Randomized Leader Election Protocols in Noisy Radio Networks with a Single Transceiver*

Jacir Luiz Bordim[1], Yasuaki Ito[2], and Koji Nakano[2]

[1] Department of Computer Science, University of Brasilia, 70910-900 Brasilia - DF, Brazil

[2] School of Engineering, Hiroshima University, Kagamiyama, Higashi-Hiroshima, 739-8527, Japan

Abstract. In this work, we present leader election protocols for single-hop, single-channel noisy radio networks that do not have collision detection (CD) capabilities. In most leader election protocols presented so far, it is assumed that every station has the ability to transmit and monitor the channel at the same time, it requires every station to be equipped with two transceivers. This assumption, however, is unrealistic for most mobile stations due to constraints in cost, size, and energy dissipation. Our main contribution is to show that it is possible to elect a leader in an anonymous radio network where each station is equipped with a single transceiver. We first present a leader election protocol for the case the number n of stations is known beforehand. The protocol runs in $O(\log f)$ time slots with probability at least $1 - \frac{1}{f}$ for any $f > 1$. We then present a leader election protocol for the case where n is not known beforehand but an upper bound u of n is known. This protocol runs in $O(\log f \log u)$ time slots with probability at least $1 - \frac{1}{f}$ for any $f > 1$. We also prove that these protocols are optimal. More precisely, we show that any leader election protocol elect a leader with probability at least $1 - \frac{1}{f}$ must run in $\Omega(\log f)$ time slots if n is known. Also, we proved that any leader election protocol elect a leader with probability at least $1 - \frac{1}{f}$ must run in $\Omega(\log f \log u)$ time slots if an upper bound u of n is known.

1 Introduction

In recent years, wireless and mobile communications have seen an explosive growth both in terms of the number of services provided and the types of technologies that have become available. Indeed, cellular telephony, radio paging, cellular data, and even rudimentary cellular multimedia services have become commonplace and the demand for enhanced capabilities will continue to grow into the foreseeable future [1,4,5,11,13,24]. It is anticipated that in the not-so-distant future, mobile users will be able to access their data and other services such as electronic mail, video telephony, stock market news, map services, electronic banking, while on the move [5,13,15]. In a time slot, a station can transmit or listen to the channel using a transceiver. Note that, a transceiver can

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perform one of the transmitting and listening operations in a time slot. Should a station need to do both operations at the same time, two transceivers are necessary. However, this assumption is unrealistic as most mobile devices are usually equipped with a single transceiver due to stringent constraints in size and power consumption.

Unlike the well-studied cellular systems that assume the existence of a robust infrastructure, radio networks must be rapidly deployable, possibly multihop, self-organizing, and capable of multimedia service support. Radio networks suit well the needs specific to disaster-relief, search-and-rescue, law-enforcement, collaborative computing, and other special-purpose applications [9,10,14,17,18].

A radio network is a distributed system with no central arbiter, consisting of n radio transceivers, henceforth referred to as *stations*. We assume that the stations are identical and cannot be distinguished by serial or manufacturing number. As customary, time is assumed to be slotted and all the stations have a local clock that keeps synchronous time, perhaps by interfacing with a GPS system. The stations are assumed to have the computing power of a usual laptop computer; in particular, they all run the same protocol and can generate random bits that provide local data on which the stations may perform computations.

We employ the commonly-accepted assumption that when two or more stations are transmitting on a channel in the same time slot, the corresponding packets *collide* and are lost. In terms of their collision detection capabilities, the radio networks come in three flavors. In the radio network with *collision detection* (CD) the status of the channel is:

NULL: if no station transmitted on the channel in the current time slot,
SINGLE: if one station transmitted on the channel in the current time slot,
COLLISION: if two or more stations transmitted in the current time slot.

In the radio network with no collision detection (no-CD) the status of a radio channel is:

NOISE: if either no station transmitted or two or more stations transmitted in the current time slot, and
SINGLE: if one station transmitted in the current time slot.

In other words, the radio network with no-CD cannot distinguish between no transmissions on the channel and the result of two or more stations transmitting at the same time. Several workers have argued that from a practical standpoint the no CD assumption makes a lot of sense since in many situations, especially in the presence of noisy channels, the stations cannot distinguish between the no transmit case and the collision of several packets that arises when several stations attempt to broadcast at once [2,3].

Note that, if a station has two transceivers, it can send a packet and can detect the status of the channel in the same time slot. However, if a station with a single transceiver sends a packet, it cannot detect the status of the channel.

The *leader election* problem asks to designate one of the stations as *leader*. In other words, after performing the leader election protocol, exactly one station learns that it was elected leader, while the remaining stations learn the identity of the leader elected. The leader election problem is fundamental, for many protocols rely directly or indirectly, on the presence of a leader in a network [21,25]. Further, once a leader is available, the radio network with CD can be simulated by the radio network with no-CD with a constant factor slowdown [16].

It is customary to address the leader election problem on the radio network in three different scenarios:

known n (Scenario 1): Every station knows in advance the number n of stations;

known upper bound of n (Scenario 2): The upper bound u of n is known in advance. More specifically, there exists a positive integer u such that $u \leq n$ is guaranteed, and every station knows u.

unknown n (Scenario 3): The number n of stations is not known beforehand.

It is intuitively clear that the task of leader election for Scenario 1 is the easier and the hardest in Scenario 3, with Scenario 2 being in-between the two.

Several randomized protocols for single-channel radio networks have been presented in the literature. Metcalfe and Boggs [19] presented a simple leader election protocol for the radio network with no-CD for known n that is guaranteed to terminate in $O(1)$ expected rounds. For unknown n, several protocols have been proposed for the radio network with CD and no-CD. Willard [25] showed that the leader election on the radio network with CD can be solved in $\log\log n + o(\log\log n)$ expected time slots. Later, Nakano and Olariu [21] presented two leader election protocols for the radio network with CD that terminate in $O(\log n)$ time slots with probability at least $1 - \frac{1}{n}$ and in $O(\log\log n)$ time slots with probability at least $1 - \frac{1}{\log n}$. Recently, Nakano and Olariu [22] improved the protocol of [21] showing that the leader election on the radio network with CD can be performed in $\log\log n + 2.78\log f + o(\log\log n + \log f)$ time slots with probability at least $1 - \frac{1}{f}$ for every $f > 1$. Hayashi *et al.* [16] proposed a leader election protocol for the radio network with no-CD that terminates in $O((\log n)^2)$ time slots with probability at least $1 - \frac{1}{n}$. Nakano and Olariu [23] have presented that a leader can be elected in $O(\log f)$ time slots with probability at least $1 - \frac{1}{f}$ for every $f > 1$ if every station knows the number of stations.

All of the above protocols assumes that every station is equipped with two transceivers, and transmit and monitor the channel at the same time. This assumption, however, is unrealistic for most mobile stations due to constraints in cost, size, and energy dissipation. Quite recently, we have shown that, even if every station is equipped with a single transceiver, a leader can be elected in $\log\log n + o(\log\log n) + O(\log f)$ time slots with probability at least $1 - \frac{1}{f}$ for every $f > 1$ in the radio network with collision detection capabilities (CD) [6] .

Our main contribution is to show that it is possible to elect a leader in an anonymous radio network with no collision detection capabilities (no-CD) where each station is equipped with a single transceiver. We first present a leader election protocol for the case the number n of stations is known beforehand. The protocol runs in $O(\log f)$ time slots with probability at least $1 - \frac{1}{f}$ for any $f > 1$. We then present a leader election protocol for the case where n is not known beforehand but an upper bound u of n is known. This protocol runs in $O(\log f \log u)$ time slots with probability at least $1 - \frac{1}{f}$ for any $f > 1$. We prove that these protocol are optimal. More precisely, we show that any leader election protocol elect a leader with probability at least $1 - \frac{1}{f}$ must run in $\Omega(\log f)$ time slots if n is known. Also, we have proved that any leader election protocol elect a leader with probability at least $1 - \frac{1}{f}$ must run in $\Omega(\log f \log u)$ time slots if an upper bound u of n is known.

2 A Refresher of Basic Probability Theory

This section offers a quick review of basic probability theory results that are useful for analyzing the performance of our randomized leader election protocols. For a more detailed discussion of background material we refer the reader to [20].

Throughout, $\Pr[A]$ will denote the probability of event A. For a random variable X, $E[X]$ denotes the expected value of X. Let X be a random variable denoting the number of successes in n independent Bernoulli trials with parameters p and $1-p$. It is well known that X has a *binomial distribution* and that for every r, $(0 \le r \le n)$,

$$\Pr[X = r] \;=\; \binom{n}{r} p^r (1-p)^{n-r}.$$

Further, the expected value of X is given by

$$E[X] = \sum_{r=0}^{n} r \cdot \Pr[X = r] = np.$$

For all $n \ge 2$, we have the inequality

$$\frac{1}{4} \le (1 - \frac{1}{n})^n < \frac{1}{e} < (1 - \frac{1}{n})^{n-1} \le \frac{1}{2},$$

where $e = 2.71828 \cdots$ is the base of the natural logarithm. For later reference, we state the following result.

Lemma 1. *Let X be a random variable taking on a value smaller than or equal to $x(f)$ with probability at most f, $(0 \le f \le 1)$, where x is a non-decreasing function. Then, $E[X] \le \int_0^1 x(f)df$.*

3 Randomized Leader Election for Known n (Scenario 1)

The main goal of this section is to provide leader election protocols for radio networks where the number n of stations in known beforehand.

Let U be a set of all stations. We assume that U has at least two stations, that is, $|U| = n \geq 2$. If U has a single station, the unique station can be elected as a leader immediately without any broadcast and computation.

Let A and B be disjoint subsets of U, that is, $A \subseteq U$, $B \subseteq U$, $A \cap B = \emptyset$, and $|U| = n$ holds. Also, let $C = U - A - B$ be the complement of $A \cup B$. The following protocol `Leader_Election(`A`,`B`)` finds a leader in three time slots if $|A| = |B| = 1$ and a single station station in A is declared as a leader.

Protocol `Leader_Election(`A`,`B`)`
Time Slot 1: Every station in A broadcasts on the channel. Stations in B and C monitor the channel.
Time Slot 2: Every station in B broadcasts on the channel if the status of the channel at time slot 1 is SINGLE. Stations in A and C monitor the channel.
Time Slot 3: Every station in A broadcasts on the channel if the status of the channel at time slot 2 is SINGLE. Stations in B and C monitor the channel.

Clearly, if $|A| = 1$ and $|B| = 1$ then the status of the channel in both time slots 2 and 3 is SINGLE. Otherwise, that is, $|A| \neq 1$ or $|B| \neq 1$ then the status of the channel in these time slots is NOISE. Thus, if the status of the channel in time slot 2 is SINGLE, a single station in A declared as a leader and stations in C learn that a leader is elected and they are not leader. If the status of the channel in time slot 3 is SINGLE the unique station in B learns that a leader has been elected.

The readers may think that the first time slots are sufficient to elect a leader and time slot 3 is not necessary. Note that all stations in U need to know if the leader has been elected. Thus, we need time slot 3 to let stations in B learn the identity of the leader elected.

The following protocol `Election(`n`)` elects a leader.

Protocol `Election(`n`)`
Step 1 Every station flips a fair coin and belongs to A with probability $\frac{1}{n}$.
Step 2 Every station in $U - A$ flips a fair coin and belongs to B with probability $\frac{1}{n-1}$.
Step 3 Execute `Leader_Election(`A`,`B`)`.

Steps 1 and 2 need no broadcast time slots, and Step 3 uses three time slots. Thus, `Randomized_Election(`n`)` runs in three time slots. Also, we can prove that $|A| = |B| = 1$ with probability at least $\frac{1}{e^2}$ as follows. Since $|A|$ follows the n independent Bernoulli trials with parameter $\frac{1}{n}$, from (1) the probability that $|A| = 1$ is

$$\Pr[|A| = 1] = \binom{n}{1}\frac{1}{n}(1 - \frac{1}{n})^{n-1} = (1 - \frac{1}{n})^{n-1} > \frac{1}{e}.$$

Suppose that $|A| = 1$. Similarly, the probability that $|B| = 1$ is

$$\Pr[|A| = 1 \mid |B| = 1] = \binom{n-1}{1}\frac{1}{n-1}(1 - \frac{1}{n-1})^{n-2} = (1 - \frac{1}{n-1})^{n-2} > \frac{1}{e}.$$

Thus, the probability that $|A| = |B| = 1$ is

$$\Pr[|A| = |B| = 1] = \Pr[|A| = 1] \cdot \Pr[|A| = 1 \mid |B| = 1] > \frac{1}{e^2}.$$

Therefore, a single trial of `Election(`n`)` elects a leader with probability at least $\frac{1}{e^2}$.

Suppose that `Election(`n`)` is repeated until $|A| = |B| = 1$ and a leader is elected. We will evaluate the number of time slots spent to elect a leader. Suppose that `Election(`n`)` are repeated t times. All of the t executions of `Election(`n`)` fail to elect a leader leader is at most $(1 - \frac{1}{e^2})^t$. It follows that with probability exceeding $1 - (1 - \frac{1}{e^2})^t$ the protocol elects a leader in at most t time slots. Note that $1 - \frac{1}{e^2} = 0.86466 \cdots$. Let f be a real number satisfying $\frac{1}{f} = (1 - \frac{1}{e^2})^t$. Then, $t = O(\log f)$ holds. Hence, the protocol terminates, with probability exceeding $1 - \frac{1}{f}$, in $O(\ln f)$ time slots. Thus, we have the following result.

Theorem 1. `Election(`n`)` *succeeds in electing a leader with probability at least* $\frac{1}{e^2}$. *Also, by repeating* `Election(`n`)` *a leader can be elected in* $O(\log f)$ *time slots, with probability at least* $1 - \frac{1}{f}$ *for any* $f > 1$.

From Lemma 1, the expected running time slots of `Election(`n`)` is $\int_0^1 \log f df = O(1)$.

We also prove the optimality of `Election(`n`)`. To complete the leader election, the status of the channel must be SINGLE in at least one time slot. Let U ($|U| = n \geq 2$) be a set of all stations. Suppose that every station broadcast with probability p in the first time slot. Let X be the random variable denoting the number of stations that broadcast to the channel. Then, the status of the channel is SINGLE with probability

$$\Pr[X = 1] = \binom{n}{1} p(1-p)^{n-1} = np(1-p)^{n-1}.$$

The derivative of $\Pr[X = 1]$ for p is

$$\frac{d\Pr[X = 1]}{dp} = n(1-p)^{n-1} - np(1-p)^{n-2} = n(1-np)(1-p)^{n-2}$$

We have $\frac{d\Pr[X=1]}{dp} = 0$ when $np = 1$. Thus, $\Pr[X = 1]$ is the maximum when $np = 1$ and $\Pr[X = 1] \leq \frac{1}{2}$ for every n (≥ 2) and p ($0 \leq p \leq 1$). The equality holds when $n = 2$ and $p = \frac{1}{2}$. Therefore, the status of the channel is SINGLE with probability no more than $\frac{1}{2}$. We will show that, for any leader election protocol in the radio network with no CD, the status of the channel is SINGLE with probability at most $\frac{1}{2}$ in every time slot.

In the leader election protocol, every station can have a history which is represented by a sequence of bits, it can have no other information. At the beginning of the k-th time slot, every station has a history of $k - 1$ bits such that, if it broadcast in j-th ($1 \leq j \leq k - 1$) time slot, the j-th bit is 1, and if it did not broadcast, the j-th bit is 0. Then, the leader election protocol can be simply represented by a function $p : \{0,1\}^* \to [0,1]$, where $\{0,1\}^*$ denotes the set of all bits of length at least 0 and $[0,1]$ denotes a set of all real numbers from 0 to 1. For example, if a station has broadcast at time slot 1 and has not broadcast at time slots 2 and 3, then it broadcasts at time slot 4 with probability $p(100)$. Let $q : \{0,1\}^* \to [0,1]$ be the function such that a station has a history h with probability $q(h)$. Clearly, $q(\epsilon) = 1$, $q(0) = p(0)$, $q(1) = p(1)$, $q(00) = q(0) \cdot (1 - p(0))$, $q(01) = q(0) \cdot p(0)$, where ϵ denotes a sequence of bits with length 0. In general, for every $h \in \{0,1\}^*$ and $x \in \{0,1\}$,

$$q(h0) = q(h) \cdot (1 - p(h))$$
$$q(h1) = q(h) \cdot p(h)$$

holds. The probability $s(k)$ that a particular station broadcasts at time slot k is

$$s(k) = \sum_{h \in \{0,1\}^{k-1}} q(h) \cdot p(h).$$

Clearly, for all $k \geq 1$, $0 \leq s(k) \leq 1$ holds. Let X_k be the random variable denoting the number of stations that broadcast at time slot k. Then, the probability that the status of the channel at time slot k is SINGLE is

$$\Pr[X_k = 1] = \binom{n}{1} s(k)(1 - s(k))^{n-1} \leq \frac{1}{2}.$$

Therefore, the status of the channel is SINGLE with probability at most $\frac{1}{2}$ for every time slot until the leader is elected. Thus, the leader election protocol represented by p runs in t time slots with probability at least $\frac{1}{2^t}$. Thus, we have,

Theorem 2. *Any leader election protocol that elects a leader with probability at least $1 - \frac{1}{f}$ need to run in $\Omega(\log f)$ time slots.*

From Theorem 2, the leader election protocol for Theorem 1 is optimal.

4 Randomized Leader Election for Known Upper Bound u (Scenario 2)

The main purpose of this section is to develop a randomized leader election protocol for an n-station radio network under the assumption that an upper bound u of the number n of stations is known beforehand. However, the actual value of n is not known. We assume that $n \geq 2$, because if $n = 1$, the leader election is not possible.

Let $n_1, n_2, \ldots, n_k$ be a sequence of positive numbers. The following protocol `Election`$(n_1, n_2, \ldots, n_k)$ is a generalization of `Election`(n).

Protocol `Election`$(n_1, n_2, \ldots, n_k)$
for $i = 1$ **to** k **do**
  begin
    Execute Steps 1 to 3 of `Election`(n_i);
    Terminate the protocol **if** the leader is elected;
  end

For simplicity, we assume that the upper bound u is a power of two. If u is not a power of two, we can choose a minimum u' such that $u' > u$ and u' is a power of two. Clearly, such u' is an upper bound of n. Our randomized leader election protocol for Scenario 2 simply executes `Election`$(2^1, 2^2, \ldots, 2^{\log u})$.

Let us evaluate the probability that `Election`$(2^1, 2^2, \ldots, 2^{\log u})$ succeeds in electing a leader. Let i be an integer such that $2^{i-1} < n \leq 2^i$. Suppose that `Leader_Election`(2^i) is executed. The probability that $|A| = 1$ is

$$\begin{aligned}\Pr[|A| = 1] &= \binom{n}{1}\frac{1}{2^i}(1-\frac{1}{2^i})^{n-1}\\ &= \frac{n}{2^i}((1-\frac{1}{2^i})^{2^i-1})^{\frac{n-1}{2^i-1}}\\ &> \frac{1}{2e} \quad (\text{from } 2^{i-1} < n \text{ and } n-1 \leq 2^i - 1).\end{aligned}$$

Similarly, we can prove

$$\Pr[|A| = 1 \mid |B| = 1] > \frac{1}{2e}.$$

in the same manner. Thus, the probability that $|A| = |B| = 1$ is

$$\Pr[|A| = |B| = 1] = \Pr[|A| = 1] \Pr[|A| = 1 \mid |B| = 1] > \frac{1}{4e^2}.$$

Therefore, a single trial of `Election`(2^i) elects a leader with probability at least $\frac{1}{4e^2}$ provided that $2^{i-1} < n \leq 2^i$. For every n such that $2 \leq n \leq u$, there exists an integer i $(1 \leq i \leq \log u)$ such that $2^{i-1} < n \leq 2^i$. Therefore, we have,

Lemma 2. *For every n such that $2 \leq n \leq u$, protocol* `Election`$(2^1, 2^2, \ldots, 2^{\log u})$ *succeeds in electing a leader in $O(\log u)$ time slots with probability at least $\frac{1}{4e^2}$.*

We further generalize `Election` for infinite sequences. Let $n_1, n_2, \ldots$ be an infinite sequence. Protocol `Election`$(n_1, n_2, \ldots)$ is defined as follows:

Protocol `Election`$(n_1, n_2, \ldots)$
for $i = 1$ **to** ∞ **do**
  begin
    Execute Steps 1 to 3 of `Election`(n_i);
    Terminate the protocol **if** the leader is elected;
  end

Let $D^1_{\log u}$ be a sequence $2^1, 2^2, \ldots, 2^{\log u}$, and $D^{k+1}_{\log u} = D^k_{\log u} \cdot D^1_{\log u}$ for all $k \geq 1$, where "$\cdot$" denotes the operator of concatenation of two sequences. Clearly, $D^k_{\log u}$ has $k \log u$ integers. Also, let $D^\infty_{\log u}$ be the infinite sequence $D^1_{\log u} \cdot D^1_{\log u} \cdots$. We have proved that in Lemma 2, `Election`$(D^1_{\log u})$ elects a leader in three time slots with probability $\frac{1}{4e^2}$. Thus, `Election`$(D^k_{\log u})$ fails to elect a leader in no more than $3k \log u$ time slots with probability at most $(1 - \frac{1}{4e^2})^k$. Let $\frac{1}{f} = (1 - \frac{1}{4e^2})^k$. Then, $3k \log u = O(\log f \log u)$. Therefore, we have,

Theorem 3. *Protocol* `Election`($D^{\infty}_{\log u}$) *elects a leader in* $O(\log f \log u)$ *time slots with probability at least* $1 - \frac{1}{f}$ *for any* $f > 1$.

From Lemma 1, the expected running time slots of `Election`(n) is $\int_0^1 \log f \log u df = O(\log u)$.

We will show that, any leader election protocol for Scenario 2 need to run in $\Omega(\log u)$ time slots to elect a leader with probability at least $\frac{1}{2}$.

Let $X(n, p)$ denote the random variable denoting the number of stations that have broadcast if each of the n stations broadcast with probability p. The probability that the status of the channel is SINGLE with probability

$$\Pr[X(n,p) = 1] = \binom{n}{1} p^1(1-p)^{n-1} = np(1-p)^{n-1}.$$

Let us evaluate the upper bounds of $\Pr[X(2^1, p) = 1]$,$\Pr[X(2^2, p) = 1], \ldots,$ $\Pr[X(2^{\log u}, p) = 1]$ and then compute $\Pr[X(2^1, p) = 1] + \Pr[X(2^2, p) = 1] + \cdots + \Pr[X(2^{\log u}, p) = 1]$. Let $m = \frac{1}{p}$. For simplicity, we assume m is an integer and a power of two. It is easy to show the upper bounds when this is not the case. Since $\Pr[X(n, p) = 1] = np(1 - p)^{n-1} \leq np$, we have

$$\begin{aligned}
&\Pr[X(2^1, p) = 1] + \Pr[X(2^2, p) = 1] + \cdots + \Pr[X(2^{\log m - 1}, p) = 1] \\
&= 2^1 p + 2^2 p + \cdots + 2^{\log m - 1} p = 2^1/m + 2^2/m + \cdots + 2^{\log m - 1}/m < 1.
\end{aligned}$$

If $n = m$ then,

$$\Pr[X(2^{\log m}, p) = 1] = mp(1-p)^{m-1} = (1 - \frac{1}{m})^{m-1} < \frac{1}{2}.$$

Also, for every $n > m$, we have

$$\begin{aligned}
&\Pr[X(2^{\log m+1}, p) = 1] + \Pr[X(2^{\log m+2}, p) = 1] + \Pr[X(2^{\log m+3}, p) = 1] + \cdots \\
&= \Pr[X(2^1 m, p) = 1] + \Pr[X(2^2 m, p) = 1] + \Pr[X(2^3 m, p) = 1] + \cdots \\
&= 2^1 mp(1-p)^{2^1 m - 1} + 2^2 mp(1-p)^{2^2 m - 1} + +2^3 mp(1-p)^{2^3 m - 1} + \cdots \\
&= mp(1-p)^{m-1}(2^1(1-p)^{2^1 m - m} + 2^2(1-p)^{2^2 m - m} + 2^3(1-p)^{2^3 m - m} + \cdots) \\
&< \frac{1}{2}(2^1(\frac{1}{e})^{2^1 - 1} + 2^2(\frac{1}{e})^{2^2 - 1} + 2^3(\frac{1}{e})^{2^3 - 1} + \cdots) < 1.
\end{aligned}$$

Therefore, $\Pr[X(2^1, p) = 1] + \Pr[X(2^2, p) = 1] + \cdots + \Pr[X(2^{\log u}, p) = 1] < \frac{5}{2}$.

Let $s(k)$ be the probability that a particular station broadcast at time slot k. To elect a leader with probability at least $\frac{1}{2}$ in t time slots for all $n = 2^1, 2^2, \ldots, 2^{\log u}$,

$$\sum_{k=1}^{t} \Pr[X(2^1, s(k)) = 1] + \Pr[X(2^2, s(k)) = 1] + \cdots + \Pr[X(2^{\log u}, s(k)) = 1] \geq \frac{\log u}{2}$$

must hold. However, the left hand side of the inequality is at most $\frac{5}{2}t$. Therefore, since $\frac{5}{2}t \geq \frac{\log u}{2}$, we have $t = \Omega(\log u)$. Hence, to elect a leader with probability $1 - \frac{1}{f}$ for all $n = 2^1, 2^2, \ldots, 2^{\log u}$, any leader election protocol need to run in $\Omega(\log f \log u)$ time slots.

Theorem 4. *Any leader election protocol runs that elects a leader with probability at least* $1 - \frac{1}{f}$ *need to run in* $\Omega(\log f \log u)$ *time slots.*

Therefore, protocol for Theorem 3 is optimal.

5 Conclusions

In this work, we have presented leader election protocols for single-hop, single-channel noisy radio networks that do not have collision detection (CD) capabilities. Also, we have assumed that every station is equipped with a single transceiver. We presented a leader election protocol for the case the number n of stations is known beforehand. that runs in $O(\log f)$ time slots with probability at least $1 - \frac{1}{f}$ for any $f > 1$. We then presented a leader election protocol for the case where n is not known beforehand but an upper bound u of n is known. This protocol runs in $O(\log f \log u)$ time slots with probability at least $1 - \frac{1}{f}$ for any $f > 1$. We also proved that these leader elections are optimal.

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