Observer-Based H_∞ Controller Designs for T-S Fuzzy Systems

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Abstract. H_∞control designs for T-S fuzzy systems have been studied, based on the observers, the systems which composed of the fuzzy observers and the error systems are proposed. Some new sufficient conditions which guarantee the quadratical stability and the existence of the state feedback H_∞ control for the systems are proposed. The condition in the theorem 3 is in the form of a matrix inequality, which is simple and converted to the LMIs that can be solved by using MATLAB.

1 Introduction

T-S(Takagi-Sugeno)[1] fuzzy systems are nonlinear systems described by a set of IF-THEN rules which give a local linear representation of an underlining system. Feng et al. [2] and Cao et al. [3], [4] have proved that the T-S fuzzy system can approximate any continuous functions in a compact set of Rn at any preciseness, and that the method based on linear uncertain system theory can convert the stability analysis of a fuzzy control system to the stability analysis of linear time-varying extreme subsystems. This allows the designers to take advantage of conventional linear system to analyze and design the fuzzy control systems.

H_∞ control has been an attractive research topic since the last decade. Some papers have discussed the H_∞ feedback control for fuzzy systems. They deal with a state feedback control design that requires all system states to be measured. In many cases, this requirement is too restrictive. The existence of state feedback H_∞ control in some papers need to find a common symmetry and positive matrix satisfying the fuzzy subsystems. So the conditions are conservative. A new quadratically stable condition which is simple and relaxed is proposed in [5], and a new observer design for the T-S fuzzy system and two new conditions of the existence of H_∞ control based on the observers are also proposed, which are simple and in the forms of linear matrix inequalities which can be directly solved by using MATLAB. In this paper, a method which is different from that in [5] to deal with the control problems of T-S fuzzy systems is proposed.

The condition is in the form of a matrix inequality which can be converted to the LMIs and solved by using MATLAB, and the condition is relaxed.

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The conditions of the existence of H_∞ control of T-S fuzzy systems are proposed in [6,7], considering the following T-S fuzzy systems:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\xi)(A_i x(t) + B_{1i}\omega(t) + B_{2i}u(t)) \\ z(t) &= \sum_{i=1}^{r} \mu_i(\xi)(C_i x(t) + D_i u(t))\end{aligned} \tag{1}$$

Theorem 1. *For a given constant $\gamma > 0$, if there exist matrices F_i, X, X_{ij}, where X is a positive-definite matrix, X_{ii} are symmetry matrices, $X_{ji} = X_{ij}^T, i = 1, ..., r$, satisfy the following matrix inequalities:*

$$A_i^T X + F_i^T B_{2i}^T X + X A_i + X B_{2i} F_i + \frac{1}{\gamma^2} X B_{1i} B_{1i}^T X < X_{ii} \tag{2}$$

$$\begin{aligned}&A_i^T X + F_j^T B_{2i}^T X + F_i^T B_{2j}^T X + A_j^T X + X A_i + X B_{2i} F_j + X B_{2j} F_i + X A_j \\ &+\tfrac{1}{\gamma^2} X B_{1i} B_{1j}^T X + \tfrac{1}{\gamma^2} X B_{1j} B_{1i}^T X \le X_{ij} + X_{ij}^T, i \ne j\end{aligned} \tag{3}$$

$$H_k = \begin{bmatrix} X_{11} & \cdots X_{1r} & C_1^T + F_k^T D_1^T \\ \vdots & \ddots \vdots & \vdots \\ X_{r1} & \cdots X_{rr} & C_r^T + F_k^T D_r^T \\ C_1 + D_1 F_k & \cdots C_r + D_r F_k & -I \end{bmatrix} < 0, k = 1, ..., r \tag{4}$$

then the state feed-back

$$u(t) = \sum_{j=1}^{r} \mu_j(\xi(t)) F_j x(t) \tag{5}$$

makes the following system stable with the H_∞ performance bound with γ:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\xi)\mu_j(\xi)(A_i x(t) + B_{2i} F_j x(t) + B_{1i}\omega(t)) \\ z(t) &= \sum_{i=1}^{r} \mu_i(\xi)\mu_j(\xi)(C_i + D_i F_j) x(t)\end{aligned} \tag{6}$$

The conditions of existence of H_∞ control in theorem 1 are in the forms of matrix inequalities, but it is restricted by condition (3)(need to consider $i \ne j$), that is, a common symmetry positive matrix satisfying the fuzzy subsystems is still to be find, so the conditions of theorem 1 are still conservative. Based on the theorem 1, new conditions are proposed in [5]:

Theorem 2. *For a given constant $\gamma > 0$, if there exist matrices M_i, Z, Z_{ij}, where Z is a positeve-definite matrix, Z_{ii} are symmetric matrices, $Z_{ji} = Z_{ij}^T, i = 1, ..., r$, satisfy the following LMIs:*

$$Z A_i^T + M_i^T B_{2i}^T + A_i Z + B_{2i} M_i + \frac{1}{\gamma^2} B_{1i} B_{1i}^T < Z_{ii} \tag{7}$$

$$\begin{aligned} &ZA_i^T + M_j^T B_{2i}^T + M_i^T B_{2j}^T + ZA_j^T + A_i Z + B_{2i} M_j + B_{2j} M_i + A_j Z \\ &+ \frac{1}{\gamma^2} B_{1i} B_{1j}^T + \frac{1}{\gamma^2} B_{1j} B_{1i}^T \leq Z_{ij} + Z_{ij}^T, i \neq j \end{aligned} \quad (8)$$

$$H_k = \begin{bmatrix} Z_{11} & \cdots & Z_{1r} & ZC_1^T + M_k^T D_1^T \\ \vdots & \ddots & \vdots & \vdots \\ Z_{r1} & \cdots & Z_{rr} & ZC_r^T + M_k^T D_r^T \\ C_1 Z + D_1 M_k & \cdots & C_r Z + D_r M_k & -I \end{bmatrix} < 0, k = 1, 2, ..., r \quad (9)$$

then the state feed-back (5) makes (6) stable with the H_∞ performance bound with γ.

The conditions of existence of H_∞ control in theorem 2 are in the forms of LMIs, but it is still restricted by (8) (as (3)), so the conditions of theorem 2 are also conservative.

In [8], first, the new systems are given based on the observers, and the error of the systems is considered, then the controllers are designed to obtain the H_∞control performance of the systems. But the conditions need to find a common symmetry and positive matrix P. So the conditions are still conservative.

In [9], the new systems are proposed based on fuzzy performance evaluator(FPE), and the disturbance rejection is added to the FPE, only the control performance of the error systems are considered, that is, the controllers are designed to obtain the H_∞ control performance of the error systems.

The paper is organized as follow: in section 2, we propose the systems based on the observers, and the error of systems is considered at the same time. In section 3, the controllers and the error matrices are designed to make the systems which composed of the observers and the error systems satisfy the given H1 control performance, especially, the condition in the theorem 3 is in the form of a matrix inequality, which is simple and does not need to find a common symmetry and positive definite matrix satisfying each subsystems(in fact, we consider the interactions among the fuzzy subsystems), so the condition is relaxed. In section 4, the designing approaches of the observers are propose. In section 5, An example is present to show the effectiveness of the results. The conclusion was made in section 6.

2 The Stability of T-S Fuzzy Systems

Consider the following T-S fuzzy systems:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\xi)(A_i x(t) + B_{1i}\omega(t) + B_{2i}u(t)) \\ y &= \sum_{i=1}^{r} \mu_i(\xi) C_i x(t) \end{aligned} \quad (10)$$

where $x(t) \in R^n$ is the state variable, $y(t) \in R^q$ is the output variable, $\omega(t) \in R^l$ is the disturbance variable, $u(t) \in R^m$ is the input variable, $A_i \in R^{n\times n}, B_{1i} \in R^{n\times l}, B_{2i} \in R^{n\times m}andC_i \in R^{q\times n}$ are premise variables. It is assumed that the premise variables do not depend on the control and disturbance variables. Where $\mu_i(\xi(t)) = \frac{\beta_i(\xi(t))}{\sum_{j=1}^{r} \beta_j(\xi(t))}$,$\beta_i(\xi(t)) = \prod_{i=1}^{p} M_{ij}(\xi(t))$,$M_{ij}(\cdot)$is the membership function of the fuzzy setM_{ij}. Obviously, we have $\sum_{i=1}^{r} \mu_i(\xi(t)) = 1$,$\mu_i(\xi(t)) > 0, i = 1, ..., r, \forall t$.

Definition 1. *For (10), when $\omega(t) \equiv 0, u(t) \equiv 0$, if there exist $\alpha > 0$ and a symmetry positive definite matrix X such that*

$$\dot{V}(x(t)) \le -\alpha x^T(t)x(t) \tag{11}$$

where $V(x(t)) = x^T(t)Xx(t)$, then (10) is called quadratically stable.

Based on the observers, we can obtain

$$\begin{aligned} \dot{\bar{x}} &= \sum_{i=1}^{r} \mu_i(\xi)(A_i\bar{x} + B_{2i}u + G_i(y - \bar{y})) \\ \bar{y} &= \sum_{i=1}^{r} \mu_i(\xi)C_i\bar{x} \end{aligned} \tag{12}$$

the state feedback is

$$u(t) = \sum_{j=1}^{r} \mu_j(\xi)K_j\bar{x} \tag{13}$$

in (12) and (13), $G_i \in R^{n\times q}, K_i \in R^{m\times n} (i = 1, 2, ..., r)$ are the feedback matrices of output error and the state feedback matrices.

Let $e(t) = x - \bar{x}$ ($e(t)$is the systems' error), then

$$\dot{\bar{x}} = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi)\mu_j(\xi) \left[(A_i + B_{2i}K_j)\bar{x} + G_iC_j\ e(t)\right] \tag{14}$$

$$\dot{e}(t) = \dot{x} - \dot{\bar{x}} = \sum_{i=1}^{r} \mu_i(\xi)A_ie - \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi)\mu_j(\xi)G_iC_je + \sum_{i=1}^{r} \mu_i(\xi)B_{1i}\omega \tag{15}$$

and let $\tilde{x} = \begin{pmatrix} \bar{x} \\ e \end{pmatrix}$, $\tilde{A}_{ij} = \begin{pmatrix} A_i + B_{2i}K_j & G_iC_j \\ 0 & A_i - G_iC_j \end{pmatrix}$, $\tilde{B}_{1i} = \begin{pmatrix} 0 & 0 \\ 0 & B_{1i} \end{pmatrix}$, $\tilde{\omega} = \begin{pmatrix} 0 \\ \omega \end{pmatrix}$,then from (14) and (15), we can obtain

$$\dot{\tilde{x}} = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i(\xi)\mu_j(\xi)(\tilde{A}_{ij}\tilde{x} + \tilde{B}_{1i}\tilde{\omega}) \tag{16}$$

Theorem 3. *If there exist matrices $K_i, G_i, i = 1, 2, ..., r$ and a symmetry positive definite matrix X such that*

$$Q = \begin{bmatrix} \varLambda_{11}^T P + P\varLambda_{11} & \ldots & \varLambda_{1r}^T P + P\varLambda_{1r} \\ \ldots & \ddots & \ldots \\ \varLambda_{r1}^T P + P\varLambda_{r1} & \ldots & \varLambda_{rr}^T P + P\varLambda_{rr} \end{bmatrix} < -\alpha I, (\alpha > 0) \tag{17}$$

where $\Lambda_{ii} = \tilde{A}_{ii}, 2\Lambda_{ij} = \tilde{A}_{ij} + \tilde{A}_{ji}, \tilde{A}_{ij} = \begin{pmatrix} A_i + B_{2i}K_j & G_iC_j \\ 0 & A_i - G_iC_j \end{pmatrix}, i,j = 1,2,...,r$ *($\Lambda_{ij} = \Lambda_{ji}$), then the state feedback (13) makes the closed-loop system (16) quadratically stable when* $\omega(t) \equiv 0$.

Proof. Let $\Lambda_{ii} = \tilde{A}_{ii}, 2\Lambda_{ij} = \tilde{A}_{ij} + \tilde{A}_{ji}, \omega(t) \equiv 0(\tilde{\omega}(t) \equiv 0)$, we construct Lyapunov function $V(t) = \tilde{x}^T(t)P\tilde{x}(t)$, then

$$\begin{aligned}
\dot{V}(\tilde{x})) &= \sum_{i}^{r} \mu_i^2(\xi)\tilde{x}^T(\tilde{A}_{ii}^T P + P\tilde{A}_{ii})\tilde{x} \\
&+ 2\sum_{i=1}^{r}\sum_{i<j}^{r} \mu_i(\xi)\mu_j(\xi)\tilde{x}^T((\frac{\tilde{A}_{ij}+\tilde{A}_{ji}}{2})^T P + (\frac{\tilde{A}_{ij}+\tilde{A}_{ji}}{2})P)\tilde{x} \\
&= \begin{pmatrix} \mu_1\tilde{x} \\ \vdots \\ \mu_r\tilde{x} \end{pmatrix}^T \begin{pmatrix} \Lambda_{11}^T P + P\Lambda_{11} & \cdots & \Lambda_{1r}^T P + P\Lambda_{1r} \\ \vdots & \ddots & \vdots \\ \Lambda_{r1}^T P + P\Lambda_{r1} & \cdots & \Lambda_{rr}^T P + P\Lambda_{rr} \end{pmatrix} \begin{pmatrix} \mu_1\tilde{x} \\ \vdots \\ \mu_r\tilde{x} \end{pmatrix} \\
&\le -\alpha\tilde{x}^T(t)\tilde{x}(t)
\end{aligned}$$

□

Corollary 1. *If there exist matrices* $K_i, G_i, i = 1,2,...,r$ *and a symmetry positive definite matrix* X *such that*

$$\begin{bmatrix} \Lambda_{11}^T P + P\Lambda_{11} & ... & \Lambda_{1r}^T P + P\Lambda_{1r} \\ ... & \ddots & ... \\ \Lambda_{r1}^T P + P\Lambda_{r1} & ... & \Lambda_{rr}^T P + P\Lambda_{rr} \end{bmatrix} < 0 \tag{18}$$

where $\Lambda_{ii} = \tilde{A}_{ii}, 2\Lambda_{ij} = \tilde{A}_{ij} + \tilde{A}_{ji}, \tilde{A}_{ij} = \begin{pmatrix} A_i + B_{2i}K_j & G_iC_j \\ 0 & A_i - G_iC_j \end{pmatrix}, i,j = 1,2,...,r$ *($\Lambda_{ij} = \Lambda_{ji}$), then the state feedback (13) makes the closed-loop system (16) quadratically stable when* $\omega(t) \equiv 0$.

3 The H_∞ Controller Based on the Observer

In the closed-loop system (16), $\tilde{\omega}$ is the outside disturbance, it will destroy the robust performance of the control systems, even makes the systems unstable. So we use the H_∞ form to measure the robust performance of the systems, and the controllers $u(t)$ are designed to make the robust performance of the closed-loop system better.

Considering the following H_∞ control performance,

$$\int_0^{t_f} \tilde{x}^T(t)\tilde{Q}\tilde{x}(t)dt \le \tilde{x}^T(0)P\tilde{x}(0) + \rho^2\int_0^{t_f} \tilde{\omega}^T(t)\tilde{\omega}(t)dt \tag{19}$$

where t_f is the control final time, $\rho > 0$ is a constant, $P$and $\tilde{Q}$ is symmetry positive matrix. Our aim is to design fuzzy controllers to make the closed-loop system (16) satisfy the given H_∞ control performance (19). For the closed-loop system (16), we construct Lyapunov function $V(t) = \tilde{x}^T(t)P\tilde{x}(t)$, then we have

$$\dot{V}(t) \le \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi)\mu_j(\xi)\tilde{x}^T(\tilde{A}_{ij}^T P + P\tilde{A}_{ij} + \tfrac{1}{\rho^2}P\tilde{B}_{1i}\tilde{B}_{1i}^T P)\tilde{x} + \rho^2\tilde{\omega}^T\tilde{\omega} \qquad (20)$$

Lemma 1. *For the closed-loop system (16), if there exist a common matrix $P = P^T > 0$ such that*

$$\tilde{A}_{ij}^T P + P\tilde{A}_{ij} + \frac{1}{\rho^2}P\tilde{B}_{1i}\tilde{B}_{1i}^T P + \tilde{Q} < 0, i, j = 1, ..., r \qquad (21)$$

then for the given constant $\rho > 0$, system (16) can obtain the H_∞ control performance (19).

Proof. from (20) and (21), we have $\dot{V}(t) < -\tilde{x}^T Q\tilde{x} + \rho^2\tilde{\omega}^T\tilde{\omega}$, and integrating both sides of the above inequalities from 0 to t_f, we have

$$\int_0^{t_f}\tilde{x}^T(t)\tilde{Q}\tilde{x}(t)dt \le \tilde{x}^T(0)P\tilde{x}(0) + \rho^2\int_0^{t_f}\tilde{\omega}^T(t)\tilde{\omega}(t)dt$$

□

Theorem 4. *For the closed-loop system (16), if there exist a constant $\alpha > 0$ and a common matrix $P = P^T > 0$ such that*

$$\begin{array}{l}\tilde{A}_{ij}^T P + P\tilde{A}_{ij} + \frac{1}{\rho^2}P\tilde{B}_{1i}\tilde{B}_{1i}^T P < -\alpha I \;\; (\alpha > 0) \\ i, j = 1, ..., r\end{array} \qquad (22)$$

then for the given constant $\rho > 0$, system (16) can obtain the H_∞ control performance(19).

Proof. If we let $\tilde{Q} = \tilde{Q}^T > 0, \lambda_{\max}(\tilde{Q}) = \alpha$,then (22) is converted to (21). □

Theorem 5. *For the closed-loop system (16), if there exist $K_i, G_i, i = 1, 2, ..., r$ and $P = P^T > 0$ such that*

$$Q = \begin{bmatrix} Q_{11} & \cdots & Q_{1r} \\ \vdots & \ddots & \vdots \\ Q_{r1} & \cdots & Q_{rr}\end{bmatrix} < 0 \qquad (23)$$

where $Q_{ij} = X\Lambda_{ij}^T + \Lambda_{ij}X + \frac{1}{\rho^2}B_{ij}$, $i, j = 1, 2, ..., r$, $X = P^{-1}$,$\Lambda_{ij} = \frac{\tilde{A}_{ij}+\tilde{A}_{ji}}{2}$,

$$B_{ij} = \frac{\tilde{B}_{1i}\tilde{B}_{1i}^T + \tilde{B}_{1j}\tilde{B}_{1j}^T}{2}, \tilde{B}_{1i} = \begin{pmatrix} 0 & 0 \\ 0 & B_{1i}\end{pmatrix}$$

$$\tilde{A}_{ij} = \begin{pmatrix} A_i + B_{2i}K_j & G_iC_j \\ 0 & A_i - G_iC_j\end{pmatrix} , i, j = 1, 2, ..., r$$

then for the given constant $\rho > 0$, system (16) can obtain the H_∞ control performance(19).

Proof. From (20), we have

$$\dot{V}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi)\mu_j(\xi)\tilde{x}^T(\tilde{A}_{ij}^T P + P\tilde{A}_{ij} + \frac{1}{\rho^2}P\tilde{B}_{1i}\tilde{B}_{1i}^T P)\tilde{x} + \rho^2\tilde{\omega}^T\tilde{\omega}$$
$$= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi)\mu_j(\xi)\tilde{x}^T(\frac{(\tilde{A}_{ij}+\tilde{A}_{ji})^T}{2}P + P\frac{(\tilde{A}_{ij}+\tilde{A}_{ji})}{2})\tilde{x}$$
$$+ \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\xi)\mu_j(\xi)\tilde{x}(\frac{1}{\rho^2}P\frac{\tilde{B}_{1i}\tilde{B}_{1i}^T + \tilde{B}_{1j}\tilde{B}_{1j}^T}{2}P)\tilde{x} + \rho^2\tilde{\omega}^T\tilde{\omega}$$
$$= \begin{pmatrix}\mu_1\tilde{x}\\ \vdots \\ \mu_r\tilde{x}\end{pmatrix}\widetilde{Q}\begin{pmatrix}\mu_1\tilde{x}\\ \vdots \\ \mu_r\tilde{x}\end{pmatrix} + \rho^2\tilde{\omega}^T\tilde{\omega}$$

where $\widetilde{Q} = \begin{bmatrix} \Lambda_{11}^T P + P\Lambda_{11} + \frac{1}{\rho^2}PB_{11}P & \cdots & \Lambda_{1r}^T P + P\Lambda_{1r} + \frac{1}{\rho^2}PB_{1r}P \\ \vdots & \ddots & \vdots \\ \Lambda_{r1}^T P + P\Lambda_{r1} + \frac{1}{\rho^2}PB_{r1}P & \cdots & \Lambda_{rr}^T P + P\Lambda_{rr} + \frac{1}{\rho^2}PB_{rr}P \end{bmatrix}$.

If $\widetilde{Q} < 0$,then from theorem 4 and corollary 2, there exist a symmetry positive matrix $\tilde{Z}$ such that

$$\widetilde{Q} < -\tilde{Z} \tag{24}$$

and for the given constant $\rho > 0$, system (16) can obtain the H_∞ control performance (19). Obviously, $\widetilde{Q} < 0$ is equivalent to the matrix inequality (23). □

4 Design of the H_∞ Controllers

In this section, we will deal with the design of the H_∞ controllers. We let $X = diag\{X_1\ X_2\}$, $X = X^T > 0$, and let

$$N_{ij} = X_1(A_i + A_j)^T + (A_i + A_j)X_1 + (B_{2i}F_j)^T + (B_{2i}F_j) + (B_{2j}F_i)^T + (B_{2j}F_i)$$

$$M_{ij} = X_2(A_i + A_j)^T + (A_i + A_j)X_2 - X_2(G_iC_j + G_jC_i)^T - (G_iC_j + G_jC_i)X_2 + \frac{1}{\rho^2}(B_{1i}B_{1i}^T + B_{1j}B_{1j}^T)$$

$$H_{ij} = (G_iC_j + G_jC_i)X_2$$

(23) is equivalent to the following matrix inequality

$$Q = \begin{pmatrix} \begin{pmatrix} N_{11} & H_{11} \\ H_{11}^T & M_{11} \end{pmatrix} & \cdots & \begin{pmatrix} N_{1r} & H_{1r} \\ H_{1r}^T & M_{1r} \end{pmatrix} \\ \vdots & \ddots & \vdots \\ \begin{pmatrix} N_{r1} & H_{r1} \\ H_{r1}^T & M_{r1} \end{pmatrix} & \cdots & \begin{pmatrix} N_{rr} & H_{rr} \\ H_{rr}^T & M_{rr} \end{pmatrix} \end{pmatrix} < 0 \tag{25}$$

where $K_i = F_i X_1^{-1}, i = 1, 2, ..., r$ is the state feedback matrices of the observers, $G_i, i = 1, 2, ..., r$ is the error feedback matrices. (25) is not LMI. It can not be solved directly by using MATLAB. So we need to quote the following lemma proved in [10]:

Lemma 2. *For the given symmetrical matrix* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ *where the dimension of* S_{11} *is* $r \times r$*. Then the following conditions are equivalent*

(1)$S < 0$;

(2)$S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3)$S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

We first solve the error feedback matrices $G_i, i = 1, ..., r$. From (25) and lemma 2, we have

$$M_{ii} = X_2 A_i^T + A_i X_2 - X_2 C_i^T G_i^T - G_i C_i X_2 + \frac{1}{\rho^2} B_{1i} B_{1i}^T < 0 \tag{26}$$

$$i = 1, 2, ..., r$$

from lemma 2 again, we have

$$\begin{pmatrix} (A_i^T X_3 + X_3 A_i - (H_i C_i)^T - (H_i C_i) & X_3 B_{1i} \\ B_{1i}^T X_3 & -\frac{1}{\rho^2} I \end{pmatrix} < 0, i = 1, 2, ..., r \tag{27}$$

where $X_3 = X_2^{-1}$, now (27) are LMIs. By using MATLAB, we have $X_3, G_i = X_3^{-1} H_i (i = 1, ..., r)$. With $G_i, i = 1, 2, ..., r$ already known, (25) become a LMI, and by using MATLAB, we can obtain $X_1, X_2, K_i = F_i X_1^{-1} (i = 1, ..., r)$. So the H_∞ controllers are obtained.

5 Simulation

We consider the following problem of balbancing an inverted pendulum on a cart. The equations for the pendulum are

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{g \sin(x_1) - amlx_2^2 \sin(2x_1) - a \cos(x_1) u}{\frac{4l}{3} - aml \cos^2(x_1)} + \omega \end{aligned}$$

Where x_1 denotes the angle of the pendulum from the vertical, x_2 is the angular velocity, $g = 9.8 m/s^2$ is the gravity constant, ω is the external disturbance variable which is a sinusoidal signal, $\omega = \sin(2\pi t)$. m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, and u is the force applied to the cart. a=$1/(m + M)$. We choose m=2.0kg, M=8.0kg, $2l$=1.0m in the simulation. The following fuzzy model is used to design state feedback fuzzy controllers:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{2} \mu_i(x(t))(A_i x(t) + B_{1i}\omega(t) + B_{2i}u(t)) \\ y(t) &= \sum_{i=1}^{2} \mu_i(x(t)) C_i x(t) \end{aligned}$$

where

$$A_1 = \begin{bmatrix} 0 & 1.0000 \\ 17.2941 & 0 \end{bmatrix}, B_{21} = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix}, B_{11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_1 = [1, 0],$$

$$A_2 = \begin{bmatrix} 0 & 1.0000 \\ 12.6305 & 0 \end{bmatrix}, B_{22} = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix}, B_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_2 = [1, 0].$$

We use the following membership functions:

$$\mu_1(x_1) = \left(1 - \frac{1}{1 + \exp(-7(x_1 + \frac{\pi}{4})}\right), \mu_2(x_1) = 1 - \mu_1(x_1)$$

let $\rho = 1$, from (27) we have

$$G_1 = [3.2586, 26.9493]^T, G_2 = [3.2586, 22.2857]^T$$

Since $G_i, i = 1, 2$ are already known, (25) is a LMI, using MATLAB we have

$$X_1 = 1.0e + 003 * \begin{bmatrix} 1.1266 & -1.5615 \\ -1.5615 & 2.5377 \end{bmatrix}, X_2 = \begin{bmatrix} 4.3139 & 7.1103 \\ 7.1103 & 44.7897 \end{bmatrix},$$

$$F_1 = 1.0e + 005 * [1.3256, -0.7481], F_2 = 1.0e + 005 * [1.7632, -1.9974],$$

$$K_1 = [522.1859, 291.8400], K_2 = [322.3536, 119.6480].$$

Fig.1 shows the response of system (16) for an initial condition $x_1 = 70°, x_2 = 1$, and $\omega(t) = \sin 2\pi t$.

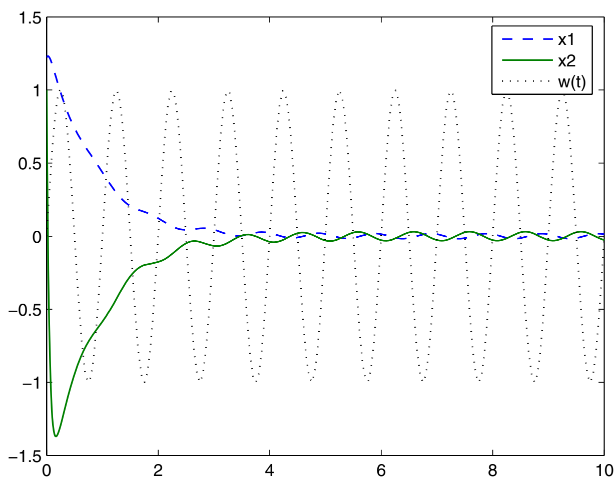

Fig. 1. $\omega(t) = \sin 2\pi t$ initial condition $x_1 = 70°, x_2 = 1$

6 Conclusion

This paper mainly concerns about the systems which are composed of the observers and the error systems. Some new sufficient conditions which guarantee the quadratically stable and the existence of the state feedback H_∞ control for the systems are proposed. The condition in the theorem 3 is in the form of a matrix inequality, which is simple and holistic, and the condition is relaxed. Finally, the condition is converted to the LMIs which can be solved by using MATLAB.

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