Stability Analysis and Controller Design of Discrete T-S Fuzzy System*

Jianxiang Yu, Songtao Zhang, and Guang Ren

Marine Engineering College, Dalian Maritime University, Dalian 116026, P.R. China
{yujianxiang, zst0626}@163.com, reng@dlmu.edu.cn

Abstract. For the conservative and difficulty of checking the stability of discrete T-S fuzzy control system with the common Lyapunov function approach and the fuzzy Lyapunov function approach, a fuzzy controller is designed to acquire globally asymptotical stability for discrete fuzzy system with the method of parallel distributed compensation (PDC) after the definition of a piecewise fuzzy Lyapunov function. Then a new sufficient condition to check the stability of closed-loop discrete T-S fuzzy system is proposed and proved. This condition is less conservative and difficult than above approaches. At last, a simulation example shows that the approach is effective.

1 Introduction

Recently, there has been a rapidly growing interest in the stability issues of T-S fuzzy systems. Most of stability conditions in terms of the common Lyapunov function [1] or the fuzzy Lyapunov function [2] are both conservative, since the common positive definite matrix $\mathbf{P}$ should satisfy r (rules' number) Lyapunov inequalities in the former, or a set of local matrices $\mathbf{P}_1, \mathbf{P}_2, \cdots \mathbf{P}_r$ should satisfy r^2 Lyapunov inequalities in the latter.

In order to overcome the shortcoming of the above two conditions, this paper proposes a new sufficient condition to check the stability of closed-loop discrete T-S fuzzy system based on the definition of a discrete piecewise fuzzy Lyapunov function. This condition only needs to satisfy the condition of fuzzy Lyapunov approach in each maximal overlapped-rule group. Therefore, the proposed condition is less conservative and difficult than former two approaches. A fuzzy controller is designed to acquire globally asymptotical stability for discrete fuzzy system with the method of parallel distributed compensation (PDC). A simulation example shows the approach is effective.

2 Main Result

A discrete T-S fuzzy model can be written as follows:

$$R_i : \text{IF } x_1(k) \text{ is } M_1^i, \text{and} \cdots, \text{and } x_n(k) \text{ is } M_n^i, \text{ THEN } \mathbf{X}(k+1) = \mathbf{A}_i \mathbf{X}(k) + \mathbf{B}_i \boldsymbol{u}(k), \tag{1}$$

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where i=$1,2,\cdots,r$, r and n are the numbers of rules and state variables respectively, $\boldsymbol{X}^{\mathrm{T}}(k)=\left[x_1(k),x_2(k),\cdots,x_n(k)\right]$ is the state vector, M_j^i (j=$1,\cdots,n$) is the fuzzy set. By the singleton fuzzifier, the product inference engine and center average defuzzification, the final output of (1) is inferred as:

$$\mathbf{X}(k+1)=\sum_{i=1}^{r}h_i\left(k\right)\mathbf{A}_i\mathbf{X}(k)+\mathbf{B}_i\boldsymbol{u}(k), \tag{2}$$

where $h_i\left(k\right)=\prod_{j=1}^{n}M_j^i\left(\mathbf{X}_j(k)\right)\Big/\sum_{i=1}^{r}\prod_{j=1}^{n}M_j^i\left(\mathbf{X}_j(k)\right)$.

Parallel distributed compensation (PDC) is a simple and natural design technique for a T-S fuzzy model (1). For (1), let $\boldsymbol{K}_i$ denote the state feedback gain of the ith local model, then the global model of a fuzzy controller can be inferred as follows:

$$\boldsymbol{u}(k)=-\sum_{i=1}^{r}h_i(k)\boldsymbol{K}_i\boldsymbol{X}(k) \tag{3}$$

In this paper, all of discussions and results are aimed at the prescribed the concepts of standard fuzzy partition (SFP) and maximal overlapped-rule group (MORG) as [3].
Definition 1: For a fuzzy system described by (2) with SFP inputs, if any of overlapped-rules groups is described as $g_c\left(c=1,2,\cdots,f\right)$, a discrete piecewise fuzzy Lyapunov function is defined as

$$V\left(\mathbf{X}(k)\right)=\mathbf{X}^{\mathrm{T}}(k)\mathbf{P}(k)\mathbf{X}(k),\ \ \mathbf{P}(k)=\sum_{c=1}^{f}\lambda_c\mathbf{P}_c(k), \tag{4}$$

where $\lambda_c\left(\mathbf{X}(k)\right)=\begin{cases}1 & \mathbf{X}(k)\in g_c\\ 0 & \mathbf{X}(k)\notin g_c\end{cases}$, $\sum_{c=1}^{f}\lambda_c\left(\mathbf{X}(k)\right)=1$, $\mathbf{P}_c(k)=\sum_{i\in L_c}h_i(k)\mathbf{P}_i$, f denotes the number of overlapped-rules groups, and L_c ={the sequence numbers of rules included in g_c }.

Theorem 1. For a fuzzy control system described by (2) and (3), if the input variables adopt SFPs, and let $\boldsymbol{G}_{ik}=\mathbf{A}_i-\mathbf{B}_i\boldsymbol{K}_k$, then the equilibrium of the closed-loop fuzzy control system is asymptotically stable in the large if there exist positive definite matrices $\mathbf{P}_i$ (or $\mathbf{P}_l$) in each MORG such that

$$\boldsymbol{G}_{ik}^{\mathrm{T}}\mathbf{P}_l\boldsymbol{G}_{ik}-\mathbf{P}_i<0\ ,\ i,k,l\in \{\text{the sequence numbers of rules included in } G_q\}, \tag{5}$$

where G_q denotes the qth MORG, q=$1,2,\cdots,\prod_{j=1}^{n}\left(m_j-1\right)$, and m_j denotes the number of fuzzy partition of the jth input variable.

Proof. The proof is similar to the proof of Theorem 4 in [3].

3 Numerical Example

In this section, a two-dimensional mapping Henon is chosen to illustrate the stability examination and the controller design for a T-S system in detail. The system state equation of the two-dimensional mapping Henon is following:

$$x(k+2)=1+bx(k)-ax^2(k+1). \tag{6}$$

When $a=1$ and $b=-3$ the system takes on a chaos state[4].

Impose a force on the two-dimensional mapping Henon, then equation (7) is following:

$$x(k+2)=1+bx(k)-ax^2(k+1)+u. \tag{7}$$

In order to model (7), a T-S fuzzy system is considered as follows:

$$R_i: \text{IF } x_1(k) \text{ is } M_1^i \text{ and } x_2(k) \text{ is } M_2^i \text{ THEN } \boldsymbol{X}(k+1)=\mathbf{A}_i\boldsymbol{X}(k)+\mathbf{B}_i\boldsymbol{u} \tag{8}$$

where $i=1,2,\cdots,9$. Let $x_1(k)=x(k)$ and $x_2(k)=x_1(k+1)$. The fuzzy partitions of $x_1(k)$ and $x_2(k)$ shown in Fig. 1 are $F_1^t(x_1(k))$ and $F_2^s(x_2(k))$ $(t,s=1,2,3)$ respectively, and conform to the conditions of SFP, and

$$M_1^1=M_1^2=M_1^3=F_1^1,\ M_1^4=M_1^5=M_1^6=F_1^2,\ M_1^7=M_1^8=M_1^9=F_1^3,$$
$$M_2^1=M_2^4=M_2^7=F_2^1,\ M_2^2=M_2^5=M_2^8=F_2^2,\ M_2^3=M_2^6=M_2^9=F_2^3.$$

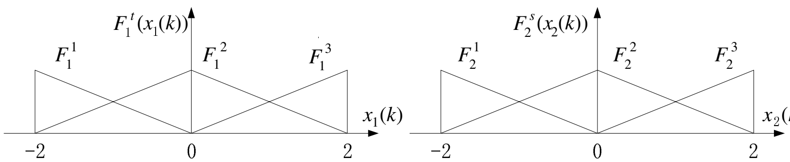

Fig. 1. The fuzzy partitions with $q_1=q_2=3$

We select the closed-loop eigenvalues of the 9 local linear subsystems via state feedback to be: $\mathbf{P}_1=\mathbf{P}_2=\mathbf{P}_3=\mathbf{P}_4=\mathbf{P}_5=\mathbf{P}_6=\mathbf{P}_7=\mathbf{P}_8=\mathbf{P}_9=[0.75 \quad 0.75]$. The state feedback gain of the local linear subsystems can be derived from Ackermann's formula.

We can conclude that this fuzzy system is stable by Theorem 1, for we have found 9 common positive definite matrices in the 4 MORGs satisfying the condition of Theorem 1 via the LMI approach.

We simulate the fuzzy system (7) using various initial conditions. The simulation result shows that this system is stable under all initial conditions. The system state responses under the initial condition of $\boldsymbol{X}_0(k)=[-1 \quad 1]^{\mathrm{T}}$ are shown in Fig. 2.

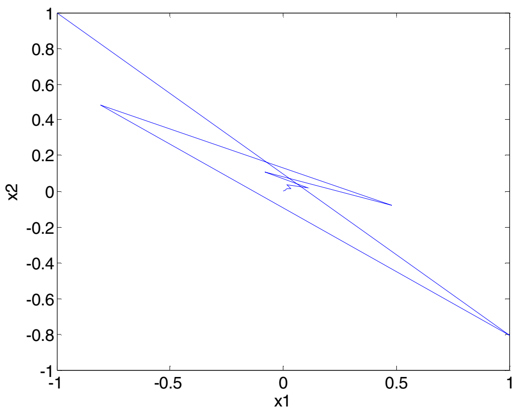

Fig. 2. The system state responses under the initial condition of $X_0(k)=[-1 \quad 1]^T$

4 Conclusions

This paper has contributed to the stability analysis and design of discrete T–S fuzzy control system. Based on the definition of a piecewise fuzzy Lyapunov function, a new stability condition of the closed-loop discrete T-S fuzzy control system is proposed. Our approach only needs to satisfy the condition of fuzzy Lyapunov approach in each MORG. This method can greatly reduce the conservatism and difficulty of the former stability analysis approaches. A design method of a T-S fuzzy controller is proposed by using the method of PDC. The simulation results show that this approach is effective.

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