdulkareem Al-Muslmani, Meshal Ahmad Saad Al-Qulaity,
and Sultan Falah Alzulrah Al-Shahrani

Abstract

Relative gravimeters are calibrated for calibration factors relating observable units to gravity. The calibration correction factors with respect to the instrument calibration factors are estimated by measurements at gravity calibration baseline (GCB) stations. GCB is a gravity network where known gravity differences are compared to those measured by relative gravimeters. The GCB in the Kingdom of Saudi Arabia has endpoints in Jeddah and Taif. The endpoints were observed by FG5 (#111) and A10 (#029) absolute gravimeters in 2013–2014. Twelve new sites in between the endpoints were installed in the late 2014. There are two stations (center/inside and ex-center/outside) at each GCB sites, hence the GCB includes 28 stations. Absolute gravity (AG) at the GCB stations were observed simultaneously by two A10X (#021, #023) absolute gravimeters. The stations were also tied simultaneously by four CG5 relative gravimeters. Besides gravity gradient at each of the stations was measured by two CG5s. The gravity measurements were completed from December 2014 to January 2015. The total uncertainty of A10X is smaller than $6 \mu\text{Gal}$. The uncertainties of gravity gradient and tie measurements are smaller than $2 \mu\text{Gal/m}$ and $5.4 \mu\text{Gal}$, respectively. Comparisons of ties observed by CG5 and A10X result in differences less than $9 \mu\text{Gal}$. The GCB network is adjusted by weighted constraint least squares. Estimated uncertainty of the gravity differences is $1\text{--}2 \mu\text{Gal}$. The gravity differences between the endpoints of the calibration line at Jeddah and Taif is $430.678 \pm 0.002 \text{ mGal}$.

Keywords

A10X • Adjustment • Baseline • Calibration • CG5 • Gravity network • Micro-g LaCoste

1 Introduction

The response of gravimeters is not always linear and may also change with aging of the meters due to relaxation of its mechanic system and environmental effects. So relative gravimeters should be properly calibrated (scaled) before performing field work and regularly calibrated when utilized over long period. The most important calibration item is

the mathematical model that relates observable units to gravity units. The model may consist of nonlinear calibration factors. The calibration correction factors with respect to the instrument calibration factors are obtained by performing calibration line measurements where large known differences in gravity are compared to those measured by the relative gravimeters.

Relative gravimeters available in the market have various measurement and mechanical systems. In order to combine measurements by different type of the gravimeters, the gravimeters are calibrated along a baseline installed in the field or in laboratory. The calibration line outside consists of absolute gravity at the high density points measured by absolute and/or relative gravimeters in order to investigate

G. Arslan (✉) • M.E. Ayhan • B.S.A. Al-Muslmani •
M.A.S. Al-Qulaity • S.F.A. Al-Shahrani
General Commission for Survey, Riyadh, Kingdom of Saudi Arabia
e-mail: a.gokhan@gcs.gov.sa

nonlinearity in the calibration factor covering the worldwide range. It is also required spanning long gravity range, large height and latitude difference between the points, including high density points in fixed gravity and height intervals accessible in short driving distance as possible (Barlow 1967; Torge 1989).

United States Geologic Survey (USGS) installed 15 calibration lines in order to span two gals in the west of USA (Barnes et al. 1969). USGS also installed a gravity calibration line along road between Jeddah and Al-Hada/Taif by using four Lacoste & Romberg relative gravimeters in 1980. The line includes six stations at which gravity and height differences between the end stations are 503.873 mGal and 2007.5 m, respectively. The gravity interval between stations is about 100 mGal (Gettings 1985). However, presently, the calibration stations do not exist in the field. Therefore General Commission for Survey (GCS) installed a new gravity calibration baseline (GCB) which is described in Sect. 2. Pre-calibration of the gravimeters, absolute and relative gravity measurements at the calibration sites, reduction of the data are discussed in Sect. 3. Quality control of gravity measurements and comparison of absolute and relative ties are described in Sect. 4. In Sect. 5, adjustment of the GCB network is explained. Finally conclusions are given in Sect. 6.

2 Gravity Calibration Baseline (GCB): Site Selection and Monuments

Calibration line follows the road between Jeddah and Taif, and consists of fourteen (14) sites: two (2) existent AG sites at the endpoints and twelve (12) new sites (Fig. 1). At each site, an ex-center (outside) station within 20 m from the center (inside) station is located for the sake of the site is destroyed. The new sites are selected to be easily accessible all the times from existing roads in driving time as short as possible between the sites for observations preferable without being affected by local traffic. The sites are located over pre-Cambrian basement rocks (Gettings 1985) in environmentally quiet area providing long-term permanence. A concrete pad sized 80 cm × 80 cm and 50 cm depth, which is suitable for both relative and absolute gravity measurements, is installed at both center and ex-center stations. A marker is installed on the concrete pad (Fig. 2).

Jeddah and Taif absolute gravity sites were installed and observed by FG5 (#111) and A10 (#029) absolute gravimeters between December 2013 and February 2014 (Ayhan et al. 2015). The endpoints of the calibration line are almost at the same latitude so the main reason of the gravity span about 431 mGal is height difference (1,521 m). The

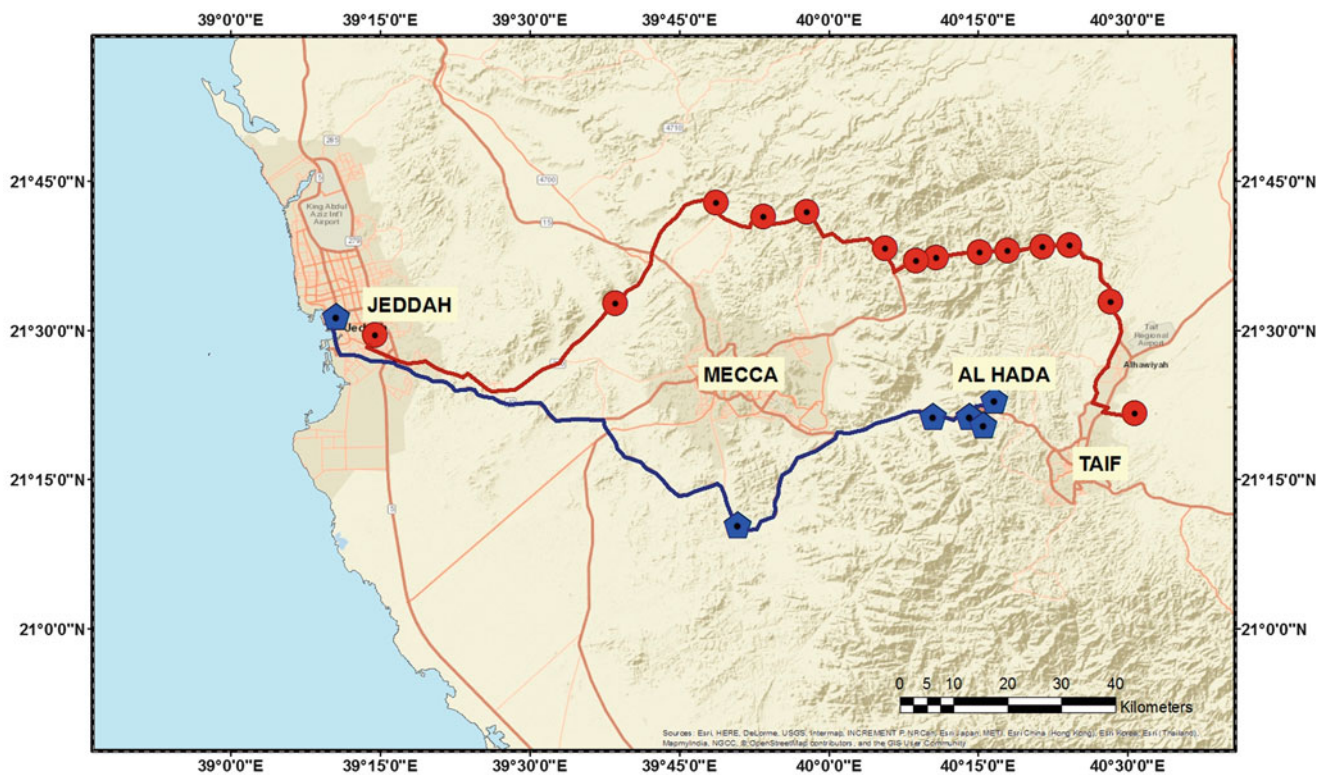


Fig. 1 GCB site locations. Red circles and line are the stations of GCS GCB and the road followed by GCS GCB and blue pentagons are the stations of USGS GCB and the road followed by USGS GCB, respectively

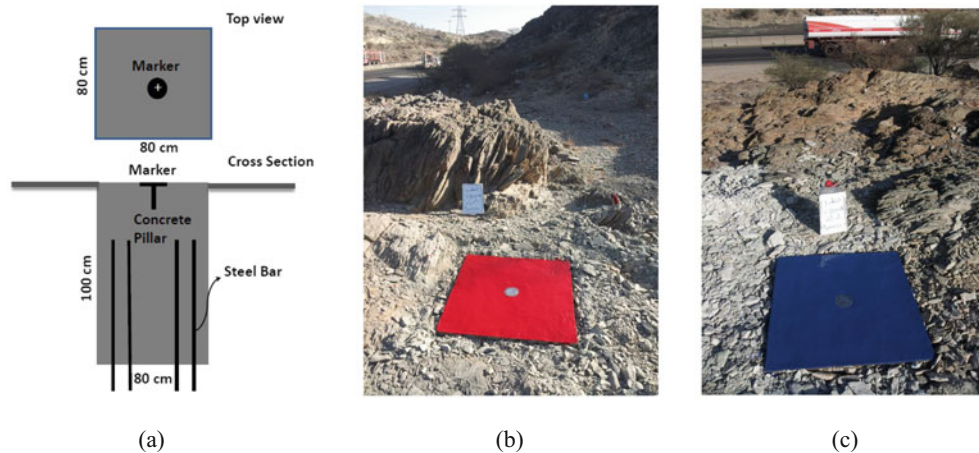


Fig. 2 Monument at calibration stations. (a) Cross section of concrete pillar (b) Pillar in *red* at center station (c) Pillar in *blue* at ex-center station

sites were installed from December 2014 to February 2015. Coordinates of all stations were measured in static mode by GNSS dual frequency receivers. A level line along the Jeddah-Taif road and connections to the calibration stations are also planned. In order to quantify stability of the stations repeat GNSS, leveling and absolute gravity measurements are required in future.

3 Measurements

GCB stations are measured by two A10X absolute gravimeters and four CG5 relative gravimeters, which were tested and calibrated before and after shipping. Micrograv and ‘g9’ software are used for processing and reducing CG5 and A10X measurements, respectively (Gettings 2009).

3.1 Calibration of Gravimeters

A10X portable absolute gravimeters which are improved version of A10 are used in absolute gravity measurements. Klopping (2015) reported that the precision and accuracy of the A10X are better than A10 uncertainties by 25% on ideal surfaces and by as much as 50% on soft surfaces, and repeatability of $\pm 3 \mu\text{Gal}$ for A10X. A10Xs were checked in the Micro-g LaCoste Laboratory in Boulder, Colorado before shipping, which include: laser calibrations, rubidium oscillators, T2 Scope collimation, A10X beam verticality and A10X laser output. The A10X gravimeters have been read on a concrete pad at Micro-g LaCoste head office where many FG5 absolute gravity standard readings have been made. In order to verify gravimeter accuracy stability after shipping, before starting of absolute survey, two A10Xs (#021, #023) were checked in Jeddah absolute gravity station measured by FG5 in the early 2014 (Ayhan et al. 2015).

The repeatability of CG5 is declared as $5 \mu\text{Gal}$ by the manufacturer (<http://www.scintrexltd.com/>). Before shipping, six CG5s (#40052, #40057, #40058, #41210, #41211, #126) were calibrated on the Rocky Mountain calibration line in Denver (Barnes et al. 1969). Before starting the relative gravity survey, calibration of the CG5s was checked between Riyadh and Kharj FG5 absolute gravity sites (Ayhan et al. 2015). After finishing the first calibration loops, all CG5s showed remarkable discrepancies, which suggest the need of the modifications of the used calibration factors. However, calibration factor modification utilized larger gravity difference than that of Riyadh and Kharj (33 mGal). Thus, it has been decided to re-calibrate the gravimeters between Jeddah and Taif points measured by A10X. Calibration measurements were processed using least squares software. Estimated drift and calibration factor for each CG5 are given in Table 1.

3.2 Absolute Measurements

The absolute gravity measurements at 28 GCB stations (14 center and 14 ex-center) were carried out by using Micro-g Lacoste (MGL) provided two A10Xs (#021 and #023) between December 2014 and January 2015.

The absolute gravity measurement is carried out at least ten evenly spaced sets each of which includes about 120 drops. All the sets take place 30 min. A10X repeatability for drops in one set is site dependent ($50 \mu\text{Gal}$ at quite site). Repeatability of sets is smaller than $\pm 3 \mu\text{Gal}$, and total uncertainty of observed absolute gravity is smaller than $\pm 6 \mu\text{Gal}$. Two (2) ten sets of 120 drops are obtained in row by two A10Xs at two different time over both the center and ex-center stations.

The real-time absolute gravity processing was performed in the field by ‘g9’ software package provided by MGL. ‘g9’

Table 1 CG5 Calibration and drift factors estimated between Jeddah and Taif A10X stations

CG5 gravimeters	Calibration factor (mGal/CU)	Drift factor (mGal/day)
CG5 #40052	1.000124 ± 0.000004	0.141 ± 0.004
CG5 #40057	0.999859 ± 0.000002	0.030 ± 0.002
CG5 #40058	0.999844 ± 0.000001	0.022 ± 0.001
CG5 #126	0.999976 ± 0.000001	0.107 ± 0.001
CG5 #41210	0.999840 ± 0.000001	0.013 ± 0.001
CG5 #41211	0.999948 ± 0.000004	-0.007 ± 0.004

CU counter unit

processing incorporates the least-squared fit of the time-distance pairs for each drop with gravity gradient speed of light effects removed. Solid earth tide (Tamura), inelastic response to tides, ocean loading (FES2004), and ambient barometric and temperature corrections all counted for in real time processing. In post-processing, corrections of polar motion and height of instruments were applied (<http://www.microglacoste.com/software.php>). In correction of height of instrument, the absolute gravity values at each station were reduced from A10X reference mass height (82 cm) to reference heights (25 cm and on the marker). Because CG5 reference mass height from the marker was fixed at 25 cm by fixed tripod and aimed to compare absolute and relative gravity differences at the height.

Gravity gradient measurements at each station were carried out by multiple observations with up/down transfer by using two CG5 gravimeters and a tripod at three levels (25 cm, 75 cm, 125 cm) to parabolic (second order) estimation. The difference of gradient readings from mean at each level is obtained smaller than $2 \mu\text{Gal/m}$.

3.3 Relative Measurements

The relative gravity measurements between the GCB stations (ties) were carried out by using four CG5 relative gravimeters between January 2015 and February 2015. Each tie measurement between center—center or center—ex-center stations includes minimum three loop sequences (the difference sequence).

The gravimeters were set up one after another over the marker at one station and loop measurements were completed in 12 h in 1 day. In order to provide relaxation of the gravimeter, minimum 15 min time series and minimum ten acceptable readings at each station for one instrument were obtained. One acceptable reading period is at least 60 s. Standard deviation of acceptable readings from the mean is smaller than $\pm 2 \mu\text{Gal}$. In order to correct and reduce gravity readings, ambient temperature, air pressure and the height of instrument (from bottom of the instrument to the top of the marker) are measured and stored. The instrument was oriented in the same way to the magnetic north by

digital compass to avoid magnetic effect and the operator was at a distance of about 10 m from the gravimeters. A portable shutter was used to protect the instrument from wind, sunlight.

Pre-processing of CG5 measurements was carried by using both software embedded in CG5 and Micrograv (Gettings 2009). Reductions and corrections applied in pre-processing are calibration, ambient temperature, continuous tilt correction, auto rejection and seismic noise filter, earth tide correction, staircase drift correction, readout rejection and reference mass height correction. After pre-processing, we found tie uncertainty smaller than $\pm 5.4 \mu\text{Gal}$.

4 Comparison of Absolute and Relative Gravity Measurements

Absolute gravities observed at the same station by the two A10Xs are compared, and the differences are shown in Fig. 3. The differences are expected to be smaller than $8.5 \mu\text{Gal}$ which is satisfied almost at each station. The differences are mostly within $5 \mu\text{Gal}$, and scattered around the mean of $2.0 \pm 2.2 \mu\text{Gal}$, which may be caused by the calibration difference of the two A10Xs.

Each of the ties between centre-centre and centre-excentre is observed independently by using four CG5s. The mean of the four measurements are computed for each ties. Then the difference of a tie by each CG5 from the mean is calculated and shown in Fig. 4. The differences are almost within $5 \mu\text{Gal}$. We found the mean of $0.1 \pm 4.4 \mu\text{Gal}$, $0.4 \pm 5.8 \mu\text{Gal}$ and $1.6 \pm 3.0 \mu\text{Gal}$ for CG5 #40057, #41211 and CG5 #41210, respectively. However CG5s #40052, #40058 and #126 used replacing each other due to instability reveal the mean $2.7 \pm 7.0 \mu\text{Gal}$. The RMS of the differences for all CG5s is $\pm 5.4 \mu\text{Gal}$, which may indicate a measure of uncertainty for CG5. The large differences are likely outliers which would be detected in the adjustment.

Absolute gravity measurement at a station is an individual observation. In order to check absolute gravity measurement at one station, another absolute gravimeter can be used at the same point or pair of points at which absolute gravity observed can be tied by relative gravity measurements. For

Fig. 3 Differences of absolute gravity by A10X #021 and A10X #023 at each station

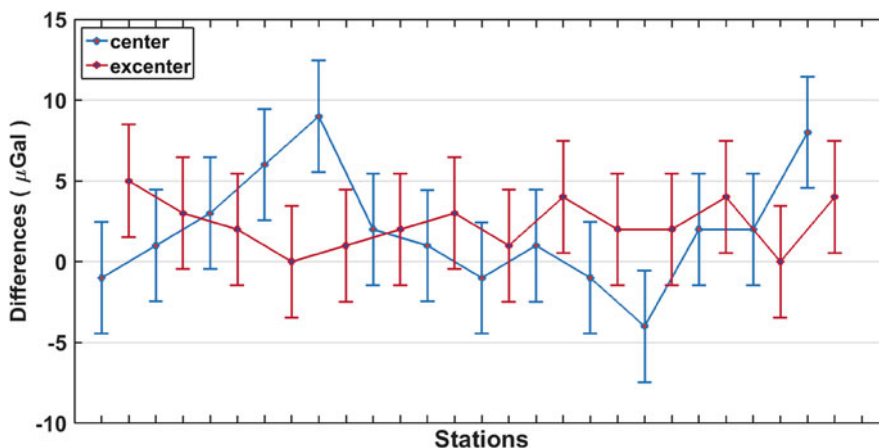


Fig. 4 Differences of four CG5s tie from the mean for each tie

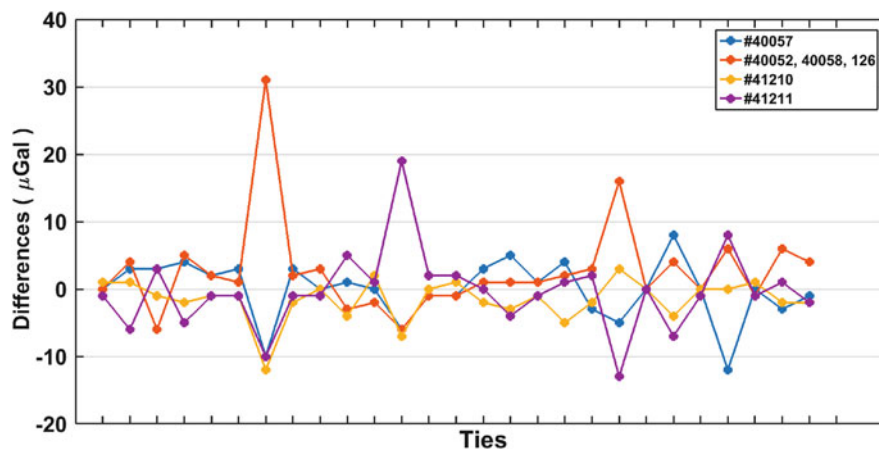
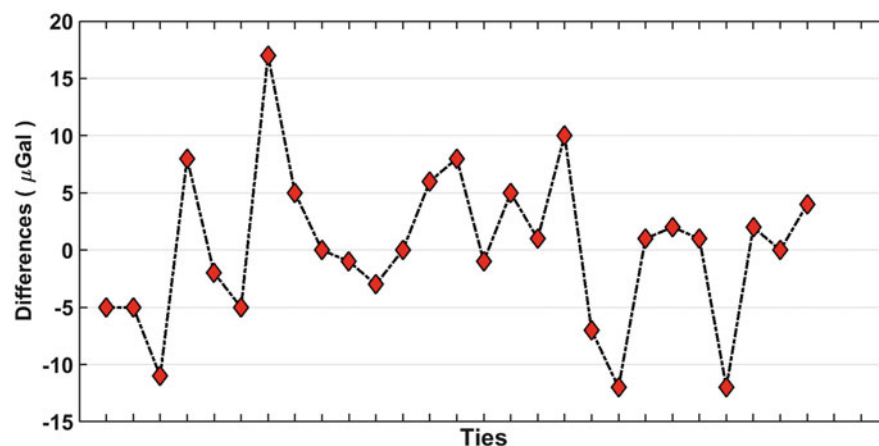


Fig. 5 The differences between the mean ties by four CG5s and tie by two A10Xs



the latter case the gravity differences between two points by absolute and relative gravity measurements are compared. This comparison provides quality check of the two types of measurements and coincidence of the gravimeter’s calibration. For this aim we compared the mean ties obtained by both averaging the four CG5 tie measurements and the two A10X tie measurements. The differences shown in Fig. 5 are scattered around zero in the range of $\pm 10 \mu\text{Gal}$, with RMS of $\pm 6.6 \mu\text{Gal}$.

Absolute gravity at the endpoints of the calibration line (AG0050, AG0061) were observed by MGL FG5 (#111) in the early 2014 (Ayhan et al. 2015). Furthermore the endpoints are tied directly by using three CG5s. Cumulative sums of centre-centre gravity measurements by four CG5 and their mean are also calculated. Gravity differences (ties) between the endpoints measured by CG5, A10X and FG5 are listed in Table 2 for comparisons. Loop closures are calculated by subtracting the tie by FG5 from the ties by

Table 2 Tie between the endpoints (AG0050–AG0061) and loop closures

Gravimeters	CG5 ^a	CG5 ^b	A10X(#021) ^c	A10X(#023) ^c	FG5(# 111) ^c
Tie	430.671	430.6875	430.676	430.671	430.681
Loop closure	−0.010	0.0065	−0.005	−0.010	0.000

^aMean of cumulative sums by four CG5

^bMean of tie measurements by three CG5s

^cTie obtained by subtracting absolute gravity observed by A10X/FG5 at the endpoints Loop closures are calculated by subtracting tie by FG5(#111) from the ties by CG5^a, CG5^b, A10X(#021) & (# 023). (in mGal)

CG5s and A10Xs and are given in the last row of the Table. Loop closures for A10Xs are within 10 μ Gal which is in the uncertainty level of A10X and FG5 when we consider their total uncertainties (Ayhan et al. 2015). Loop closure of 6.5 μ Gal for CG5 tie is at the level of CG5 uncertainty whereas cumulative CG5 tie gives −10 μ Gal loop closure which is assumed to be caused by cumulative uncertainties along the calibration stations.

5 Adjustment

5.1 The Model

The observation equations of tie (gravity difference) and absolute gravity measurements are given below:

$$\Delta l_{ij} + v_{ij} = g_j - g_i + \Delta F(\Delta z_{ij}) + \Delta D(\Delta t_{ij}) \quad (1)$$

$$l_i + v_i = g_i \quad (2)$$

where Δt_{ij} is time difference of measurements, Δz_{ij} is the gravimeter reading difference in counter unit (CU), Δl_{ij} is the corrected gravity difference between station i and j , v_{ij} is residual, g_i is unknown gravity at station i , ΔF is the polynomial calibration correction function and ΔD is the polynomial drift function of gravimeter, l_i is absolute gravity measurement at point i and v_i is its residual. Observation equations in matrix form are given below:

$$\mathbf{L}^b + \mathbf{V} = \mathbf{A}\mathbf{X} \quad (3)$$

$$\mathbf{L}_g + \mathbf{V}_g = \mathbf{A}_g\mathbf{X} \quad (4)$$

where; \mathbf{L} is vector of measurements, \mathbf{V} is vector of residuals, \mathbf{A} is design matrix, \mathbf{X} is vector of unknowns (gravity values g_i and gravimeter parameters (scale factor and drift constants)). Observations are assumed uncorrelated and weight of the observations is defined by $p_{ij} = 1/\sigma_{ij}^2$ and $p_i = 1/\sigma_i^2$ for gravity difference and absolute gravity, respectively. Matrix \mathbf{A} has rank defect one in gravity networks so at least one fix point (constraint) is required. Absolute gravity

observations in Eq. (4) are introduced as constraints and weighted constraint least squares solution of the network is obtained. After obtaining estimates of unknowns, residuals and their uncertainties, the global test and Pope's outlier test are applied in order to clean data set and verify the mathematical model (Hwang et al. 2002).

5.2 GCB Network Adjustment

GCB Network consists of 28 station, 112 tie measurements by CG5 and 56 absolute gravity measurements by MGL A10X. Network configuration is shown in Fig. 6. As it is seen in the figure we include tie measurements between the endpoints of the calibration line (AG050 and AG061) which are observed by three CG5s in 2015. So this network provides a long loop to control internally both the CG5 and A10X measurements.

The network adjustment was achieved by using in house software in matlab based on the theory explained in the previous section. We excluded 17 tie measurements by CG5 and six absolute gravity measurements as outliers. The gravity differences and their uncertainties are calculated based on the estimated absolute gravity at the stations between center-center and center-excenter stations, and given in Table 3.

6 Discussions

The standard deviation of CG5 relative measurements and total uncertainty of MGL A10X absolute gravity measurements are found smaller than $\pm 5.4 \mu$ Gal and about $\pm 6 \mu$ Gal, respectively. Estimated uncertainty for ties between the GCB stations is $\pm 1-2 \mu$ Gal which is remarkable improved.

The sites are not located at fix driving distance interval. Gravity interval between the sites is 60 mGal for four ties and between 3 and 40 mGal for the rest (Fig. 7a). Height differences between sites (center-center) vary from about 50 to 280 m (Fig. 7b). It was planned initially fix height difference about 100 m between the sites so that about 30 mGal gravity interval. However, this requirement was not be able to achieve in site selection. The gravity differences between center and ex-center stations vary between 0.043

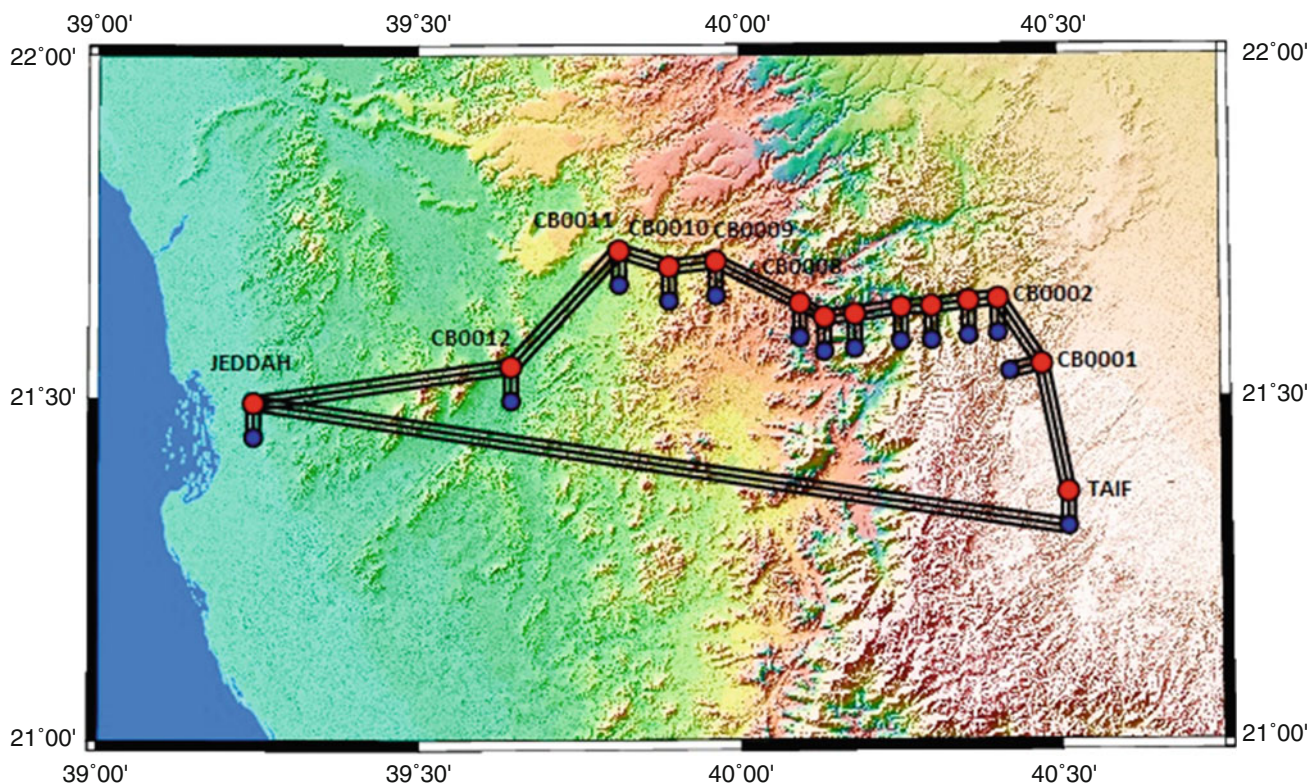


Fig. 6 GCB Network. Red and blue solid cycles are center and ex-center stations respectively. Black line is tie measurement

Table 3 The estimated ties (gravity differences) between center-center and center-excenter stations

Center-center	Tie estimates (mGal)(mGal)	Center-excenter	Tie estimates (mGal)(mGal)
AG050–CB120	63.279 ± 0.002	AG050–AG051	0.567 ± 0.002
CB120–CB110	21.813 ± 0.001	CB120–CB121	0.216 ± 0.001
CB110–CB100	19.366 ± 0.002	CB110–CB111	0.058 ± 0.001
CB100–CB090	14.599 ± 0.002	CB100–CB101	0.259 ± 0.003
CB090–CB080	58.531 ± 0.001	CB090–CB091	0.343 ± 0.001
CB080–CB070	18.143 ± 0.001	CB080–CB081	−0.043 ± 0.001
CB070–CB060	3.862 ± 0.001	CB070–CB071	0.151 ± 0.002
CB060–CB050	26.991 ± 0.001	CB060–CB061	0.024 ± 0.001
CB050–CB040	14.079 ± 0.001	CB050–CB051	0.027 ± 0.001
CB040–CB030	32.324 ± 0.001	CB040–CB041	0.086 ± 0.001
CB030–CB020	57.272 ± 0.001	CB030–CB031	−0.295 ± 0.001
CB020–CB010	62.030 ± 0.001	CB020–CB021	0.033 ± 0.001
CB010–AG060	38.498 ± 0.001	CB010–CB011	−0.179 ± 0.002
AG050–AG061	430.678 ± 0.002	AG060–AG061	−0.109 ± 0.001

and 0.567 mGal. Considering height and gravity difference between the endpoints, rate of gravity variation along the baseline is 0.284 mGal/m, which is close to free air gradient.

GCB stations were installed on the right side of the road in the direction from Jeddah to Taif. All the stations are located closed to the road which is separated in some segments by a barrier so that reaching some stations may be difficult.

7 Conclusions

The gravity differences between absolute measurements by using two A10X are found mostly smaller than 5 μGal. In general; the differences between observed gravity differences by each CG5 and the average of four gravity differences for

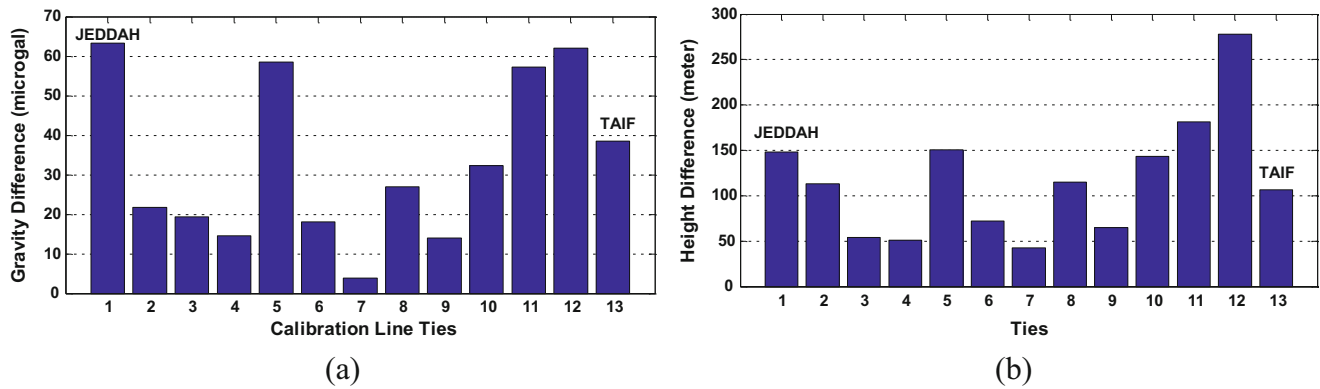


Fig. 7 (a) Gravity differences between sites. (b) Height differences between sites

each tie are within 5 μGal . The relative and absolute gravity differences between center-center stations are coincident within 10 μGal with RMS of $\pm 6.6 \mu\text{Gal}$. Comparisons of the gravity differences between the endpoints of the baseline reveal that CG5 and A10X measurements are coincident with FG5 measurement within 10 μGal .

The estimated gravity differences vary between 3.861 and 63.280 mGal for the center-center stations and -0.295 and 0.567 mGal for the center-excenter stations. The GCB between Jeddah and Taif is appropriate for calibration of relative and absolute gravimeters, and satisfies a scale standard in gravity surveys by public institutes and private companies.

Acknowledgments We appreciate support of GCS for the project. Thanks to Jeff Kanney from MGL, Boulder, Colorado who did seriously A10X measurements. We also thank to field engineers and operators from Magee Geophysical Services, Reno, USA, for their precise field works of CG5 measurements.

References

- Ayhan ME et al (2015) Absolute gravity measurements by using FG5 and A10 absolute gravimeters in the Kingdom of Saudi Arabia. *Arab J Geosci* 8:6199–6209
- Barnes DF, Oliver HW, Robbins SL (1969) Standardization of gravimeter calibrations in the geological survey. *EOS* 50(10):526–527
- Barlow BC (1967) Gravity meter calibration in Australia. Bureau of Mineral Resources, Geol. and Geophy., Rept. No.: 122. pp 48
- Gettings ME (1985) Gravity base ties and gravimeter calibration in western Saudi Arabia. Open-File Report 85-26, Department of Interior, U.S. Geological Survey, USA
- Gettings P (2009) High precision gravity measurements. Department of Geology & Geophysics, University of Utah, Salt Lake City
- Hwang C, Wang CG, Lee LH (2002) Adjustment of relative gravity measurements using weighted and datum-free constraints. *Comput Geosci* 28:1005–1015
- Klopping F (2015) A10X dropper development and applications. MGL in-house White Paper
- Torge W (1989) *Gravimetry*. de Gruyter. Berlin. 465 pp