Recent Developments in Learning and Competition with Finite Automata (Extended Abstract)

Abraham Neyman

Institute of Mathematics and Center for the Study of Rationality,
The Hebrew University of Jerusalem, Givat Ram, Jerusalem 91904, Israel
aneyman@math.huji.ac.il
www.ratio.huji.ac.il/neyman

Consider a repeated two-person game. The question is how much smarter should a player be to effectively predict the moves of the other player. The answer depends on the formal definition of effective prediction, the number of actions each player has in the stage game, as well as on the measure of smartness. Effective prediction means that, no matter what the stage-game payoff function, the player can play (with high probability) a best reply in most stages. Neyman and Spencer [4] provide a complete asymptotic solution when smartness is measured by the size of the automata that implement the strategies.

Let $G = \langle I, J, g \rangle$ be a two-person zero-sum game; I and J are the set of actions of player 1 and player 2 respectively, and $g : I \times J \to \mathbb{R}$ is the payoff function to player 1. Consider the repeated two-person zero-sum game $G(k, m)$ where player 1's possible strategies are those implementable by an automaton with k states and player 2's possible strategies are those implementable by an automaton with m states. We say that player 2 can *effectively predict* the moves of player 1 if for every reaction function $r : I \to J$ player 2 has a strategy (in $G(k, m)$) such that for every strategy of player 1 the expected empirical distribution of the action pairs (i, j) is essentially supported on the set of action pairs of the form $(i, r(i))$. A recent result of Neyman and Spencer characterizes the asymptotic relation of $m = m_k$ and k so that player 2 can effectively predict the moves of player 1. This asymptotic relation is: $\liminf \frac{\log m_k}{k}$, as k goes to infinity, is at least the minimum of $\log |I|$ and $\log |J|$. It follows that the value of $G(k, m_k)$ converges to $\max_{i \in I} \min_{j \in J} g(i, j)$ as $k \to \infty$ and $\liminf_{k \to \infty} \frac{\log m_k}{k} \geq \min(\log |I|, \log |J|)$.

An open problem (see [2]) is the quantification of the feasible "level of prediction" when the limit of $\frac{\log m_k}{k}$ equals θ and $0 < \theta < \min(\log |I|, \log |J|)$. For example, do the values of $G(k, m_k)$ converge as $k \to \infty$ and $\lim_{k \to \infty} \frac{\log m_k}{k} = \theta$, and, for those values of θ for which the limit exists, what is the limit of the values as a function of the stage game G and θ? It is known that the value of $G(k, m_k)$ converges, as $m_k \geq k \to \infty$ and $\frac{\log m_k}{k} \to 0$, to the value of the stage game [1].

The level of prediction, where player 1 is either (an uncertain periodic) nature or a player that does not observe the moves of player 2, has a complete asymptotic characterization [3]. The value of the two-person zero-sum repeated game, where

P. Spirakis et al. (Eds.): WINE 2006, LNCS 4286, pp. 1–2, 2006.

player 1's possible strategies are those implementable by oblivious automata of size k and player 2's possible strategies are those implementable by automata of size m, converges, as k goes to infinity and $\frac{\log m}{k}$ goes to $\theta \geq 0$, to a limit $v(\theta)$. The limit $v(\theta)$ is characterized by the data of the stage game $G = \langle I, J, g \rangle$. It equals the maxmin of $E_Q g(i,j)$ where the max is over all mixed stage actions p and the min is over all distributions Q on action pairs with marginal p on I, denoted Q_I, and $H(Q_I) + H(Q_J) - H(Q) \leq \theta$, where H is the entropy function. This result remains intact when player 2's possible strategies are those implementable by automata with time-dependent mixed actions and mixed transitions.

Another question is how long it takes the smarter player to effectively predict the moves of the other player. We study this question by analyzing the T-stage repeated game $G^T(k,m)$ where player 1's (respectively, player 2's) possible strategies are those implementable by an automaton with k (respectively, m) states. It is known that when player 2 is "supersmart" ($m = \infty$) and $T \gg k \log k$, player 2 can effectively predict the moves of player 1 [5]. Formally, the values of the two-person zero-sum games $G^{T_k}(k,\infty)$ converge to $\max_{i \in I} \min_{j \in J} g(i,j)$ as $k \to \infty$ and $\limsup_{k \to \infty} \frac{k \log k}{T_k} = 0$. It is conjectured in [2] that the values of the two-person zero-sum games $G^{T_k}(k,\infty)$ converge to the value of the stage game G as $k \to \infty$ and $\limsup_{k \to \infty} \frac{k \log k}{T_k} = \infty$.

References

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