A Formal Characterization of Vagueness and Granularity for Context-Aware Mobile and Ubiquitous Computing*

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Abstract. In this article, a formal approach for modeling central concepts of context-awareness in ubiquitous and mobile computing is introduced. The focus is on an appropriate handling of issues of vagueness and granularity in ubiquitous computing environments. A formalization of perceptual and sensory uncertainty and a characterization of granularity are applied for modeling three central aspects of context-awareness: context as retrieved from sensors, context for representing relevance, and context as unfocussed background information. The notions are developed and demonstrated with respect to the special case of spatial contexts, but are sufficiently general to also cover other types of context. Use of the characterized concepts is motivated with an example of ongoing work on ontology design for ubiquitous computing environments.

1 Introduction

Vagueness and uncertainty arising from limited sensory accuracy and the dynamic of an environment flexibly adapting during user interaction pose central challenges to context modeling and ontology design for ubiquitous computing environments. Several definitions for context and context-awareness exist [5, 10, 25, 26] resulting in different perspectives and approaches to establishing context-awareness in computing environments. Nevertheless, central aspects and challenges have been identified. Three aspects of context constitute the conceptual basis for this article: context as retrieved from sensors, context for representing relevance, and context as unfocussed background information.

Context retrieved from sensors provides information about the context of the user in the physical world [26]. A central challenge in modeling information from sensors in an application is how to model the uncertainty resulting from the inevitable limitations of accuracy.

Context for representing relevance makes advanced human-computer interface techniques, such as *proximate selection* [25], possible. A representation of

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relevance in a context can help anticipate which objects are more likely to be desired next by a user, so that the user needs less effort for retrieving information that is closely related to the currently displayed information. Additionally, a representation of relevance can be used to improve efficiency of an application.

Context as unfocussed background information is information of which a user is aware, but which is currently not in the focus. Similarly to irrelevant information, background information can be disregarded in processing. This is used in AI systems to reduce complexity [14]. However, a change in background information entails a change of context, whereas a change in irrelevant information is inconsequential.

The aims of this article are to present a formal modeling for granularity and vagueness as core concepts underlying these three central aspects, and to use this modeling to reveal formal links that can be used in future context-aware applications. The formal concepts described in the article can be applied to different types of context [26]. However, spatial context was chosen as an example domain of special relevance, since it is not only a parameter for interaction with a user but also influences effectivity and efficiency of ubiquitous computing environments conceived as systems for (spatially) distributed computing [30]. Consequentially, spatial context has been a focus of interest with respect to technical [15] as well as application-oriented questions [8], and the representation of spatial context has emerged to be a problem of sufficient complexity to justify a more detailed theoretical analysis.

Structure of the Article. In Sect. 2, formal properties common to uncertainty resulting from limited sensory accuracy and perceptual vagueness are studied. Uncertainty is represented as an interval on a scale, which can be computed from a sensor reading and a range of accuracy, given a desired precision. Section 3 gives an outline of a theoretical framework for modeling spatial contexts, which is based on a mereotopological notion of regions. The formal notions of context for representing relevance and of context as unfocussed background are then illustrated for the example of spatial contexts in Sect. 4. In Sect. 5, a method for developing and modifying a granular spatial context representation for ubiquitous computing environments is sketched. A summary and an outlook on ongoing research are given in Sect. 6.

2 Sensory Input and Perceptual Vagueness

Vagueness resulting from the limited accuracy of sensors is not only a problem for ubiquitous computing and robotics, the human perceptual system has to handle similar restrictions. In this section, formal links between sensory uncertainty and perceptual vagueness are traced back to results from basic measurement theory. Applicability of the formal notions is demonstrated with composition tables for qualitative reasoning[1] about uncertain perceptions and measurements.

[1] For an introduction to qualitative reasoning cf. Cohn and Hazarika [9], Galton [13]; for a discussion on the role of composition tables for ontologies cf. Eschenbach [12].

2.1 Perceptual Vagueness

Vagueness in the human perceptual systems has been studied in psychophysical experiments on distinguishability, e.g., of colors or lengths (for an overview cf. [22], p. 671): subjects were shown two lines successively or in horizontal alignment, so that direct comparison of lengths was not possible, and were then asked to judge, whether the lines had been of the same lengths or of different lengths. The experiments showed that lengths of less than a certain difference could not be distinguished. In comparison with a line of 10 cm, e.g., a line of 10.5 cm was judged to have the same length, and subjects could not indicate which of the two lines was shorter, whereas a line of 11 cm was noticeably longer. Accordingly, this difference is called the *just noticeable difference* (JND).

The mathematical properties of the relations *perceivably smaller* and *of indistinguishable length* as given by the experiments can be formally characterized by a semiorder ($\prec$) and an indistinguishability relation ($\approx$), respectively. Axioms A1–A4 give the characterization proposed in [31]. Semiorders are irreflexive (A1). Given two ordered pairs of values, either the smaller of the first is smaller than the larger of the second pair, or the smaller of the second pair is smaller than the larger of the first pair (A2). If three values are ordered according to $\prec$, then every further value is smaller than the largest of the triple or larger than the smallest (A3). Two values are indistinguishable, iff they cannot be ordered (A4).[2]

$$\forall x : \neg x \prec x \tag{A1}$$

$$\forall x_1, x_1', x_2, x_2' : x_1 \prec x_1' \wedge x_2 \prec x_2' \rightarrow x_1 \prec x_2' \vee x_2 \prec x_1' \tag{A2}$$

$$\forall x_1, x_2, x_3, x : x_1 \prec x_2 \wedge x_2 \prec x_3 \rightarrow x_1 \prec x \vee x \prec x_3 \tag{A3}$$

$$\forall x, x' : x \approx x' \leftrightarrow \neg(x \prec x' \vee x' \prec x) \tag{A4}$$

The relations $\approx$, $\prec$ and its inverse relation $\succ$ are mutually exclusive and exhaustive relations on the domain of possible lengths. For reasoning about these relations, the composition table 1(a) can be used, e.g.: if x is unnoticeably smaller than y (row: $x \prec y$) and y is indistinguishable from z (column: $y \approx z$), then we can infer that x must be perceivably smaller than, or indistinguishable from z (entry: $\prec, \approx$; to be read as: $x \prec z \vee x \approx z$).

A particularly interesting property of semiorders is that we can obtain more certain information from uncertain information by subsequent observations: if x_1 in reality is smaller than x_2, then, given all possible lengths, there is an x that is large enough to be distinguishably larger than x_1, but still not perceivably different from x_2; cf. van Deemter [32] for a discussion of this property and its usage with respect to linguistic notions of context. Using this property, the relation $\approx$ can be split up into three relations $=$, $\preccurlyeq$ (*unnoticeably smaller*) and its inverse relation $\succcurlyeq$, so that also inferable relations between lengths can be

[2] In order to abbreviate formulae and to reduce the number of brackets, the scope of quantifiers is to be read as maximal, i.e. until the end of a formula, or until the closing bracket of a pair of brackets containing the quantifier. Additionally, the following precedence applies: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \stackrel{def}{\Leftrightarrow}$.

Table 1. Composition table with three relations: smaller ($\prec$), indistinguishable ($\approx$), larger ($\succ$) (a) and composition table with more specific inferred information (b) derived from Tab. 2. The asterisk ($*$) represents a situation in which no information can be inferred, i.e., any of the relations is possible.

	$\prec$	$\approx$	$\succ$
$\prec$	$\prec$	$\prec, \approx$	$*$
$\approx$	$\prec, \approx$	$*$	$\approx, \succ$
$\succ$	$*$	$\approx, \succ$	$\succ$

(a)

	$\prec$	$\approx$	$\succ$
$\prec$	$\prec$	$\prec, \precsim$	$*$
$\approx$	$\prec, \precsim$	$*$	$\succsim, \succ$
$\succ$	$*$	$\succsim, \succ$	$\succ$

(b)

represented. The relation $\precsim$ holds between two values x_1 and x_2, iff the two are indistinguishable, and there is a third value that is noticeably larger than x_1 but still indistinguishable from x_2; in this case, x_1 is smaller, but not noticeably smaller, than x_2 (D1):

$$x_1 \precsim x_2 \overset{def}{\Leftrightarrow} x_1 \approx x_2 \wedge \exists x : x_1 \prec x \wedge x_2 \approx x \tag{D1}$$

Accordingly, we obtain a composition table of five exhaustive and mutually exclusive relations between lengths (Tab. 2). Comparison of (a) and (b) in Tab. 1 shows how the use of $\precsim$ contributes to the elimination of vagueness in continued observation and inference: in four cases in which (a) contains indistinguishability, inference of more specific information is possible in (b).

Table 2. Composition table with five exhaustive and mutually exclusive relations: smaller ($\prec$), unnoticeably smaller ($\precsim$), equal ($=$), unnoticeably larger ($\succsim$), larger ($\succ$). The asterisk ($*$) represents a situation in which no information can be inferred.

	$\prec$	$\precsim$	$=$	$\succsim$	$\succ$
$\prec$	$\prec$	$\prec$	$\prec$	$\prec, \precsim$	$*$
$\precsim$	$\prec$	$\precsim, \prec$	$\precsim$	$\succsim, =, \precsim$	$\succsim, \succ$
$=$	$\prec$	$\precsim$	$=$	$\succsim$	$\succ$
$\succsim$	$\precsim, \prec$	$\succsim, =, \precsim$	$\succsim$	$\succ, \succsim$	$\succ$
$\succ$	$*$	$\succsim, \succ$	$\succ$	$\succ$	$\succ$

2.2 Uncertainty from Sensors

We can now compare uncertainty from perception with uncertainty from sensors by showing that accuracy intervals are a model of the axiomatic characterization presented above.

The vagueness associated with sensory input can be specified by an interval of accuracy and a precision expressed as a percentage [15], e.g.: a temperature sensor giving $10\,°C$ with an accuracy of $\pm 1\,°C$ at a precision of $95\,\%$ means that the true temperature is in the interval $[9\,°C, 11\,°C]$ with a probability of $95\,\%$. Conversely, we could state that a value of $10\,°C$ in the world will in $95\,\%$ of all cases result in a sensory reading in the interval $[9\,°C, 11\,°C]$. For simplifying the following discussion, the two perspectives are called *sensor perspective* and *world perspective*, respectively. In the specification of the accuracy of a sensor reading, a sensor perspective is assumed; the notion of indistinguishability in Sect. 2.1, in contrast, indicates a world perspective, as it refers to objects in the world. Furthermore, the discussion focusses on accuracy, with precision being regarded as a fixed value specifying overall reliability of a system. A more detailed treatment is beyond the scope of this article.

For sensors such as the thermometer described above, the domain of possible sensor readings can be given as the set I_T of closed, convex intervals of unit length within a range $[t_{\min}, t_{\max}]$ on $\mathbb{R}$:

$$I_T = \{[t-1, t+1] \mid t \in [t_{\min}, t_{\max}]\}$$

Chalmers et al. [7], assuming a sensor perspective, present a similar approach for reasoning about context in the presence of uncertainty, which is based on arbitrary intervals. However, the full set of 13 interval relations [1] is not necessary for the domain of I_T, since no interval of I_T can contain another interval of I_T. In fact, the five relations, which correspond to the relations $\prec$, $\preccurlyeq$, $=$, $\succcurlyeq$, $\succ$ are sufficient, as I_T can be shown to be a model of the axiomatic characterization presented above with the following interpretations for $\prec$ and $\preccurlyeq$:[3] $\prec_{I_T}$ holds between two unit intervals iff the first ends before the second begins, $\preccurlyeq_{I_T}$ holds iff the second interval starts after, but within the duration of the first interval.

$$\prec_{I_T} \stackrel{def}{=} \{([t_1-1, t_1+1], [t_2-1, t_2+1]) \mid t_1+1 < t_2-1\}$$
$$\preccurlyeq_{I_T} \stackrel{def}{=} \{([t_1-1, t_1+1], [t_2-1, t_2+1]) \mid t_1-1 < t_2-1 \leq t_1+1\}$$

The differences between the two perspectives show, if we look at the relations between values in the world which can be inferred from sensory readings. The statement $[t_1-1, t_1+1] \preccurlyeq [t_2-1, t_2+1]$ under a world perspective means for the actual values t_1 and t_2 in the world: $t_1 < t_2$. Under a sensor perspective the statement $[t'_1-1, t'_1+1] \preccurlyeq [t'_2-1, t'_2+1]$ for sensor readings t'_1 and t'_2 entails only vague information about the actual values in the world: $t'_1 < t'_2$ and thus $[t'_1-1, t'_1+1] \preccurlyeq [t'_2-1, t'_2+1]$ in the domain of sensory values entails only $[t_1-1, t_1+1] \approx [t_2-1, t_2+1]$ for the measured actual values. Using composition table (b) of Tab. 1, knowledge regarding $\preccurlyeq$ in the world domain can be obtained with multiple measurements. For actual applications, however, imprecision limits the maximally usable number of measurements.

Uncertainty resulting from limited sensory accuracy is not only a critical challenge for the representation and processing of information about the physical context of a user in ubiquitous and mobile computing. Likewise, knowledge about contextual parameters that are assumed by a user is accessible only indirectly to an application. Accordingly, the relations of semiorder and indistinguishability were employed to model granularity as a dynamically changing parameter

[3] The proof follows along the lines of the one given in [31]. Cf. also [29].

of context in human-computer interaction [29]. In [29], the example of temporal granularity was examined. An extension to the more complex case of spatial granularity is described in Sect. 4.

3 Spatial Context

Two models of location are employed in ubiquitous and mobile computing [8]: *geometric* and *symbolic* location models. This distinction has both technical and semantic aspects. Hightower and Borriello [15] accordingly differentiate between two types of spatial information a location system may provide: the coordinate-based information provided, e.g., by GPS is called *physical position*; *symbolic location* information, in contrast, is used in location systems that employ sensors that only determine whether or not objects are in a certain stationary or mobile area. Concerning information processing, the retrieved information is numerical in the first case and boolean or textual – e.g. the ID of an object detected by the sensor – in the second case.

This technical distinction is mirrored on the semantic level: coordinate-based information can directly be interpreted spatially, if the used reference system and resolution are known; in contrast, the spatial information in symbolic location systems – i.e. how different sensor areas are spatially related – has to be provided externally, either during installation of the system or via inference from coordinate-based information. Concerning resolution, coordinate-based information, e.g. obtained from GPS, has a certain limited accuracy, whereas symbolic location systems, such as the system described by Schilit and Theimer [24], can be organized in a hierarchical manner based on the relation of spatial containment, so as to provide arbitrarily fine spatial distinctions. However, the spatial notion of resolution or size is usually not represented in symbolic location systems.

Location-aware systems for heterogeneous environments need to incorporate both sources of location information. Hybrid location models have been specified to address this need [18, 21]. The formal framework proposed in the following sections is related to these approaches, and provides a theoretical foundation for improving hybrid location models. The characterization of regions given in Sect. 3.1 provides the basic relations of containment and overlap used in the symbolic location model; in Sect. 3.2, rudimentary notions of resolution or grain-size are added to this framework, in order to make it compatible with coordinate-based location models and to allow stratification according to grain-size. The resulting framework for hybrid location models, similar to the one of Leonhardt [21], is based on the relations of containment and overlap, but additionally contains representations for resolution and size. A location model equipped with methods to handle granularity (Sect. 4) can be key to improving scalability and interoperability of location-aware systems. Section 5 illustrates this claim with the sketch of a non-partitioning stratification methodology.

3.1 Regions

A mereotopological framework is chosen as a foundation for characterizing regions [2, 6, 23]: the basic relation C of mereotopology, stating that two regions are connected, can be characterized as a reflexive (A5) and symmetric relation (A6); the relation $\sqsubseteq$ holding between a region and its parts can then be defined in terms of C (D2): x is part of y, iff every region that is connected to x is also connected to y.

$$\forall x : C(x, x) \tag{A5}$$

$$\forall x, y : C(x, y) \rightarrow C(y, x) \tag{A6}$$

$$x \sqsubseteq y \overset{def}{\Leftrightarrow} \forall z : C(x, z) \rightarrow C(y, z) \tag{D2}$$

This rudimentary foundation suffices for present purposes, as $\sqsubseteq$ gives the basic ordering constraints on the sizes of regions that are used in the following: it can be shown that $\sqsubseteq$ is a reflexive, antisymmetric relation. For a thorough treatment of mereotopological ontological questions, however, a more elaborate framework would be needed [2, 6, 23].

3.2 Extended Locations: Regions with a Unique Size

In order to develop a notion of grain-size, an ordering relation $\leq$ (*smaller or of equal size*) describing basic size constraints between two regions is used. Following Dugat et al. [11], a suitable relation $\leq$ can be axiomatized as a reflexive (A7) and transitive relation (A8) holding, inter alia, between a region and its parts (A9). The relation can be used to define a notion of congruence of a special class of regions – here called: *extended locations* –, upon which Dugat et al. following Borgo et al. [4] then build a geometry of spheres. Extended locations are characterized with a predicate L as a special class of regions on which $\leq$ yields a linear order (A10):

$$\forall x : x \leq x \tag{A7}$$

$$\forall x, y, z : x \leq y \land y \leq z \rightarrow x \leq z \tag{A8}$$

$$\forall x, y : x \sqsubseteq y \rightarrow x \leq y \tag{A9}$$

$$\forall x, y : L(x) \land L(y) \rightarrow x \leq y \lor y \leq x \tag{A10}$$

Additional relations $\equiv$ (*same size*) and $<$ (*smaller*) can be defined:

$$x \equiv y \overset{def}{\Leftrightarrow} x \leq y \land y \leq x \tag{D3}$$

$$x < y \overset{def}{\Leftrightarrow} x \leq y \land \neg y \leq x \tag{D4}$$

Spheres are one example of a class of regions that adhere to the requirements for extended locations L, since two spheres can always be ordered according to their diameter. In contrast to the spheres of Dugat et al. [11] however, the extended locations are not restricted further in shape or topology. Additionally, the notion of size is used here for illustrating the partial order $\leq$ on regions, but a formal characterization of size as related to distance in a metric sense would require further restrictions and is beyond the scope of this paper; for geometric characterizations cf. [4, 11, 27, 28]. The main advantage of the less restrictive formalization chosen here is that it encompasses a broad range of models and therewith modeling alternatives for developers of location aware systems, as illustrated in Sect. 5 below.

4 Spatial Relevance and Spatial Background

Notions of context for representing relevance and of context as unfocussed background information can be formally specified based on a characterization of granularity [29]. Mechanisms for representing, and reasoning about, granularity are a means to filter and simplify complex domains, so as to focus on the currently relevant objects and attributes [16]. Spatial granularity is largely determined by the concept of grain-size. It can be used to restrict the set of objects under consideration to the subset of objects having at least a certain minimal size. A further component of spatial relevance is proximity. Objects which are within a certain range of currently relevant objects are more likely to be relevant than remote objects. This concept is fundamental for the interface technique of proximate selection [25]. Linking the concept of ranges of proximity to the notion of grain-size, we can ensure that the number of objects currently under consideration can be kept small: as we focus on a smaller area, i.e. *zoom* into a scene, objects further away become irrelevant, and smaller details become relevant.[4]

Based on the notion of extended locations, a stratification of space into an ordering of levels of granularity can now be characterized. The characterization is based upon the primitive relation $\mid\!\lhd$ between extended locations, with $x \mid\!\lhd y$ denoting that x is a grain location of the context location y. The grains thus represent the smallest possibly relevant locations, whereas the context location provides the maximal range of proximity and determines the background: an object that is contained in a location smaller than a grain can be classified as irrelevant; an object containing the context location can be classified as a background object, since all objects relevant in the context lie within its region.

Axiom A11 states that grains are ubiquitous within a context location: every region connected to a context location that has a grain is also connected to a grain. It is worth noting that this axiom is the only ontological axiom in a narrow sense, as it actually guarantees existence of regions under certain conditions. From an application point of view, the axiom demands minimum requirements on availability of fine-grained location services. The second axiom (A12) states that grains and context locations are extended locations, and that the grains of a context location are contained in the location. Axiom A13 gives the central restriction on the ordering of levels of granularity: grains are ordered in the same way as their respective context locations and vice versa. As a consequence, all grains of a context location have equal extension (1).

[4] For a discussion on the photo metaphor see §1.6 in [13]; for empirical evidence regarding phenomena of granularity in spatial imagery see Kosslyn [20].

$$\forall c, g, x : C(x, c) \wedge g \mathrel{|\!\lhd} c \rightarrow \exists g' : g' \mathrel{|\!\lhd} c \wedge C(x, g') \tag{A11}$$

$$\forall c, g : g \mathrel{|\!\lhd} c \rightarrow L(g) \wedge L(c) \wedge g \sqsubseteq c \wedge g \neq c \tag{A12}$$

$$\forall c_1, c_2, x_1, x_2 : x_1 \mathrel{|\!\lhd} c_1 \wedge x_2 \mathrel{|\!\lhd} c_2 \rightarrow [c_1 < c_2 \leftrightarrow x_1 < x_2] \tag{A13}$$

$$\forall c, x, y : x \mathrel{|\!\lhd} c \wedge y \mathrel{|\!\lhd} c \rightarrow x \equiv y \tag{1}$$

The ordering on levels of granularity can be characterized with relations $\prec$ and $\approx$: c_1 is of *finer granularity* than c_2, iff there is a grain of c_2 that is larger than c_1 (D5); c_1 is of *compatible granularity* with c_2, iff c_1 is not smaller than any grain of c_2 and c_2 is not smaller than any grain of c_1 (D6). The predicate *CL* (*proper context location*) selects those extended locations which have grain locations (D7).

$$c_1 \prec c_2 \overset{def}{\Leftrightarrow} \exists g : g \mathrel{|\!\lhd} c_2 \wedge c_1 < g \tag{D5}$$

$$c_1 \approx c_2 \overset{def}{\Leftrightarrow} \forall g_1, g_2 : g_1 \mathrel{|\!\lhd} c_1 \wedge g_2 \mathrel{|\!\lhd} c_2 \rightarrow g_1 \leq c_2 \wedge g_2 \leq c_1 \tag{D6}$$

$$CL(x) \overset{def}{\Leftrightarrow} \exists g : g \mathrel{|\!\lhd} x \tag{D7}$$

It can be shown (Sect. A) that, if restricted to the class of *CL*-locations (D7), $\prec$ actually is a semiorder with $\approx$ as a relation of indistinguishability, as the use of the symbols suggests. The axiomatization thus supports representation of the vagueness associated with the notion of granularity as a parameter of interaction with a user: the actual granularity conceptualized by the user, like the actual values measured by a sensor, can be modeled as an indirectly accessible parameter.

The above axioms are neutral with respect to the question whether space is partitioned by grain locations. Axiom A11 demands that context locations having a grain location are completely covered by grains, but allows for grains to overlap. Axiom A13 does not restrict this either. For a partitioning approach to modeling spatial granularity cf. Bittner and Smith [3].

5 Application: Stratification of a Ubiquitous Computing Environment

With the ordering on levels of granularity being anchored in the containment hierarchy, a context management system that keeps containment information can be modified to handle information about levels of granularity: first, constraints on the sizes of regions have to be extracted; second, sizes which are particularly important throughout the whole domain of application have to be identified; these sizes are then used in the third step to stratify the domain. If the third step has been performed in a consistent way, further regions and strata can be flexibly incorporated into the system, when new location sensing components are to be added to an environment.

Step 1: Size Constraints. A consistent hierarchy of sizes on which to base the stratification of a domain can be obtained from a given containment hierarchy.

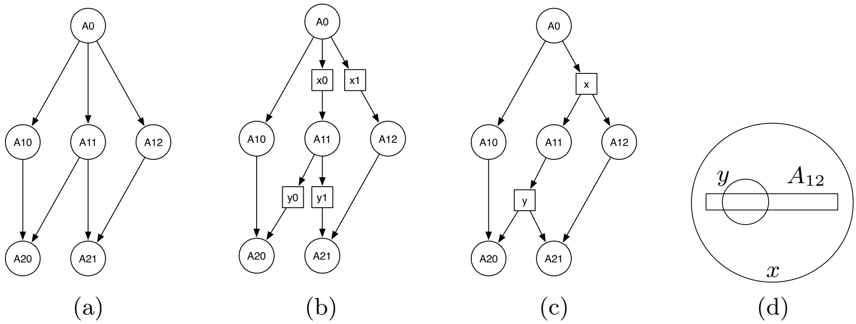

Fig. 1. A containment hierarchy (relation $\sqsubseteq$) describing a set of symbolic locations (a) can be enriched with extended locations representing possible coordinate-based locations based on knowledge about the extension of regions (b). After unification of equally extended locations ($\equiv$), we obtain a corresponding hierarchy of sizes (relation $\leq$, c). The region A_{12} is an example of a region that is contained in an extended location x, but neither contains nor is contained in any location of the same extension as y (d).

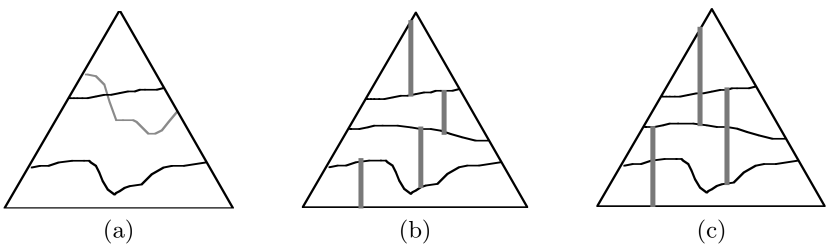

Fig. 2. Schematic of the graph of a containment hierarchy (horizontal lines indicate extended locations of the same equivalence class with respect to $\equiv$, vertical lines illustrate the extent of strata): alternative (gray) for stratifying the upper part of the hierarchy (a), partitioning stratification (b), and non-partitioning stratification (c).

All necessary information on size constraints, as specified by the above axioms, can then be derived by replacing every occurrence of $\sqsubseteq$ with $\leq$ as justified by (A9). Figure 1(a) shows a simple example for a containment hierarchy.

Step 2: Extended Locations. The procedure of the first step is sufficient for modeling arbitrary fixed containment structures. The need to introduce extended locations arises when mobile devices and location sensors providing coordinate-based locations are used. The region representing a specific GPS-signal, for instance, can be specified as a circle corresponding to the accuracy of the signal around the currently measured GPS-coordinate (Sect. 2). If this accuracy and with it the size of the region does not change with the place where it is measured – or if the accuracy changes, but is known – the mobile device provides an absolute measure for comparing disjoint spatial regions. If the extensions of the regions collected in step 1 are known, we can compute the regions corresponding to possible measurements of a coordinate-based location sensor as the extended locations and enter these into the hierarchy of sizes (for a discussion of methods cf. Leonhardt [21] on semi-symbolic location hierarchies). Since the extended locations have to be linearly ordered with respect to $\leq$, the classes of regions that are extended locations of the same equivalence class with respect to $\equiv$ have to be selected carefully, so as to avoid inconsistencies. Figure 2(a) shows a schematic of a $\leq$-hierarchy with two possible alternatives (crossing lines) for separating the upper part of the hierarchy, between which a developer would have to chose.

Step 3: Stratification. With the extended locations entered in the $\leq$-graph, a stratification of the domain of sizes can be derived. Figure 2 illustrates two possible options for stratifying a containment hierarchy with three size levels: in (b), four non-overlapping levels of granularity were generated, whereas, in (c), three overlapping levels of granularity were chosen. A non-overlapping stratification has the advantage of providing smaller strata; an overlapping stratification allows for modeling smooth transitions between levels of granularity [29].

Step 4: Modification. A difficult problem in software development is how to ascertain a sufficient flexibility of used representations, so that later refactoring can be avoided or kept to a minimum. The proposed formal structure supports this effort in so far as results from previous steps are not affected when the structure is changed: a change in the stratification (step 3) does not entail redevelopments at earlier steps. In fact, new strata of granularity can simply be added to an existing granularity structure, because the strata are not required to partition the domain of sizes. Likewise, adding new fixed regions (step 1) or mobile sensors (step 2) requires only local updating in the $\leq$-graph.

6 Outlook and Conclusions

This article presented a formal comparison of perceptual and sensory vagueness and a characterization of granularity applied to the domain of spatial contexts. The characterizations were used to model central aspects of context-awareness: uncertainty of contextual information was modeled using the notion of indistinguishability; context-dependent relevance was represented with the concept of granularity. Granularity provides the notion of grain-size – determining the smallest represented details – as well as the notion of context location – specifying a maximal range of proximity and the unfocussed background of current interactions with a user.

The proposed spatial framework can be used for reasoning as well as for specification purposes: as a characterization of space as obtained from sensors in a heterogeneous ubiquitous computing environment, it can be employed in specifying, checking, and proving availability and reliability of spatially distributed services, e.g., for safety critical applications; as a logical language for describing spatial layouts, it can be used for reasoning, e.g., in tools for diagnosis and automatic configuration of ubiquitous computing environments.

With respect to semantic web applications and related developments, the proposed language can be employed for describing the local spatial ontology of a ubiquitous computing environment, so that the configuration of an environment can be communicated to a user's devices. Both availability of services in the environment and necessary devices on the user's side contribute to the actual spatial configuration of which the user's mobile devices and services in the environment can make use.

Future work includes the application of the specified concepts and methodology for defining a flexible and extensible ontology for Ubiquitous Smart Spaces. This ontology has to incorporate a broad variety of different location sensing technologies, such as: a stationary IR-based sensor system for location and orientation tracking [19], and a computer vision based system for gesture recognition [17].

Nevertheless, application of the proposed framework is not restricted to modeling spatial context. The mereotopological basis is neutral with respect to dimensionality, so that spaces of arbitrary dimensionality can be represented. The formalization can be applied for any context modeling domain that uses both coordinate-based information with fixed resolutions and symbolic information in a containment hierarchy.

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A Proof: Definition D5 Defines a Semiorder on *CL*-Locations

$$\forall c : CL(c) \rightarrow \neg c \prec c \tag{2}$$

$$\begin{aligned}\forall c_1, c_1', c_2, c_2' : CL(c_1) \wedge CL(c_1') \wedge CL(c_2) \wedge CL(c_2') \wedge c_1 \prec c_1' \wedge c_2 \prec c_2' \\ \rightarrow c_1 \prec c_2' \vee c_2 \prec c_1'\end{aligned} \tag{3}$$

$$\begin{aligned}\forall c_1, c_2, c_3, c : CL(c) \wedge CL(c_1) \wedge CL(c_2) \wedge CL(c_3) \wedge c_1 \prec c_2 \wedge c_2 \prec c_3 \\ \rightarrow c_1 \prec c \vee c \prec c_3\end{aligned} \tag{4}$$

$$\forall c, c' : CL(c) \wedge CL(c') \rightarrow (c \approx c' \leftrightarrow \neg(c \prec c' \vee c' \prec c)) \tag{5}$$

Irreflexivity (2) follows by transitivity (A8) from the irreflexivity of $<$.

Proof (3). Assume four *CL*-locations c_1, c_1', c_2, c_2' with $c_1 \prec c_1'$, $c_2 \prec c_2'$, and $\neg c_1 \prec c_2'$ given. Then by (D5), there are grain locations g_1' of c_1' and g_2' of c_2' with $c_1 < g_1'$ and $c_2 < g_2'$. With the third condition $\neg c_1 \prec c_2'$, we know that no grain of c_2' is larger than c_1, and thus particularly $\neg c_1 < g_2'$. By linearity of $\leq$ (A10) on extended locations and (D4) follows $g_2' \leq c_1$. Using transitivity (A8) the following order can be inferred: $c_2 < g_2' < g_1'$. And thus (D5): $c_2 \prec c_1'$.

Proof (4). Assume four *CL*-locations c_1, c_2, c_3, c with $c_1 \prec c_2$, $c_2 \prec c_3$. Then there exist grain locations g_2 of c_2 and g_3 of c_3 with $c_1 < g_2$ and $c_2 < g_3$ (D5). Using linearity (A10), we know that $c \leq c_2$ or $c_2 \leq c$ holds.

For the case $c \leq c_2$, we infer from $c_2 < g_3$ by transitivity (A8) that $c < g_3$ and thus $c \prec c_3$.

For the case $c_2 \leq c$, we infer by (A13) and the requirement that c be a *CL*-location (D7), that c has a grain location g, so that $g_2 \leq g$. By transitivity (A8) and $c_1 < g_2$ this entails that $c_1 < g$ and thus $c_1 \prec c$.

Proof (5). Assume c and c' are *CL*-locations. We then know that they have grain locations g and g', respectively, and that for all such grain locations follows: $g < c$ and $g' < c'$ (A12) and (A9), since by (A13) all grains of a context location have the same size, i.e. behave in the same way with respect to $\leq$. The theorem then follows directly from (D6) and (D5), since $g \leq c'$ and $g' \leq c$ holds iff $\neg c' < g$ and $\neg c < g'$ (D4).