New Robust Stability Criterion for Uncertain Fuzzy Systems with Fast Time-Varying Delays

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Abstract. This paper considers the robust stability of uncertain T-S fuzzy system with time-varying delay. A new delay-dependent stability condition for the system is derived in terms of LMIs. The constraint on the time-varying delay function is removed, which means that a fast time-varying delay is allowed. Numerical example is given to illustrate the effectiveness and less conservativeness of the developed techniques.

1 Introduction

Recently, people have paid more and more attention on the robust stability of T-S fuzzy system with time-varying delay. Unfortunately, the existing results always assume that the time-varying delay function is continuously differentiable and its derivative is smaller than one, see [1] for example, which is a rigorous constraint. Therefore, it is interesting but challenging to develop the robust stability condition without any constraint on the time-varying delay.

In this paper, the problem of robust stability for uncertain T-S fuzzy system with time-varying delay is investigated. Based on the Lyapunov functional approach and Leibniz-Newton formula, a new delay-dependent criteria is presented in terms of linear matrix inequalities (LMIs) which can be easily solved by efficient interior-point algorithm. The derivative of the time-varying delay function may be larger than one. Numerical example is given to illustrate that the obtained results are less conservative than the existing results in the literature.

2 Problem Formulation

Consider the following uncertain T-S fuzzy system, the ith rule of this T-S fuzzy model is of the following form:

Plant Rule i:

IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_p(t)$ is M_{ip} THEN

$$\dot{x}(t) = (A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))x(t - \tau(t)), \tag{1}$$

$$x(t) = \varphi(t), \quad t \in [-\bar{\tau}, 0], \quad i = 1, 2, \cdots, r, \tag{2}$$

where $z_1(t)$, $z_2(t)$, $\cdots$, $z_p(t)$ are the premise variables, and M_{ij}, $j = 1, 2, \cdots, p$ are fuzzy sets, $x(t)$ is the state variable, r is the number of if-then rules, $\tau(t)$ is

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the time-varying delay satisfies $0 \leq \tau(t) \leq \bar{\tau}$ and $\dot{\tau}(t) \leq d$, $\varphi(t)$ is a vector-valued initial condition. The parametric uncertainties $\Delta A_i(t)$, $\Delta B_i(t)$ are time-varying matrices with appropriate dimensions, which are defined as follows:

$$\Delta A_i(t) = D_{1i}F_{1i}(t)E_{1i}, \quad \Delta B_i(t) = D_{2i}F_{2i}(t)E_{2i}, \quad i = 1, 2, \cdots, r, \tag{3}$$

where D_{1i}, E_{1i}, D_{2i}, E_{2i} are known constant real matrices with appropriate dimensions and $F_{1i}(t)$ and $F_{2i}(t)$ are unknown real time-varying matrices with Lebesgue measurable elements bounded by:

$$F_{1i}^T(t)F_{1i}(t) \leq I, \quad F_{2i}^T(t)F_{2i}(t) \leq I, \quad i = 1, 2, \cdots, r. \tag{4}$$

By fuzzy blending, the overall fuzzy model is inferred as follows:

$$\begin{aligned}\dot{x}(t) &= \frac{\sum_{i=1}^{r} \omega_i(z(t))[(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))x(t-\tau(t))]}{\sum_{i=1}^{r} \omega_i(z(t))} \\ &= \sum_{i=1}^{r} \mu_i(z(t))[\bar{A}_i(t)x(t) + \bar{B}_i(t)x(t-\tau(t))],\end{aligned} \tag{5}$$

with $\omega_i(z(t)) = \prod_{l=1}^{p} M_{il}(z_l(t))$, $\mu_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{r} \omega_i(z(t))}$, $\bar{A}_i(t) = A_i + \Delta A_i(t)$, $\bar{B}_i(t) = B_i + \Delta B_i(t)$, $M_{il}(z_l(t))$ is the membership degree of $z_l(t)$ in M_{il}. It is assumed that $\omega_i(z(t)) \geq 0$, $i = 1, 2, \cdots, r$, $\sum_{i=1}^{r} \omega_i(z(t)) > 0$ for all t, so we have $\mu_i(z(t) \geq 0$ and $\sum_{i=1}^{r} \mu_i(z(t)) = 1$.

Remark 1. In many existing papers, the assumption $\dot{\tau}(t) \leq d < 1$ is needed, see [1] for example, but in this paper, this constraint is not necessary, which means that a fast time-varying delay is allowed.

3 Main Results

In this section, we will derive a delay-dependent robust stability condition for the following uncertain fuzzy system (6).

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} \mu_i(z(t))\bar{A}_i(t)x(t) + \sum_{i=1}^{r} \mu_i(z(t))\bar{B}_i(t)x(t-\tau(t)) \\ &= A(t)x(t) + B(t)x(t-\tau(t))\end{aligned} \tag{6}$$

where $A(t) = \sum_{i=1}^{r} \mu_i(z(t))\bar{A}_i(t), B(t) = \sum_{i=1}^{r} \mu_i(z(t))\bar{B}_i(t)$.

Theorem 1. *The uncertain fuzzy system (6) is robustly asymptotically stable, if there exist symmetric positive definite matrices P, Q, R and real matrices P_1, P_2, N_1, N_2 and scalar $\varepsilon_{1i} > 0$, $\varepsilon_{2i} > 0$, $i = 1, 2, \cdots, r$, such that the following LMI holds:*

$$\Sigma = \begin{bmatrix} \Sigma_{11} & P - P_1^T + A_i^T P_2 & P_1^T B_i - N_1^T + N_2 & -N_1^T & P_1^T D_{1i} & P_1^T D_{2i} \\ \star & -P_2^T - P_2 + \bar{\tau}^2 R & P_2^T B_i & 0 & P_2^T D_{1i} & P_2^T D_{2i} \\ \star & \star & \Sigma_{33} & -N_2^T & 0 & 0 \\ \star & \star & \star & -R & 0 & 0 \\ \star & \star & \star & \star & -\varepsilon_{1i} I & 0 \\ \star & \star & \star & \star & \star & -\varepsilon_{2i} I \end{bmatrix} < 0, \tag{7}$$

where $\Sigma_{11} = P_1^T A_i + A_i^T P_1 + Q + N_1 + N_1^T + \varepsilon_{1i} E_{1i}^T E_{1i}$, $\Sigma_{33} = -(1-d)Q - N_2 - N_2^T + \varepsilon_{2i} E_{2i}^T E_{2i}$.

In all matrices, "$\star$" denotes the symmetric terms in a symmetric matrix

Proof. Choose the following positive definite Lyapunov functional:

$$V(t) = x^T(t)Px(t) + \int_{t-\tau(t)}^{t} x^T(s)Qx(s)ds + \bar{\tau} \int_{-\bar{\tau}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta. \quad (8)$$

Then, taking the derivative of $V(t)$ along the trajectory of system (6), and using the Lemma 1 in [2], we have

$$\begin{aligned}\dot{V}(t) \leq\ & 2x^T(t)P\dot{x}(t) + 2[x^T(t)P_1^T + \dot{x}^T(t)P_2^T] \times [-\dot{x}(t) + A(t)x(t) \\ & + B(t)x(t-\tau(t))] + x^T(t)Qx(t) - (1-d)x^T(t-\tau(t))Qx(t-\tau(t)) \\ & + \bar{\tau}^2 \dot{x}^T(t)R\dot{x}(t) - \left[\int_{t-\tau(t)}^{t} \dot{x}(s)ds\right]^T R \int_{t-\tau(t)}^{t} \dot{x}(s)ds.\end{aligned}$$

Using Leibniz-Newton formula, we have

$$2[x^T(t)N_1^T + x^T(t-\tau(t))N_2^T] \times [x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds] = 0. \quad (9)$$

Then adding up (9) to $\dot{V}(t)$, we have

$$\dot{V}(t) = \eta^T(t) \begin{bmatrix} \Theta_{11} & P - P_1^T + A^T(t)P_2 & P_1^T B(t) - N_1^T + N_2 & -N_1^T \\ \star & -P_2^T - P_2 + \bar{\tau}^2 R & P_2^T B(t) & 0 \\ \star & \star & -(1-d)Q - N_2 - N_2^T & -N_2^T \\ \star & \star & \star & -R \end{bmatrix} \eta(t),$$

where $\Theta_{11} = P_1^T A(t) + A^T(t)P_1 + Q + N_1 + N_1^T$ and $\eta(t) = [x^T(t), \dot{x}^T(t), x^T(t-\tau(t)), (\int_{t-\tau(t)}^{t} \dot{x}(s)ds)^T]^T$.

Multiplying the left and the right sides of Σ by vector $\zeta^T(t)$ and $\zeta(t)$ respectively, we have $\zeta^T(t)\Sigma\zeta(t) < 0$, where

$$\begin{gathered}\zeta(t) = [x^T(t),\ \dot{x}^T(t),\ x^T(t-\tau(t)),\ \int_{t-\tau(t)}^{t} \dot{x}^T(s)ds, \\ x^T(t)E_{1i}^T F_{1i}^T(t),\ x^T(t-\tau(t))E_{2i}^T F_{2i}^T(t)]^T.\end{gathered}$$

Noting that, for any positive scalars $\varepsilon_{1i} > 0$ and $\varepsilon_{2i} > 0$, the following inequalities hold:

$$\begin{aligned}&\varepsilon_{1i}[F_{1i}(t)E_{1i}x(t)]^T[F_{1i}(t)E_{1i}x(t)] \leq \varepsilon_{1i}x^T(t)E_{1i}^T E_{1i}x(t), \\ &\varepsilon_{2i}[F_{2i}(t)E_{2i}x(t-\tau(t))]^T[F_{2i}(t)E_{2i}x(t-\tau(t))] \\ &\qquad \leq \varepsilon_{2i}x^T(t-\tau(t))E_{2i}^T E_{2i}x(t-\tau(t)).\end{aligned}$$

Based on the above two inequalities, we obtain $\eta^T(t)\Upsilon^{(i)}\eta(t) < 0$, where

$$\Upsilon^{(i)} = \begin{bmatrix} \Upsilon_{11}^{(i)} & P - P_1^T + \bar{A}_i^T(t)P_2 & P_1^T\bar{B}_i(t) - N_1^T + N_2 & -N_1^T \\ \star & -P_2^T - P_2 + \bar{\tau}^2 R & P_2^T\bar{B}_i(t) & 0 \\ \star & \star & -(1-d)Q - N_2 - N_2^T & -N_2^T \\ \star & \star & \star & -R \end{bmatrix} < 0, \quad (10)$$

and $\Upsilon_{11}^{(i)} = P_1^T\bar{A}_i(t) + \bar{A}_i^T(t)P_1 + Q + N_1 + N_1^T$.

Therefore, $\dot{V}(t) = \sum_{i=1}^r \mu_i(z(t))\eta^T(t)\Upsilon^{(i)}\eta(t) < 0$, from Lyapunov stability theorem, we can claim that if (7) holds, then system (6) is asymptotically stable.

4 Numerical Example

In this section, we borrow the example in [1] to illustrate the less conservativeness of our results.

Example 1. Consider the uncertain fuzzy system (6) with parameters:

$$A_1 = \begin{bmatrix} -3.2 & 0.6 \\ 0 & -2.1 \end{bmatrix}, \; B_1 = \begin{bmatrix} 1 & 0.9 \\ 0 & 2 \end{bmatrix}, \; A_2 = \begin{bmatrix} -1 & 0 \\ 1 & -3 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}$$

The membership function for Rule 1 and Rule 2 are

$$M_1(x_1(t)) = \frac{1}{1 + \exp(-2x_1(t))}, \quad M_2(x_1(t)) = 1 - M_1(x_1(t)).$$

When $d = 0$, both Theorem 1 of [3] and Theorem 1 of [4] fail to verify that the system is asymptotically stable, and using Corollary 1 in [1], the upper bound of the time delay is $\bar{\tau}_{\max} = 0.58$, but using Theorem 1 in this paper, we have the upper bound of the time delay is $\bar{\tau}_{\max} = 0.6148$. Obviously, our result is less conservative than that obtained by the method in [1].

5 Conclusions

In this paper, we investigate the robust stability problem for uncertain T-S fuzzy system with time-varying delay. Based on the Lyapunov functional approach, a sufficient condition for the asymptotic stability of the uncertain fuzzy system is obtained. Numerical example illustrates the less conservativeness of our results.

References

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