# Stabilization of Multirate Sampled-Data Fuzzy Systems Based on an Approximate Discrete-Time Model

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**Abstract.** This paper studies a stabilization problem for a multirate digital control of fuzzy systems based on the approximately discretized model. In the multirate control scheme, a numerical integration scheme is used to approximately predict the current state from the state measured at the sampling points. It is shown that the multirate digital fuzzy controller stabilizing an approximate discrete-time fuzzy model would also stabilize the sampled-data fuzzy system in the sufficiently small control update time. Furthermore, some sufficient conditions for the stabilization of the approximate discrete-time fuzzy model are provided under the delta-operator frame work, which are expressed as the linear matrix inequalities (LMIs) and thereby easily tractable by the convex optimization techniques. A numerical example is demonstrated to visualize the feasibility of the developed methodology.

## 1 Introduction

Sampled-data systems are widespread more and more because most systems encountered in engineering applications are continuous while controls are implemented digitally using computers. Traditional analysis and design tools for continuous-time or discrete-time systems are unable to be directly used in the sampled-data systems. On way to address the sampled-data control is to develop a discrete-time model for the controlled, continuous-time plant and then pursue a digital controller based on the discretized model. This approach has been basically applicable for linear time-invariant (LTI) systems [8, 9, 12, 13, 14, 15, 16, 10, 11].

The nonlinear sampled-data control problem is difficult because exact discrete-time models of continuous-time processes are typically impossible to compute. From that reasons, there have been some researches focusing on the digital controller [1, 2, 3, 4, 5, 6] for Takagi–Sugeno (T-S) fuzzy systems based on their approximate discrete-time models. Although a great deal of effort has been made on digital control such as [1,2,3,4,5,6], there still exists some matters that must be worked out. The first issue is how to efficiently tackle the stability preservation. It is a very important factor to preserve the stability in the digital controller,

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but the previous methods [1,2,3] do not only assure the stability of the sampled-data fuzzy closed-loop systems but also their approximately discretized model. At this point, the results [4,5,6] provided that sufficient conditions to stabilize the approximate discrete-time model of the sampled-data fuzzy system. However, they only show that the closed-loop sampled-data system is stable under the assumption that there exists no discretization error. Next, a considerable issue is about the multirate digital control. All of the previous results [1,2,3,4,5] are applicable only to a single-rate digital control in which the sampling and the control update periods are assumed to be equal. In practical applications, however, hardware restrictions can make two periods different essentially. There have been some investigations focusing on the multirate digital control of LTI systems from several disparate perspectives [12,13,14,15,16]. However, until now, no tractable method for the multirate digital fuzzy control has been proposed, with perhaps a few exceptions [6].

In this paper, we studies a multirate digital control of fuzzy systems based on the approximate discrete-time model. It is proved that the multirate digital fuzzy controller stabilizing an approximate discrete-time fuzzy model would also stabilize the sampled-data fuzzy system in the sufficiently small control update time. Some sufficient conditions for the stabilization of the approximate discrete-time fuzzy model are provided under the delta-operator framework, which are expressed as the linear matrix inequalities (LMIs) and thereby easily tractable by the convex optimization techniques. Furthermore, we show that the discretized error approach zero as increasing the input multiplicity. From this fact, we can design the digital controller stabilize the sampled-data fuzzy system in the wide range of the sampling period by increasing the input multiplicity.

The rest of this paper is organized as follows: Section 2 briefly reviews the T–S fuzzy system. In Section 3, the stability analysis and control synthesis of the multirate sampled-data fuzzy system is included. An example of a biodynamical system of human immunodeficiency virus type 1 (HIV-1) [21,22] is provided in Section 4. Finally, Section 5 concludes this paper with some discussions.

## 2 Preliminaries

Consider the system described by the following T–S fuzzy model [17,18]:

$$\dot{x}(t) = \sum_{i=1}^{r} \theta_i(z(t))(A_i x(t) + B_i u(t)) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$, $r$ is the number of model rules, $z(t) = [z_1(t) \cdots z_p(t)]^T$ is the premise variable vector that is a function of states $x(t)$, and $\theta_i(z(t))$, $i \in \mathcal{I}_R(= \{1, 2, \cdots, r\})$ is the normalized weight for each rule, that is $\theta_i(z(t)) \geq 0$ and $\sum_{i=1}^{r} \theta_i(z(t)) = 1$.

We consider a multirate digital fuzzy system where $u(t)$ is held in constant between the (uniformly spaced) control update points. Let $T$ and $\tau$ be the sampling and the control update periods, respectively, and assume $\tau = T/N$. The multirate digital fuzzy controller takes the following form:

$$u(t) = \sum_{i=1}^{r} \theta_i(z(kT+\kappa\tau))K_i x(kT+\kappa\tau) \tag{2}$$

for $t \in [kT+\kappa\tau, kT+\kappa\tau+\tau)$, $k \times \kappa \in \mathbb{Z}_{\geqslant 0} \times \mathbb{Z}_{[0,N-1]}$, where $x(kT+\kappa\tau)$, $\kappa \in \mathbb{Z}_{[1,N-1]}$ is predicted from $x(kT)$, and the subscript '$d$' denotes the digital control. By substituting (2) into (1), the closed-loop sampled-data fuzzy system is obtained by

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i(z(t))\theta_j(z(kT+\kappa\tau))(A_i x(t) + B_i K_j x(kT+\kappa\tau)) \tag{3}$$

for $t \in [kT+\kappa\tau, kT+\kappa\tau+\tau)$, $k \times \kappa \in \mathbb{Z}_{\geqslant 0} \times \mathbb{Z}_{[0,N-1]}$. A mixture of the continuous-time and discrete-time signals occurs in the above system (3). It makes traditional analysis tools for a homogeneous signal system unable to be directly used. It is found in [1, 2, 3, 4, 5, 6, 7] that the approximate discrete-time model of (3) takes the following form:

$$\begin{aligned} x(kT+\kappa\tau+\tau) \cong & \sum_{i=1}^{r}\sum_{j=1}^{r} \theta_i(z(kT+\kappa\tau))\theta_j(z(kT+\kappa\tau)) \\ & \times (G_i + H_i K_j)x(kT+\kappa\tau) \end{aligned} \tag{4}$$

where $G_i = \mathrm{e}^{A_i\tau}$ and $H_i = (G_i - I)A_i^{-1}B_i$.

## 3 Main Results

In this section, we show that the multirate digital fuzzy controller (2) stabilizing the approximate discrete-time fuzzy model (4) would also stabilize the multirate sampled-data fuzzy system (3) in the sufficiently small control update time. Furthermore, some sufficient conditions for the stabilization of the approximate discrete-time fuzzy model (4) are provided, which are expressed as the linear matrix inequalities (LMIs).

For the practical engineering approach, we consider the multirate control scheme that utilizes a numerical integration scheme to approximately predict the current state $x(kT+\kappa\tau)$ from the state $x(kT)$ measured at the sampling points, the delayed measurements. For more detail, see [19]. At this point, redefining (4) as $w(kT+\kappa\tau+\tau) \triangleq \mathcal{F}(w(kT+\kappa\tau))$, and rewriting (2) with $x(kT+\kappa\tau)$ replaced by $w(kT+\kappa\tau)$ leads

$$u(t) = \sum_{i=1}^{r} \theta_i(z(kT+\kappa\tau))F_i w(kT+\kappa\tau) \tag{5}$$

for $t \in [kT+\kappa\tau, kT+\kappa\tau+\tau)$, $k \times \kappa \in \mathbb{Z}_{\geqslant 0} \times \mathbb{Z}_{[0,N-1]}$, where $w(kT+\kappa\tau)$ is the approximate estimate of the state $x(kT+\kappa\tau)$ based on the measurements $x(kT)$, and $w(kT+\kappa\tau) = x(kT+\kappa\tau)$ if $\kappa = 0$.

*Remark 1. Under the assumption that the premise variables vector $z(kT+\kappa\tau)$ can be computed from $x(kT+\kappa\tau)$, we can predict $w(kT+\kappa\tau)$ by the following recursive application of* (4) *defined as*

$$\begin{aligned} w(kT+\tau) &\triangleq \mathcal{F}^1(w(kT)) \\ w(kT+2\tau) &= \mathcal{F}(w(kT+\tau)) \\ &= \mathcal{F}(\mathcal{F}^1(w(kT))) \\ &\triangleq \mathcal{F}^2(w(kT)) \\ w(kT+\kappa\tau) &\triangleq \mathcal{F}^\kappa(w(kT)). \end{aligned}$$

Redefining (1) as $\dot{x}(t) \triangleq f(x(t), u(t))$, and substituting (5) into (1) leads

$$\dot{x}(t) = f(x(t), w(kT+\kappa\tau)) \tag{6}$$

for $t \in [kT+\kappa\tau, kT+\kappa\tau+\tau)$, $k \times \kappa \in \mathbb{Z}_{\geqslant 0} \times \mathbb{Z}_{[0,N-1]}$.

Now, we show that the sampled-data system (6) is also asymptotically stable in the sufficiently small $\tau$ if the approximate discrete-time model (4) is asymptotically stable, which needs the following lemmas.

**Lemma 1.** *Let $f(x,u)$ be locally Lipschitz in their arguments. The exact discrete-time model of* (6) *takes the following form:*

$$\begin{aligned} x(kT+\kappa\tau+\tau) = &\ \mathcal{F}(x(kT+\kappa\tau), w(kT+\kappa\tau)) \\ &+ \tau^2\mathcal{E}(x(kT+\kappa\tau), w(kT+\kappa\tau)) \end{aligned} \tag{7}$$

*Proof.* The proof is omitted due to lack of space.

**Lemma 2.** *Let $\mathcal{F}(x,u)$ be locally Lipschitz in their arguments. Suppose that $\|\mathcal{E}(x(kT+\kappa\tau), u(kT+\kappa\tau))\| \leqslant \delta$ for some $\delta$. Then,*

$$\|x(kT+\kappa\tau) - w(kT+\kappa\tau)\| \leqslant \frac{L_2^\kappa - 1}{L_2 - 1}\tau^2\delta \tag{8}$$

*for any $k \times \kappa \in \mathbb{Z}_{\geqslant 0} \times \mathbb{Z}_{[0,N-1]}$.*

*Proof.* The proof is omitted due to lack of space.

*Remark 2. Note that the norm of the discretization error, $\|x(kT+\kappa_0\tau+\tau) - w(kT+\kappa_0\tau+\tau)\|$ will go to zero as $\tau$ approaches zero. Hence, the approximate discrete-time model can preserve the property and structure of* (6) *by increasing $N$.*

**Theorem 1.** *The zero equilibrium $x_{eq} = [0]_{n\times 1}$ of* (6) *is asymptotically stable in the sufficiently small $\tau$ if the zero equilibrium $w_{eq} = [0]_{n\times 1}$ of the approximate discrete-time model* (4) *is asymptotically stable.*

*Proof.* The proof is omitted due to lack of space.

We now are in position to find some sufficient conditions such that the approximate discrete-time fuzzy model (4) is globally asymptotically stable in the sense of Lyapunov.

*Remark 3.* It is easy to see that $G_i \to I$ and $H_i \to \left[0\right]_{n\times m}$ as $\tau \to 0$, which signifies the eigenvalues of $G_i + H_iK_j$ gathers around one thereby weakens the numerical robustness of the related convex optimization problem.

To effectively tackle this problem, stability analysis technique based on the delta-operator is applied in this paper.

*Remark 4.* It has been shown that the delta-operator offers advantages over the shift operator, in terms of numerical robustness, i.e., lower coefficient sensitivity especially when the eigenvalues of a shift-operator-based discretized model are clustered around one, which corresponds to a fast sampling of the continuous-time representation of systems.

**Theorem 2.** *The system* (4) *is stabilizable by the controller* (2) *in the sufficiently small* $\tau$ *if there exist a matrix* $Q = Q^T \succ 0$ *and matrices* $X_{ij} = X_{ij}^T = X_{ji} = X_{ji}^T$, $M_i$ *such that*

$$\begin{bmatrix} \left(\frac{QG_{\delta i}^T + M_j^T H_{\delta i}^T + QG_{\delta j}^T + M_i^T H_{\delta j}^T + G_{\delta i}Q + H_{\delta i}M_j + G_{\delta j}Q + H_{\delta j}M_i}{2}\right) + X_{ij} & (\bullet)^T \\ \left(\frac{\tau^{\frac{1}{2}} G_{\delta i}Q + \tau^{\frac{1}{2}} H_{\delta i}M_j + \tau^{\frac{1}{2}} G_{\delta j}Q + \tau^{\frac{1}{2}} H_{\delta j}M_i}{2}\right) & -Q \end{bmatrix} \prec 0 \tag{9}$$

$$\left[X_{ij}\right]_{r\times r} \succ 0, \quad 1 \leqslant i \leqslant j \leqslant r \tag{10}$$

*where* $(\bullet)^T$ *denotes the transposed element in symmetric positions.*

*Proof.* The proof is omitted due to lack of space.

*Remark 5.* The methodology in Theorems 2 for the state-feedback control can readily be modified to establish results for more general controls, which involve output feedback control, set-point regulation, robust control [23], and so on.

**Corollary 1.** *If* $\tau \to 0$, *then the following conditions are equivalent:*

*(i) There exist* $Q = Q^T \succ 0$ *and matrices* $X_{ij} = X_{ij}^T = X_{ji} = X_{ji}^T$, $M_i$ *such that LMIs* (9) *and* (10)*of Theorem 2.*
*(ii) There exist* $Q = Q^T \succ 0$ *and matrices* $X_{ij} = X_{ij}^T = X_{ji} = X_{ji}^T$, $M_i$ *such that*

$$\begin{aligned} &QA_{\delta i}^T + M_j^T B_{\delta i}^T + QA_{\delta j}^T + M_i^T B_{\delta j}^T \\ &\quad + A_iQ + B_{\delta i}M_j + A_{\delta j}Q + B_{\delta j}M_i + 2X_{ij} \prec 0 \end{aligned} \tag{11}$$

$$\left[X_{ij}\right]_{r\times r} \succ 0, \quad 1 \leqslant i \leqslant j \leqslant r \tag{12}$$

*Proof.* The proof is omitted due to lack of space.

*Remark 6.* Note that LMIs (11) and (12) is readily derived from the a continuous-time Lyapunov stability theorem by choosing $V = x(t)^T Px(t)$, and denoting $Q = P^{-1}$ and $K_i = M_i$. Hence, we conclude that the condition (i) of Corollary 1 converges a stabilizability condition [20] for the continuous-time fuzzy system as $\tau \to 0$.

## 4 Computer Simulations

We present in this section a numerical application in order to show the effectiveness of our approach. We wish to deign the multirate digital fuzzy controller (2) with $N = 5$ for the complex nonlinear systems. The comparisons of the recent method presented in [4] are provided.

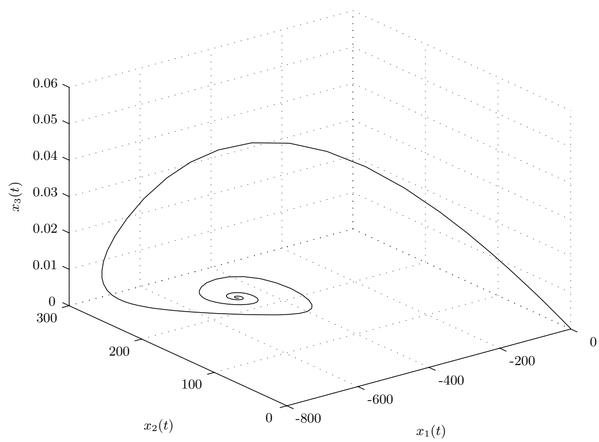


**Fig. 1.** Uncontrolled trajectory of the HIV-1 system

A biodynamic model of HIV-1 [21, 22] is given by

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \\ \dot{x_3}(t) \end{bmatrix} = \begin{bmatrix} -a_1 - a_2 x_3(t) & 0 & -a_2 b_1 \\ 0 & -a_3 + a_4 x_3(t) & a_4 b_2 \\ a_5 x_3(t) & -a_6 x_3(t) & a_5 b_1 - a_6 b_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) \tag{13}$$

where $a_1 = 0.25$, $a_2 = 50$, $a_3 = 0.25$, $a_4 = 10.0$, $a_5 = 0.01$, $a_6 = 0.006$, $b_1 = 1000$ cells/mm$^3$, and $b_2 = 550$ cells/mm$^3$. As discussed in [21, 22], the HIV-1 system has two equilibrium points, where the desired equilibrium point is the origin. Fig. 1 shows the uncontrolled trajectory of this system. The initial conditions are $x_1 = x_2 = 0$ cells/mm$^3$ and $x_3 = 10^{-4}$ (corresponding ponding to k copies/ml).

To facilitate control design, we proceed to construct two-rule fuzzy model of HIV-1 system (13). To this end, the nonlinear terms $x_3 x_1$ and $x_3 x_2$ should be expressed as

$$x_3(t)x_1(t) = \theta_1(x_3(t)) \cdot x_{3min}x_1(t) + \theta_2(x_3(t)) \cdot x_{3max}x_1(t) \tag{14}$$

$$x_3(t)x_2(t) = \theta_1(x_3(t)) \cdot x_{3min}x_2(t) + \theta_2(x_3(t)) \cdot x_{3max}x_2(t) \tag{15}$$

where $\theta_1(x_3(t)) + \theta_2(x_3(t)) = 1$ and $x \in [x_{3min}, x_{3max}]$. Here, we can reasonably determine $[x_{3min}, x_{3max}]$ as $[-0.006, 0.006]$. Solving (14) or (15) for $\theta_1$ and $\theta_2$, and then using (14) and (15) to rewrite (13) as two-rule fuzzy model, we end up with

$$\dot{x}(t) = \sum_{i=1}^{2} \theta_i(x_3(t))(A_i x(t) + B_i u(t)) \tag{16}$$

where $\theta_1(x_3(t)) = \frac{-x_3(t)+x_{3max}}{x_{3max}-x_{3min}}$ and $\theta_2(x_3(t)) = \frac{x_3(t)-x_{3min}}{x_{3max}-x_{3min}}$, and the local system and input matrices are

$$\left[\begin{array}{c|c} A_1 & B_1 \\ \hline A_2 & B_2 \end{array}\right] = \left[\begin{array}{ccc|c} -a_1 - a_2 x_{3min} & 0 & -a_2 b_1 & 0 \\ 0 & -a_3 + a_4 x_{3min} & a_4 b_2 & 0 \\ a_5 x_{3min} & -a_6 x_{3min} & a_5 b_1 - a_6 b_2 & 1 \\ \hline -a_1 - a_2 x_{3max} & 0 & -a_2 b_1 & 0 \\ 0 & -a_3 + a_4 x_{3max} & a_4 b_2 & 0 \\ a_5 x_{3max} & -a_6 x_{3max} & a_5 b_1 - a_6 b_2 & 1 \end{array}\right]$$

This fuzzy model exactly represents the biodynamics of the nonlinear HIV-1 system under $x_{3min} \leq x_3 \leq x_{3max}$. Note that the fuzzy model does not has a common $B$, i.e., $B_1 = B_2$. In general, the fuzzy controller design of the common cases is simple. To show the effect of our approach, we consider a more difficult case, i.e., we change $B_2$ as follows:

$$B_2 = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

We first seek to examine the convergence property of Theorem 2 for extremely small enough $T = 10^{-20}$ years. Using Theorem 2, we can find the multirate digital fuzzy gains

$$\left[\begin{array}{c} K_1 \\ \hline K_2 \end{array}\right] = \left[\begin{array}{ccc} 16 & -20 & -28878 \\ \hline 6 & -8 & -13004 \end{array}\right] \tag{17}$$

However, the LMIs given in [4] are infeasible due to the problem in Remark 3.

Next, we choose $T = 0.14$ years as the relatively large sampling time. Solving to Theorem 2 leads the following multirate digital gains:

$$\left[\begin{array}{c} K_1 \\ \hline K_2 \end{array}\right] = \left[\begin{array}{ccc} 0.0052 & -0.0070 & -25.6664 \\ \hline 0.0003 & -0.0010 & -8.4352 \end{array}\right] \tag{18}$$

However, the stability conditions in [4] are not strictly feasible, and then their digital gains are given by

$$\left[\frac{K_1}{K_2}\right] = \left[\frac{0.0008\ \text{-}0.0009\ \text{-}9.9082}{\text{-}0.0002\ \ 0.0001\ \ \text{-}1.2297}\right] \tag{19}$$

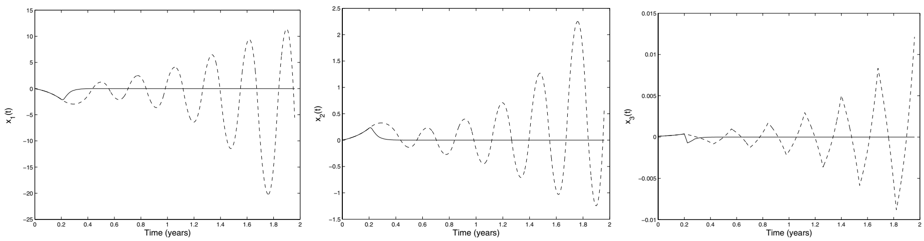


**Fig. 2.** Comparison of state responses of the controlled HIV-1 system (control input is activated at time $t = 0.2$ years): proposed (solid), [4] (dashed). The sampling period is $T = 0.14$ years.

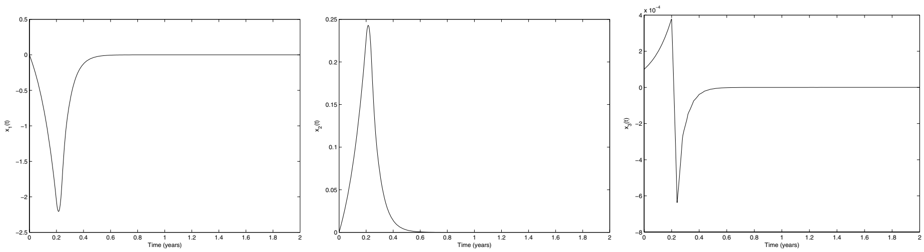


**Fig. 3.** State responses of the controlled HIV-1 system (control input is activated at time $t = 0.2$ years): proposed (solid) The sampling period is $T = 0.2$ years

Figs. 2 shows the time responses of two digitally controlled systems. As shown in the figures, the multirate digital control by the proposed method drives the trajectories to the equilibrium at the origin, while the other method fails to stabilize the system.

Another relatively longer sampling period $T = 0.2$ years is chosen. Applying Theorem 2 leads the following multirate digital gains are:

$$\left[\frac{K_1}{K_2}\right] = \left[\frac{0.0028\ -0.0037\ -18.8768}{0.0001\ -0.0005\ \ -5.6706}\right] \tag{20}$$

However, we cannot compute the feasible solution compute from the conditions in [4]. As shown in Fig. 3, the proposed controller well guarantee the stability preservation.

We emphasize that the proposed method guarantees the stability of the multirate sampled-data system in much wider range of sampling period than the previous method in which may fail to stabilize the system especially for relatively longer sampling period. This is because in the proposed method, the intersample behavior between sampling points can be considered, whereas the other approach does not.

## 5 Closing Remarks

In this paper, we have examined that a multirate digital controller that stabilize approximate discrete-time fuzzy model would also stabilize the resulting sampled-data fuzzy system in the sufficiently small control update time. To the authors' best knowledge, the proposed method is noble in several directions by considering: 1) the multirate digital control; 2) the stability of the multirate sample-data fuzzy system; 3) the stability analysis based on the delta operator. The simulation results on the HIV-1 convincingly demonstrated that it is possible to obtain the excellence performance through the proposed method.

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