A Feasibility Study on the Unification of the Italian Height Systems Using GNSS-Leveling Data and Global Satellite Gravity Models

R. Barzaghi, D. Carrion, M. Reguzzoni, and G. Venuti

Abstract

Regional height systems are usually referred to the mean sea level at a reference tide gauge. As the mean sea level gravity potential changes from place to place, regional systems refer to different equipotential surfaces and the establishment of a unified datum requires the determination of one bias per region. This is what is referred to as the height datum problem. The possibility to solve this problem by exploiting the nowadays available satellite gravity mission data, the high resolution global gravity potential models, GNSS heights, as well as leveling and gravity data has been explored. A solution strategy and a first error budget have been presented in Gatti et al. (J Geod 87(1):15–22, 2012), showing that an accuracy of about 5 cm can be globally achieved in the bias computation. In the present work, this strategy, with refinements in the error modeling, is applied to the Italian case, where different height systems are used for the mainland and Sicily and Sardinia islands.

Keywords

Height datum problem • Global gravity models • GNSS-leveling • GOCE • GRACE

1 Introduction

The availability of satellite gravity data together with accurate high resolution global gravity models have reopened the possibility to estimate regional height system biases with respect to a common global reference. Different solutions have been explored, for instance by Kotsakis et al. [\(2012\)](#page-6-0) and Rummel [\(2012\)](#page-7-0) to solve the problem either on a global or on a local scale. We started from a proposal in Gatti et al. [\(2012\)](#page-6-1) and performed a first experiment in Italy, where three different height systems are present. The unknown bias of each region reflects in the normal heights of the region itself (Rummel and Teunissen [1988\)](#page-7-1). Height anomalies obtained

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by differencing GNSS heights and normal heights contain the bias as well. The difference between these biased height anomalies and the corresponding values derived from an unbiased anomalous potential can be therefore modeled as the unknown bias plus errors. When these errors are kept low by an ad hoc combination of satellite-only and high resolution global gravity models, a sufficiently accurate (i.e. with a standard deviation below 5 cm) estimate of the unknowns can be obtained. Although not yet available, Italian normal heights of the main leveling network will be computed in the near future also thanks to a cooperation between the Istituto Geografico Militare (IGM), which is the official institution for creation and maintenance of the Italian geodetic reference network, and the Politecnico di Milano. In particular, normal heights of the GNSS national network will be derived with the corresponding height anomalies, thus allowing for the unification of the Italian height systems. In this work we report the results of a first experiment in this direction, mainly devoted to establish the feasibility of the procedure in terms of accuracy of the solution, when all the available information about the observation errors

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C. Rizos, P. Willis (eds.), *IAG 150 Years*, International Association of Geodesy Symposia 143, DOI 10.1007/1345_2015_35

is exploited. Moreover, we obtained a preliminary estimate by substituting the needed, but not yet computed normal heights, with the currently official heights, derived from the adjustment of leveling data, without applying any correction accounting for gravity. Well aware that those heights can differ from the required normal ones up to some decimeters, we took into account this systematic error with an increased a priori standard deviation of the leveling observations. We will repeat the same evaluation once the proper observables will be computed and released, but we still consider useful the results of this first attempt.

2 The Height Datum Problem and the Adopted Solution

Global satellite gravity potential models can be profitably used in the solution of the height datum problem. The problem is that of determining the biases between the potential of the equipotential surfaces chosen as regional height references $W = W_0'$ and that of the geoid $W = W_0$:

$$
\delta W^j = W_0 - W_0^j = U_0 - W_0^j \tag{1}
$$

where W_0^j is the actual potential of the mean sea surface at the action of the action in W_0 is a surface the reference tide gauge P_0^f for the region j, W_0 is equal to the normal gravity potential U_0 on the reference ellipsoid to the normal gravity potential U_0 on the reference ellipsoid. This latter potential is in turn defined by the values of the angular velocity of the Earth ω , its mass M, the ellipsoid semi-major axis a and eccentricity e , which are all well known quantities. The regional bias δW^j in potential reflects in normal heights derived from spirit leveling and gravity observations (cf. Appendix). Height anomalies derived by comparing the nowadays available unbiased ellipsoidal heights h (here unbiased stands for not affected by the potential bias), with the biased normal heights h^{*j} of Eq. [\(12\)](#page-6-2) will be biased as well: be biased as well:

$$
\tilde{\xi}^j = h - \tilde{h}^{*j} = \xi - \frac{\delta W^j}{\overline{\gamma}_{P_0^j}}.
$$
 (2)

By exploiting Bruns's formula, for a generic point P of the region j , and recalling Eq. (12) , one can write:

$$
\tilde{\xi}^j = \frac{T}{\overline{\gamma}} - h_0^{*j} \tag{3}
$$

where $\overline{\gamma}$ is the average value of the normal gravity between the ellipsoid and the telluroid, along the normal to the ellipsoid through P . Provided that an unbiased, sufficiently accurate value of the anomalous potential T is available, Eq. [\(3\)](#page-1-0) can be used to estimate the regional biases h_0^* . Note that a large uncertainty, say 10 m, in the knowledge of the normal height has a negligible effect on the evaluation of

the average values of normal gravity \bar{y} . Gatti et al. [\(2012\)](#page-6-1) proved that a proper combination of the satellite-only global model GOCO (Pail et al. [2010\)](#page-6-3) and the high resolution global model EGM2008 (Pavlis et al. [2012\)](#page-7-2) can give such an unbiased, sufficiently accurate anomalous potential, producing an overall accuracy of 5 cm on the estimated biases. More specifically, one can use satellite-only models up to a degree L, and high resolution models just for the highest degree from $L+1$ to their maximum degree H. While satellite-only models are not biased, high resolution models, computed also from gravity anomalies derived from leveling and gravity measurements, are affected by the regional height biases. Nevertheless, the indirect (i.e. through the gravity anomaly Δg) effects of those biases in the high degree coefficients can be disregarded (Gatti et al. 2012). The boundary degree L in the combination of the low resolution satellite-only model and the high resolution one has to be tuned case-by-case, mainly depending on the size of the involved regions.

3 The Least Squares Estimation: Observation Equations and Observation Error Models

The observation equation in Eq. (3) , for a point P belonging to the region j , can be rewritten in the following way:

$$
\tilde{\xi}^{j} - \frac{T^{L} + T^{H}}{\overline{\gamma}} = -h_{0}^{*j} + \nu
$$
 (4)

where T^L is the prediction of the anomalous potential at the point P derived from the satellite gravity model up to degree L, T^H is the prediction derived from EGM2008 from degree $L + 1$ up to degree H and v is the observation noise. For N_i points in J regions a linear system of $N_i \times J$ equations and J unknowns can be solved by a least squares adjustment, once the observation error covariance matrix C_v is defined. This matrix has to account for the errors in the ellipsoidal heights derived from GNSS through the covariance matrix C_h , the errors in the normal heights derived from leveling and gravity measurements through $C_{\tilde{h}^*}$, the commission errors of
the satellite only gravity model up to the degree I, through the satellite-only gravity model up to the degree L through $C_{\mathcal{T}^L}$ and those in the high resolution model from degree $L+1$ up to degree H through C_{T^H} . It results:

$$
C_{\nu} = C_h + C_{\tilde{h}^*} + C_{T^L} + C_{T^H}.
$$
 (5)

4 The Italian Case

Italy has three different height systems, one for the peninsula and two for Sicily and Sardinia, the tide gauges being in Genova, Catania and Cagliari respectively, for a total of three unknown biases. To estimate those biases, a set of 1,068 points with known GNSS ellipsoidal heights and leveling derived heights was considered. Among them, 43 points are in Sicily, 48 in Sardinia and the remaining 977 in the mainland. The heights derived from leveling measurements were obtained by a least squares adjustment of the observations without any correction accounting for gravity effects (Betti et al. [2013\)](#page-6-4). GNSS heights are referred to the ETRF89 reference frame, epoch 2006. All the data are made available to us by IGM. The evaluation of normal and orthometric corrections is in progress, also thanks to a cooperation between IGM and the Politecnico di Milano. At the available leveling points, the value of the gravity potential was computed starting from the GOCO-03S satellite-only gravity global model and EGM2008 spherical harmonic coefficients. The GOCO-03S model basically combines the ITG-Bonn GRACE solution with the time-wise GOCE one (release R3, that is the second to the last solution based on 1 year and a half GOCE data) (Pail et al. [2011;](#page-7-3) Mayer-Gürr [2006\)](#page-6-5); the coefficients were downloaded from the website of the International Center for Global Earth Models (ICGEM). Moreover, we could use the GOCO-03S order-wise block diagonal error covariance matrix, which practically brings the same information as the full error covariance matrix (Gerlach and Fecher [2012\)](#page-6-6). As for EGM2008 spherical harmonic coefficients, the error coefficient variances and a global grid of local geoid error variances are available. Consistently with GNSS data, the coefficients of the two global models are tide-free. As for leveling data, they are referred to the mean sea level at the three tide gauges of Genova, Catania and Cagliari.

5 Reference Frame Transformations

GNSS coordinates, GOCO-03S and EGM2008 data are referred to different frames with different epochs. Therefore, before combining those observations, transformations to a common frame and epoch must be performed. In order to evaluate the impact of those transformations in the GNSS observations, we started by updating the given coordinates to the most recent frame of the GOCE model. The Italian GNSS data are given in ETRF89, epoch 2006, while GOCE data are in ITRF2008, with a not specified epoch between 2010 and 2011. Both transformations from ETRF89-2006 to ITRF2008-2010 and from ETRF89-2006 to ITRF2008-2011 were performed in three steps:

- from ETRF89-2006 to ITRF89-2006 using the EUREF online tool 'ETRS89/ITRS TRANSFORMATION' [\(http://www.epncb.oma.be/_productsservices/coord_](http://www.epncb.oma.be/_productsservices/coord_trans/) [trans/\)](http://www.epncb.oma.be/_productsservices/coord_trans/);
- from ITRF89-2006 to ITRF2008-2006 using the transformations and parameters provided by the International

Table 1 Statistics of the differences between biased height anomalies computed with the ETRF89 (epoch 2006) GNSS data and with the ITRF2008 epoch 2010 (first row) and epoch 2011 (second row)

Epoch	Mean (cm)	Std (cm)	Min (cm)	Max (cm)
2010	-0.85	0.44	-2.00	0.60
2011	-0.86	0.45	-2.00	0.60

Earth Rotation and Reference Systems Service (IERS) [\(http://itrf.ensg.ign.fr/doc_ITRF/Transfo-ITRF2008_](http://itrf.ensg.ign.fr/doc_ITRF/Transfo-ITRF2008_ITRFs.txt) [ITRFs.txt\)](http://itrf.ensg.ign.fr/doc_ITRF/Transfo-ITRF2008_ITRFs.txt);

• from ITRF2008-2006 to ITRF2008 epochs 2010 and 2011, using the mean values of the Italian stations velocities again provided by IERS [\(http://itrf.ensg.ign.fr/ITRF_](http://itrf.ensg.ign.fr/ITRF_solutions/2008/doc/ITRF2008_GNSS.SSC.txt) [solutions/2008/doc/ITRF2008_GNSS.SSC.txt\)](http://itrf.ensg.ign.fr/ITRF_solutions/2008/doc/ITRF2008_GNSS.SSC.txt). In particular, the stations of Medicina, Genova, Torino I, Cagliari, Matera, Padova and Perugia were taken into account.

It resulted a change in the planimetric coordinates of about 50 cm and about 1 cm in height. In order to evaluate the impact of such differences in our observations we compared the biased height anomalies $\zeta = h - h^*$ computed
from GNSS heights referred to the original reference frame from GNSS heights referred to the original reference frame ETRF89 at epoch 2006 with the anomalies computed from GNSS heights referred to ITRF2008 at epochs 2010 and 2011. The statistics of the differences are reported in Table [1.](#page-2-0) The differences between the two considered epochs are negligible and we expect that this is true also when considering the larger time span of GRACE data. Analogous transformations for the EGM2008 data cannot be applied because the time reference of its gravity database is not available.

6 Error Budget and the Choice of the Boundary Degree L

In order to set a proper boundary degree in the combination of GOCO-03S and EGM2008 for the solution of the height datum problem, the biases accuracy was evaluated from the available error models. We assumed the set of differences between GNSS and leveling heights to have an uncorrelated noise with a standard deviation $\sigma_{\tilde{\zeta}} = 1$ cm, that is

-

$$
C_{\tilde{\zeta}} = C_h + C_{\tilde{h}^*} = \sigma_{\tilde{\zeta}}^2 \times I \tag{6}
$$

where I is the identity matrix. The error covariance matrix C_{T^L} of the set of potential values $\{T^L\}$ predicted in the GNSS-leveling points from GOCO-03S was obtained by propagation from the given order-wise block diagonal error covariance matrix. The covariance matrix C_{T^H} of the set of potential values $\{T^H\}$, computed in the same points from EGM2008, was obtained by propagation from the coefficient error variances properly rescaled accordingly to the geographical map of local geoid errors (Gilardoni et al.

Fig. 1 Correlation matrix of the total error in the case of $L = 250$. The first 977 observations are mainland points, then the 43 Sicily points and finally the 48 Sardinia points

Fig. 2 GOCO-03S and EGM2008 mean error standard deviation versus the boundary degree L and the corresponding accuracy of the mainland bias

Fig. 3 GOCO-03S and EGM2008 mean error standard deviation versus the boundary degree L and the corresponding accuracy of the Sicily bias

Fig. 4 GOCO-03S and EGM2008 mean error standard deviation versus the boundary degree L and the corresponding accuracy of the Sardinia bias

Fig. 5 Mainland, Sicily and Sardinia bias accuracies versus the boundary degree L

[2013\)](#page-6-7). The resulting error covariance matrix of Eq. [\(5\)](#page-1-1) is shown in Fig. [1,](#page-3-0) for $L = 250$. In Figs. [2,](#page-3-1) [3](#page-3-2) and [4](#page-4-0) the GOCO-03S and EGM2008 mean error standard deviations are plotted as functions of L together with the resulting bias standard deviations, for mainland, Sicily and Sardinia respectively. As it can be better appreciated in Fig. [5,](#page-4-1) where the standard deviations of the three biases are compared, different values of L do not reflect in significant variations of the parameter accuracies. Therefore, we preferred to use GOCO-03S to its highest degree $L = 250$, in order to

Table 2 Estimated biases and corresponding standard deviations for different GNSS-leveling observation error model

$\sigma_{\tilde{\zeta}} = 1 \,\mathrm{cm}$	Mainland	Sicily	Sardinia
\hat{h}_0^{*j} [cm]	82.73	73.25	95.54
$\hat{\sigma}_{h_{\alpha}^{*j}}$ [cm]	0.59	2.75	4.92
$\hat{\sigma}_0^2 = 12.78$, H_0 rejected			
$\sigma_{\tilde{r}} = 5 \,\mathrm{cm}$	Mainland	Sicily	Sardinia
\hat{h}_0^{*j} [cm]	78.98	68.72	97.11
$\hat{\sigma}_{h_0^{*j}}$ [cm]	0.51	2.62	3.17
$\hat{\sigma}_0^2 = 3.32$, H_0 rejected			
$\sigma_{\tilde{\ell}} = 10 \,\mathrm{cm}$	Mainland	Sicily	Sardinia
\hat{h}_0^{*j} [cm]	77.50	67.78	97.84
$\hat{\sigma}_{h_0^{*j}}$ [cm]	0.51	2.56	2.77
$\hat{\sigma}_0^2 = 1.37$, H_0 rejected			
$\sigma_{\tilde{\ell}} = 11 \,\mathrm{cm}$	Mainland	Sicily	Sardinia
\hat{h}_0^{*j} [cm]	77.34	67.59	97.88
$\hat{\sigma}_{h_{\alpha}}^{*}$ [cm]	0.52	2.57	2.74
$\hat{\sigma}_0^2 = 1.20$, H_0 rejected			
$\sigma_{\tilde{r}} = 12 \,\mathrm{cm}$	Mainland	Sicily	Sardinia
\hat{h}_0^{*j} [cm]	77.22	67.40	97.90
$\hat{\sigma}_{h_0^{*j}}$ [cm]	0.52	2.57	2.72
$\hat{\sigma}_0^2 = 1.07$, H_0 accepted			

reduce the height datum secondary effects in EGM2008 (cf. Sect. [2\)](#page-1-2).

7 Bias Estimation and Analysis of the Least Squares Residuals

The Italian biases of mainland, Sicily and Sardinia were finally computed via least squares. A first model assessment was then performed by analyzing the adjustment residuals. More specifically, a χ^2 test was done at a significance level of 5%, to verify the null hypothesis $H_0: \sigma_0^2 = 1$ (Koch [1987\)](#page-6-8). The test was mainly devoted to the evaluation of the observation stochastic model. In particular, we are confident on the global gravity model covariances, but we know that uncorrected leveling derived heights, completely disregarding the effects of gravity, introduce systematic errors. These errors can reach some decimeters in zones like the Alps or the Calabrian Arc, where gravity anomalies undergo to high variations. Although a deeper analysis is required, these values partly justifies the following result. We performed different adjustments of the same observations, each time adopting a different a priori height anomaly accuracy $\sigma_{\tilde{y}}$,
lacking for that which for which the y^2 test weights the will looking for that value for which the χ^2 test verifies the null
by nothesis. The result of such an approach is that an accuracy hypothesis. The result of such an approach is that an accuracy of 12 cm can be accepted. A summary of the trials is reported in Table [2.](#page-4-2) The residuals of the least squares adjustment corresponding to the last trial are reported in Fig. [6.](#page-5-0)

Fig. 6 Estimated residuals in meters of the least squares adjustment, $\sigma_{\tilde{\zeta}} = 12 \text{ cm}$

8 Discussion

A first experiment in the solution of the Italian height datum problem was performed. The main goal was to understand the feasibility of the adopted solution strategy in terms of expected accuracies of the estimates for the best possible error model currently available. The difference between biased and unbiased height anomalies in one height system region can be modeled as the sum of the unknown bias plus a random error, containing from one side the GNSS and leveling/gravity derived height accuracies, from the other side the two global model commission errors. By assuming an accuracy level of 1 cm in the GNSS/leveling data and by using the best error models nowadays available to evaluate the global model commission error covariances, it resulted an accuracy of the estimated biases below 1:5 cm, the highest error being in Sardinia for its smallest extension. This proves the feasibility of such an approach. In other words, the combination of

GOCO-03S up to its highest degree $L = 250$ and EGM2008 for the remaining higher degrees proves to have a sufficient resolution to describe the unbiased height anomalies (the omission error is sufficiently low); moreover, the effects of the biases entering EGM2008 through the free air gravity anomalies depending on biased heights can be disregarded. This approach clearly depends on the availability of accurate GNSS and normal heights. The actual Italian height values, in fact, leave high systematic errors in the observation residuals. The poor result obtained in this first estimate could be due also to local errors of the global models especially in areas like the Alps or the Calabrian Arc. In any event a partial confirmation of the proposed approach is given by the difference of biases between Sicily and mainland. This has been estimated by us to have a value of 9:81 cm, with a standard deviation of 2:57 cm. A similar value, namely 14:1 cm (personal communication), was independently derived by IGM, by means of a trigonometric connection across the Messina Strait.

Appendix: Determination of Normal Heights from Spirit Leveling and Gravity Observations

We shortly review here two possible ways for the determination of regional normal heights from gravity and leveling observations. In one case we least squares adjust potential differences of height benchmarks derived from observed gravity g and leveling increments δL [see Heiskanen and Moritz [1967,](#page-6-9) Eqs. [\(4](#page-1-3)[–3\)](#page-1-0), page 161]:

$$
\Delta W_{AB} = -\sum_{A}^{B} g \delta L \tag{7}
$$

by fixing the potential of the reference regional tide gauge to U_0 . This introduces a bias in the adjusted potential values equal to the difference between the unknown actual potential of the reference point and U_0 :

$$
\tilde{W}^j = W - \delta W^j. \tag{8}
$$

From these estimated potentials, biased normal heights will be derived as follows:

$$
\tilde{h}^{*j} = \frac{\tilde{C}}{\overline{\gamma}} = \frac{W_0 - \tilde{W}^j}{\overline{\gamma}} = \frac{U_0 - W}{\overline{\gamma}} + \frac{\delta W^j}{\overline{\gamma}} = h^* + \frac{\delta W^j}{\overline{\gamma}}
$$
(9)

where \tilde{C} are the biased geopotential numbers, and $\bar{\gamma}$ is average normal gravity between the reference ellipsoid and the telluroid along the normal to the ellipsoid through the considered benchmark.

In the other case we least squares adjust leveling increments properly corrected (Betti et al. [2013\)](#page-6-4)

$$
\Delta L_{AB} + NC = h_B^* - h_A^*,\tag{10}
$$

deriving benchmark normal heights with respect to the biased normal height of the reference point P_0^f . This height, which
we set equal to zero, by assuming that the point is on the we set equal to zero, by assuming that the point is on the geoid, is actually proportional to the potential bias:

$$
h_0^{*j} = \frac{U_0 - W_0^j}{\overline{\gamma}_{P_0^j}} = \frac{U_0 - W_0}{\overline{\gamma}_{P_0^j}} + \frac{\delta W_0^j}{\overline{\gamma}_{P_0^j}} = \frac{\delta W_0^j}{\overline{\gamma}_{P_0^j}}.
$$
 (11)

It follows that the benchmark normal heights are all biased by the normal height of the reference tide gauge:

$$
\tilde{h}^{*j} = h^* + \frac{\delta W_0^j}{\overline{\gamma}_{P_0^j}} = h^* + h_0^{*j}.
$$
 (12)

By comparing Eqs. (9) and (12) an inconsistency appears: in the first case the bias δW^j does not reflect in a constant bias in normal heights, while in the second case it does. In the first case, in fact, δW^j is divided by $\bar{\gamma}$, which depends of the point where the height is computed. On the contrary, in the second case, the bias in potential is divided by the average normal gravity of the reference point and the ratio is therefore a constant bias in height. This discrepancy can be attributed to the different approximations done in both formulas and it should be better investigated. Nonetheless, the term $\frac{\partial W}{\partial x}$ undergoes to variations that are smaller than 1 mm even for heights of $2,000 \text{ m}$, thus making the two solutions prectically equivalent. It is clear that the variation solutions practically equivalent. It is clear that the variation of $\overline{\gamma}$ does not compromise the solution of the height datum

problem as it was presented here: the least squares system, in fact, can be easily modified considering as unknowns only the involved potential biases δW_0^j and correspondingly
modifying the least squares coefficient matrix including the modifying the least squares coefficient matrix including the known $\overline{\gamma}$ coefficients.

Acknowledgements The authors would like to thank Roland Pail and Christian Gerlach for kindly providing us the GOCO-03S order-wise block diagonal error covariance matrices.

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