Binary Relation Based Rough Sets*

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Abstract. Rough set theory has been proposed by Pawlak as a tool for dealing with the vagueness and granularity in information systems. The core concepts of classical rough sets are lower and upper approximations based on equivalence relations. This paper studies arbitrary binary relation based generalized rough sets. In this setting, a binary relation can generate a lower approximation operation and an upper approximation operation. We prove that such a binary relation is unique, since two different binary relations will generate two different lower approximation operations and two different upper approximation operations. This paper also explores the relationships between the lower or upper approximation operation generated by the intersection of two binary relations and those generated by these two binary relations, respectively.

Keyword: Rough set, Lower approximation, Upper approximation, Binary relation, Fuzzy set, Granular computing.

1 Introduction

At the Internet age, more and more data are being collected and stored, thus, how to extract the useful information from such enormous data becomes an important issue in computer science. In order to cope with this issue, researchers have developed many techniques such as fuzzy set theory [40], rough set theory [18], computing with words [27,41,42,43,44], computational theory for linguistic dynamic systems [28], etc.

Rough set theory has been proposed by Pawlak [18] as a tool to conceptualize, organize and analyse various types of data in data mining. This method is especially useful for dealing with uncertain and vague knowledge in information systems. Many examples of applications of the rough set method to process control, economics, medical diagnosis, biochemistry, environmental science,

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biology, chemistry psychology, conflict analysis and other fields can be found in [1,5,6,7,8,12,13,15,16,17,19,20,21,22,23,26,29,30,31,32,33,34,45,51,52].

The classical rough set theory is based on equivalent relations, but in some situations, equivalent relations are not suitable for coping with the granularity, thus classical rough set method is extended to similarity relation based rough set [10,11,25], covering based rough sets [2,48,49,50], etc [4].

Papers [3,9,35,36,37,38,39] have done extensive research on binary relation based rough sets. In this paper, we also study general binary relation based rough sets. Our focus is on relationships between two lower approximation operations generated by two binary relations, and relationships between two upper approximation operations generated by two binary relations.

The other parts of this paper are organized as follows: In Section 2, we present the fundamental concepts and properties of the Pawlak's rough set theory, and basic definitions and properties of binary relations. Section 3 discusses binary relation based rough sets in literature. Section 4 is the major contribution of this paper. We explore the relationships between rough set generated by two relations on a universe and claim that two different binary relations will generate two different lower approximation operations and two different upper approximation operations. This paper concludes in section 5.

2 Background

2.1 Fundamentals of the Pawlak's Rough Sets

Let U be a finite set, the domain of discourse, and R an equivalent relation on U. R is generally called an indiscernability relation in rough set theory [18]. R will generate a partition $U/R = \{Y_1, Y_2, \ldots, Y_m\}$ on U where $Y_1, Y_2, \ldots, Y_m$ are the equivalent classes generated by the equivalent relation R, and, in the rough set theory, they are also called elementary sets of R. For any $X \subseteq U$ we can describe X by the elementary sets of R and the two sets

$$R_*(X) = \cup\{Y_i \in U/R | Y_i \subseteq X\}$$
$$R^*(X) = \cup\{Y_i \in U/R | Y_i \cap X \neq \phi\}$$

are called the lower and the upper approximation of X, respectively.

Let ϕ be the empty set, $-X$ the complement of X in U, from the definition of approximation sets, we have the following conclusions about them.

The properties of the Pawlak's rough sets: (1L) $R_*(U) = U$
(1H) $R^*(U) = U$
(2L) $R_*(\phi) = \phi$
(2H) $R^*(\phi) = \phi$
(3L) $R_*(X) \subseteq X$
(3H) $X \subseteq R^*(X)$
(4L) $R_*(X \cap Y) = R_*(X) \cap R_*(Y)$
(4H) $R^*(X \cup Y) = R^*(X) \cup R^*(Y)$
(5L) $R_*(R_*(X)) = R_*(X)$

(5H) $R^*(R^*(X)) = R^*(X)$
(6L) $R_*(-X) = -R^*(X)$
(6H) $R^*(-X) = -R_*(X)$
(7L) $X \subseteq Y \Rightarrow R_*(X) \subseteq R_*(Y)$
(7H) $X \subseteq Y \Rightarrow R^*(X) \subseteq R^*(Y)$
(8L) $R_*(-R_*(X)) = -R_*(X)$
(8H) $R^*(-R^*(X)) = -R^*(X)$
(9L) $\forall K \in U/R, R_*(K) = K$
(9H) $\forall K \in U/R, R^*(K) = K$

The (3L), (4L), and (8L) are characteristic properties for the lower approximation operations [14,46,47], i.e., all other properties of the lower approximation operation can be deduced from these three properties. Correspondingly, (3H), (4H), and (8H) are characteristic properties for the upper approximation operation.

2.2 Relations on a Set

In this subsection, we present some basic concepts and properties of binary relations to be used in this paper. For detailed description and proof of them, please refer to [24].

Definition 1. *(Relations) Let U be a set, $U \times U$ the product set of U and U. Any subset R of $U \times U$ is called a relation on U. For any $(x, y) \in U \times U$, if $(x, y) \in R$, we say x has relation R with y, and denote this relationship as xRy.*

For any $x \in U$, we call the set $\{y \in U | xRy\}$ the right neighborhood of x in R and denote it as $RN_R(x)$.

For any $x \in U$, we call the set $\{y \in U | yRx\}$ the left neighborhood of x in R and denote it as $LN_R(x)$.

When there is no confusion, we omit the lowercase R.

Definition 2. *(Reflexive relations) Let R be a relation on U. If for any $x \in U$, xRx, we say R is reflexive. In another word, If for any $x \in U$, $x \in RN(x)$, R is reflexive.*

Definition 3. *(Symmetric relations) Let R be a relation on U. If for any $x, y \in U$, $xRy \Rightarrow yRx$, we say R is symmetric. In another word, If for any $x, y \in U$, $y \in RN(x) \Rightarrow x \in RN(y)$, R is symmetric.*

Definition 4. *(Transitive relations) Let R be a relation on U. If for any $x, y, z \in U$, xRy, and $yRz \Rightarrow xRz$, we say R is transitive.*

Definition 5. *(Equivalent relations) Let R be a relation on U. If R is reflexive, symmetric, and transitive, we say R is a equivalent relation on U.*

3 Binary Relation Based Generalized Rough Sets

An extensive research on algebraic properties of rough sets based on binary relations can be found in paper [3,9,35,36,37,38,39]. They proved the existence of a certain binary relation for an algebraic operator with special properties, but they did not consider the uniqueness of such a binary relation. Furthermore, we consider the relationships between rough sets generated by the join of two binary relations and rough sets generated by these two binary relations, respectively. We also discuss the above issue for the intersection of two binary relations.

Definition 6. *(Rough set based on a relation [38]) Suppose R is a binary relation on a universe U. A pair of approximation operators, $L(R), H(R) : P(U) \to P(U)$, are defined by:*

$$L(R)(X) = \{x | \forall y,\ xRy \Rightarrow y \in X\}=\{x | RN(x) \subseteq X\},$$
$$H(R)(X) = \{x | \exists y \in X,\ s.t.\ xRy\}=\{x | RN(x) \cap X \neq \phi\}.$$

They are called the lower approximation operation and the upper approximation operation, respectively. The system $(P(U), \cap, \cup, -, L(R), H(R))$ is called a rough set algebra, where $\cap, \cup$, and - are set intersection, union, and complement.

Example 1. (A relation and its lower and upper approximation operations) Let $U = \{a, b, c\}$ and $R = \{(a, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$, then
$RN(\{a\}) = \{a\}$, $RN(\{b\}) = \{b, c\}$, $RN(\{c\}) = \{a, b, c\}$.
$L(R)\{a\} = \{a\}$, $L(R)\{b\} = \{\phi\}$, $L(R)\{c\} = \{\phi\}$,
$L(R)\{a, b\} = \{a\}$, $L(R)\{a, c\} = \{a\}$, $L(R)\{b, c\} = \{b\}$,
$L(R)\{a, b, c\} = \{a, b, c\}$.
$H(R)\{a\} = \{a, c\}$, $H(R)\{b\} = \{b, c\}$, $H(R)\{c\} = \{b, c\}$,
$H(R)\{a, b\} = \{a, b, c\}$, $H(R)\{a, c\} = \{a, b, c\}$, $H(R)\{b, c\} = \{b, c\}$,
$H(R)\{a, b, c\} = \{a, b, c\}$.

Proposition 1. *(Basic properties of lower and upper approximation operations [38]) Let R be a relation on U. $L(R)$ and $H(R)$ satisfy the following properties: $\forall X, Y \subseteq U$,*
(1) $L(R)(U) = U$
(2) $L(R)(X \cap Y) = L(R)(X) \cap L(R)(Y)$
(3) $H(R)(\phi) = \phi$
(4) $H(R)(X \cup Y) = H(R)(X) \cup H(R)(Y)$
(5) $L(R)(-X) = -H(R)(X)$

Proposition 2. *[38] Let R be a relation on U. If operation $L : P(U) \to P(U)$ satisfies the following properties:*
(1)$L(U) = U$ *(2)*$L(X \cap Y) = L(X) \cap (Y)$
then there exists a relation R on U such that $L = L(R)$.

Proposition 3. *[38] Let R be a relation on U. If operations $H : P(U) \to P(U)$ satisfies the following properties:*
(1)$H(\phi) = \phi$ *(2)*$H(X \cup Y) = H(X) \cup H(Y)$
then there exists a relation R on U such that $H = H(R)$.

Proposition 4. *[38] Let U be a set. If an operator $L : P(U) \to P(U)$ satisfies the following properties:*
(1)$L(U) = U$ *(2)*$L(X \cap Y) = L(X) \cap (Y)$ *(3)*$L(X) \subseteq X$
then there exists one reflexive relation R on U such that $L = L(R)$.

Proposition 5. *[38] Let U be a set. If an operator $H : P(U) \to P(U)$ satisfies the following properties:*

(1) $H(\phi) = \phi$ (2) $H(X \cup Y) = H(X) \cup H(Y)$ (3) $X \subseteq H(X)$

then there exists one reflexive relation R on U such that $H = H(R)$.

Proposition 6. *[38] Let U be a set. If an operator $L : P(U) \to P(U)$ satisfies the following properties:*

(1)$L(U) = U$ (2)$L(X \cap Y) = L(X) \cap (Y)$ (3)$L(X) \subseteq L(-L(-X))$

then there exists one symmetric relation R on U such that $L = L(R)$.

Proposition 7. *[38] Let U be a set. If an operator $H : P(U) \to P(U)$ satisfies the following properties:*

(1) $H(\phi) = \phi$ (2) $H(X \cup Y) = H(X) \cup H(Y)$ (3) $H(-H(X)) \subseteq H(-X)$

then there exists one symmetric relation R on U such that $H = H(R)$.

Proposition 8. *[38] Let U be a set. If an operator $L : P(U) \to P(U)$ satisfies the following properties:*

(1)$L(U) = U$ (2)$L(X \cap Y) = L(X) \cap (Y)$ (3)$L(X) \subseteq L(L(X))$

then there exists one transitive relation R on U such that $L = L(R)$.

Proposition 9. *[38] Let U be a set. If an operator $H : P(U) \to P(U)$ satisfies the following properties:*

(1) $H(\phi) = \phi$ (2) $H(X \cup Y) = H(X) \cup H(Y)$ (3) $H(H(X)) \subseteq H(X)$

then there exists one transitive relation R on U such that $H = H(R)$.

4 Uniqueness of Binary Relations to Generate Rough Sets

For two relations R_1 and R_2 on a set U, R_1 and R_2 will generate their respective lower approximation operations and the upper approximation operations. $R_1 \cup R_2$ is also a relation on U, so it will generate its own lower approximation operation and the upper approximation operation. Then, what is the relationships among these lower approximation operation and upper approximation operations? How about the relation $R_1 \cap R_2$? We start to answer these questions. Firstly, we consider the situation for $R_1 \cup R_2$.

Theorem 1. *Let R_1 and R_2 be two relations on U and $X \subseteq U$. $L(R_1 \cup R_2)(X) = L(R_1)(X) \cap L(R_2)(X)$ and $H(R_1 \cup R_2)(X) = H(R_1)(X) \cup H(R_2)(X)$.*

Proof. $\forall X \subseteq U$, $L(R_1 \cup R_2)(X) = \{x | \forall y \in U, x(R_1 \cup R_2)y \Rightarrow y \in X\}$

$= \{x | \forall y \in U, xR_1y \text{ or } xR_2y \Rightarrow y \in X\}$

$= \{x | \forall y \in U, xR_1y \Rightarrow y \in X\} \cap \{x | \forall y \in U, xR_2y \Rightarrow y \in X\}$

$= L(R_1)(X) \cap L(R_2)(X)$.

$H(R_1 \cup R_2)(X) = \{x | \exists y \in X, x(R_1 \cup R_2)y\}$

$= \{x | \exists y \in X, xR_1y \text{ or } xR_2y\}$

$= \{x | \exists y \in X, xR_1y\} \cup \{x | \forall y \in X, xR_2y\}$

$= H(R_1)(X) \cup H(R_2)(X)$.

Proposition 10. *Let R_1 and R_2 are two relations on U. If $R_1 \subseteq R_2$, then $L(R_2) \subseteq L(R_1)$ and $H(R_1) \subseteq H(R_2)$.*

Then, we consider the situation for $R_1 \cap R_2$.

Theorem 2. *Let R_1 and R_2 be two relations on U and $X \subseteq U$. $L(R_1)(X) \cup L(R_2)(X) \subseteq L(R_1 \cap R_2)(X)$ and $H(R_1 \cap R_2)(X) \subseteq H(R_1)(X) \cap H(R_2)(X)$.*

Proof. It is easy to prove this theorem by Proposition 10.

Example 2. (Equalities in Theorem 2 do not hold generally)
Let $U = \{a, b, c\}$, $R_1 = \{(a, a), (a, b), (b, b)\}$, and $R_2 = \{(a, a), (a, c), (c, a), (c, b), (c, c)\}$, we have
$RN_{R_1}(\{a\}) = \{a, b\}$, $RN_{R_1}(\{b\}) = \{b\}$, $RN_{R_1}(\{c\}) = \phi$.
$RN_{R_2}(\{a\}) = \{a, c\}$, $RN_{R_2}(\{b\}) = \phi$, $RN_{R_2}(\{c\}) = \{a, b, c\}$,
$R_1 \cap R_2 = \{(a, a)\}$, and
$RN_{R_1 \cap R_2}(\{a\}) = \{a\}$, $RN_{R_1 \cap R_2}(\{b\}) = \phi$, $RN_{R_1 \cap R_2}(\{c\}) = \phi$.
For $X = \{a\}$ and $Y = \{b\}$, we have
$L(R_1)(X) = \{c\}$, $H(R_1)(Y) = \{a, b\}$,
$L(R_2)(X) = \{b\}$, $H(R_2)(Y) = \{c\}$,
and
$L(R_1 \cap R_2)(X) = \{a, b, c\}$, $H(R_1 \cap R_2)(Y) = \phi$.
Thus, $L(R_1)(X) \cup L(R_2)(X) \subset L(R_1 \cap R_2)(X)$ and
$H(R_1 \cap R_2)(Y) \subset H(R_1)(Y) \cap H(R_2)(Y)$.

A relation on U will generate a lower approximation operation and an upper approximation operation, then is it possible for two different relations on U to generate the same lower approximation operation and the same upper approximation operation? We start to study this problem.

Proposition 11. *Let R_1 and R_2 are two relations on U. If $H(R_1) \subseteq H(R_2)$, then $R_1 \subseteq R_2$.*

Proof. $\forall x, y \in U$, if $(x, y) \in R_1$, $y \in RN_{R_1}(x)$, $x \in H(R_1)\{y\} \subseteq H(R_2)\{y\}$, so $RN_{R_2}(x) \cap \{y\} \neq \phi$, that means $(x, y) \in R_2$, thus $R_1 \subseteq R_2$.

Corollary 1. *Let R_1 and R_2 are two relations on U. If $H(R_1) = H(R_2)$, then $R_1 = R_2$.*

Theorem 3. *Let R_1 and R_2 are two relations on U. If $H(R_1) = H(R_2)$ if and only if $R_1 = R_2$.*

Proof. It comes from Proposition 10 and Corollary 1.
By the duality between $H(R)$ and $L(R)$, we have the following result about $L(R)$.

Proposition 12. *Let R_1 and R_2 are two relations on U. If $L(R_1) \subseteq L(R_2)$, then $R_2 \subseteq R_1$.*

Corollary 2. *Let R_1 and R_2 are two relations on U. If $L(R_1) = L(R_2)$, then $R_1 = R_2$.*

Theorem 4. *Let R_1 and R_2 are two relations on U. If $L(R_1) = L(R_2)$ if and only if $R_1 = R_2$.*

Theorem 3 and 4 show that two different binary relations will certainly generate two different lower approximation operations and two different lower approximation operations. Recall that Proposition 2 and 3 show an operator on U with two certain properties can be generated by a binary relation, we actually have proved the uniqueness of such a binary relation.

Theorem 5. *Let R be a relation on U. If operation $L : P(U) \to P(U)$ satisfies the following properties:*
(1)$L(U) = U$
(2)$L(X \cap Y) = L(X) \cap L(Y)$
then there exists one and only one relation R on U such that $L = L(R)$.

Theorem 6. *Let R be a relation on U. If operations $H : P(U) \to P(U)$ satisfies the following properties:*
(1)$H(\phi) = \phi$
(2)$H(X \cup Y) = H(X) \cup H(Y)$
then there exists one and only one relation R on U such that $H = H(R)$.

Theorem 7. *Let U be a set. If an operator $L : P(U) \to P(U)$ satisfies the following properties:*
(1)$L(U) = U$
(2)$L(X \cap Y) = L(X) \cap (Y)$
(3)$L(X) \subseteq L(-L(-X))$
then there exists one and only one symmetric relation R on U such that $L = L(R)$.

Proof. It comes from Proposition 2 and Theorem 4.

Theorem 8. *Let U be a set. If an operator $H : P(U) \to P(U)$ satisfies the following properties:*
(1) $H(\phi) = \phi$
(2) $H(X \cup Y) = H(X) \cup H(Y)$
(3) $H(-H(X)) \subseteq H(-X)$
then there exists one and only one symmetric relation R on U such that $H = H(R)$.

Proof. It comes from Proposition 3 and Theorem 3.

Theorem 9. *Let U be a set. If an operator $L : P(U) \to P(U)$ satisfies the following properties:*
(1)$L(U) = U$
(2)$L(X \cap Y) = L(X) \cap (Y)$
(3)$L(X) \subseteq L(L(X))$
then there exists one and only one transitive relation R on U such that $L = L(R)$.

Proof. It comes from Proposition 2 and Theorem 4.

Theorem 10. *Let U be a set. If an operator $H : P(U) \to P(U)$ satisfies the following properties:*
(1) $H(\phi) = \phi$
(2) $H(X \cup Y) = H(X) \cup H(Y)$
(3) $H(H(X)) \subseteq H(X)$
then there exists one and only one transitive relation R on U such that $H = H(R)$.

Proof. It comes from Proposition 3 and Theorem 3.

5 Conclusions

In this paper we have studied relationships between generalized rough sets generated by two binary relations. We proved that two different binary relations will generate two different lower approximation operations and two different upper approximation operations. As for the applications of binary relation based rough sets to knowledge discovery from database, please refer to paper [10,11,25].

We will explore the relationships between binary relation based rough sets and covering based rough sets [48] in our future works. Another future research topic is to apply binary relation based rough set theory to the computational theory for linguistic dynamic systems [28] and security [52].

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