Reliable Control of Fuzzy Descriptor Systems with Time-Varying Delay*

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Abstract. The reliable fuzzy controller design problem of T-S fuzzy descriptor systems with time-varying delay is introduced. Based on linear matrix inequality approach, a less conservative reliable controller design method is presented. The resulting fuzzy control systems are reliable in the sense that asymptotic stability is achieved not only when all control components are operating well, but also in the presence of some component failures. Moreover, the result is extended to the case of observer-based reliable fuzzy control.Two numerical examples are also given to illustrate the design procedures and their effectiveness.

1 Introduction

Reliable control is an effective approach to improve system reliability. The kernel idea of this approach is to design a fixed controller such that the closed-loop can maintain stability and performance, not only when all control components are operational, but also in the case of some admissible control component outages. In the past two decades, reliable control problems have been extensively studied by many researchers [1, 2, 3, 4].

On the other hand, many complex nonlinear systems can be expressed in a certain form of mathematical models locally. Takagi and Sugeno have proposed a fuzzy model to describe the complex systems [5]. In this T-S fuzzy model, local dynamics in different state space regions are represented by local linear systems. The overall model is obtained by 'blending' these linear models through membership functions. As a common belief, the control technique based on the T-S fuzzy model is conceptually simple and effective for the control of complex systems.

As to reliable control of T-S fuzzy systems, progress has been made in most recent years too [6, 7, 8]. In literature [6, 7], reliable controller design are based on a assumption that control component failures are modeled as outages, i.e., when a failure occurs, the signal (in the case of sensors) or the control action (in the case of actuators) simply becomes zero. In [8], a more general failure model

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is adopted for actuator failures, which studied problem that control components being failure to some extent, i.e., the failure coefficients take value in the interval $[0, 1]$. When the actuator is invalid but kept in the admissible area, the controller will stabilizes the system.

In 1999, Tanniguchi and Tanaka et al extended the T-S fuzzy model to descriptor nonlinear systems [9, 10]. They brought the concept of T-S fuzzy descriptor systems forward. But so far, the reliable control problems for T-S fuzzy descriptor system has scarcely been studied.

Time-delays often occur in many dynamic systems, it has been shown that the existence of delays usually becomes the source of instability and deteriorates performance of systems. So, it is worth to study a system with time-delay both theoretically and practicality.

In practical situations, failure of actuators often occurs. Thus, from a safety point as well as a performance point of view, an important requirement is to have a reliable control such that the stability and performance of the closed-loop system can tolerate actuator failures. In this paper, we will consider the reliable control problem of T-S fuzzy descriptor systems with time-delay.

The paper is organized as follows. Firstly, the problem is formulated. In section 3, based on the solvability of LMIs, taking account of affects of all subsystems, which gives the design method of the reliable state feedback controller. In section 4, the result obtained in section 3 is extended to the case of observer-based reliable fuzzy control. In section 5, numerical examples are used to illustrate the results. Finally, concluding remarks are made in section 6.

Notations: Matrix $X > 0$ $(X \geq 0)$ denotes that$X$ is a positive (semi-positive) definite matirx, $A > (\geq)B$ denotes $A - B > (\geq)0$. Symbol I stands for the unit matrix with appropriate dimensions.

2 Problem Formulation and Failure Model

In this section, if the uncertain system parameter information is considered, the nonlinear descriptor system can be presented as an uncertain fuzzy descriptor model with time-varying delay. The ith fuzzy rule is of the following form:

$$\begin{aligned} R_i : \text{IF } z_1 \ \text{ is } \ & N_{i1} \text{ and} \cdots z_p \text{ is } N_{ip} \text{ , THEN} \\ E\dot{x}(t) \ = \ & (A_i + \Delta A_i(t))\, x(t) + (A_{1i} + \Delta A_{1i}(t))\, x(t - \tau(t)) \\ & + (B_i + \Delta B_i(t))\, u(t) \ , \\ y(t) \ = \ & C_i x(t) \ , \\ x(t) \ = \ & \varphi(t) \ , t \in [-\tau_0, 0] \ . \quad i = 1, 2, \ldots, r \ . \end{aligned} \tag{1}$$

Where N_{ij} are the fuzzy sets, $z_1, z_2, \cdots, z_p$ are premise variables. Scalar r is the number of IF-THEN rules. $E \in R^{n\times n}$ may be singular. $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input and $y(t) \in R^l$ is the output. $A_i, A_{1i} \in R^{n\times n}, B_i \in R^{n\times m}, C_i \in R^{l\times n}, i = 1, 2, \cdots, r$. The $\tau(t)$ is the time-varying delay and satisfies $0 < \tau(t) \leq \tau_0 < \infty$, τ_0 is the upper bound of the delay. It is further assumed that $\dot{\tau}(t) \leq \beta < 1$ and β is a known constant. $\varphi(t)$ is initial vector. $\Delta A_i(t), \Delta A_{1i}(t), \Delta B_i(t)$ denote the uncertainties and take the form of $[\Delta A_i(t) \quad \Delta A_{1i}(t) \quad \Delta B_i(t)] = MF(t)[E_i \quad E_{1i} \quad E_{bi}]$, where M, E_i, E_{1i}, E_{bi} are known constant matrices and $F(t)$ is an unknown matrix function, $F(t)$ satisfying $F(t)^T F(t) \leq I$. The fuzzy descriptor model is assumed to be locally regular, i.e., exist $s_i \in C$, such that $\det(s_i E - A_i - \Delta A_i(t)) \neq 0 (i = 1, 2, \cdots, r)$.

By taking a standard fuzzy inference strategy, that is, using a singleton fuzzifier, procedure fuzzy inference and center average defuzzifier, the final fuzzy model of the systems is inferred as follows

$$E\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(z)\left[(A_i + \Delta A_i(t))\,x(t) + (A_{1i} + \Delta A_{1i}(t))\,x(t - \tau(t)) + (B_i + \Delta B_i(t))u(t)\right], \tag{2}$$

where $\lambda_i(z) = \prod_{j=1}^{p} N_{ij}(z_j) / \sum_{i=1}^{r}\prod_{j=1}^{p} N_{ij}(z_j)$, $\sum_{i=1}^{r} \lambda_i(z) = 1$. $N_{ij}(z_j)$ is the grade of membership of z_j in N_{ij}. For simplicity, $x, x_\tau, u, \lambda_i, \Delta A_i, \Delta A_{1i}, \Delta B_i$ will be used instead of $x(t), x(t - \tau(t)), u(t), \lambda_i(z), \Delta A_i(t), \Delta A_{1i}(t), \Delta B_i(t)$, respectively.

Consider a nonlinear descriptor system $E\dot{x} = f(x, u)$, where $E \in R^{n\times n}, x \in R^n, u \in R^m, \det(E) = 0$. The following definition regarding the solvability of the nonlinear descriptor system was given in [11].

Definition 1. *If for any piecewise continuous input u and initial state x_0, there always exists a unique differentiable solution x with $x(0) = x_0$, then system $E\dot{x} = f(x, u)$ is called solvable.*

The purpose of this paper is to design a reliable controller. The analysis is developed under the assumption that the T-S fuzzy descriptor system is solvable.

Lemma 1. [12] *Let $M, E, F(t)$ be real matrices of appropriate dimensions with $F^T(t)F(t) \leq I$. Then for any scalar $\varepsilon > 0$,*

$$MF(t)E + E^T F^T(t)M^T \leq \varepsilon MM^T + \varepsilon^{-1}E^T E\ .$$

3 Reliable Control Via State Feedback for T-S Model

First, the reliable fuzzy controller will be designed to stabilize system (2). The fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts and has local linear controller in the consequent parts. The ith fuzzy rule of the fuzzy controller is of the following form

$$R_i : \text{IF } z_1 \text{ is } N_{i1} \text{ and} \cdots z_p \text{ is } N_{ip}\ , \text{THEN}$$

$$u = K_i x\ ,\ i = 1, 2, \cdots, r\ .$$

For fuzzy reliable control problems, the following actuator fault model is used.

$$R_i : \text{IF } z_1 \text{ is } N_{i1} \text{ and} \cdots z_p \text{ is } N_{ip}\ , \text{THEN}$$

$$u = \Phi_{\omega i} K_i x \ , \ i = 1, 2, \cdots, r \ .$$

Where $K_i, i = 1, 2, \cdots, r$ are the local linear feedback gains. Hence, the overall fuzzy controller is given by

$$u_\omega = \sum_{i=1}^{r} \lambda_i \Phi_{\omega i} K_i x \ , \tag{3}$$

where $\Phi_{\omega i} = \text{diag} \ [\delta_{\omega i}(1), \delta_{\omega i}(2), \cdots, \delta_{\omega i}(m)]$, $\delta_{\omega i}(j) \in [0, 1]$, $i = 1, 2, \cdots, r$, $j = 1, 2, \cdots, m$. Matrix $\Phi_{\omega i}$ describes the fault extent. $\delta_{\omega i}(j) = 0$ means that the jth component in the ith local actuator is invalid, $\delta_{\omega i}(j) \in (0, 1)$ implies that the jth component is at fault in some extent and $\delta_{\omega i}(j) = 1$ denotes that the jth component operates properly. Thus, for a given diagonal matrix $\Phi_{\Omega i}, i = 1, 2, \cdots, m$. the set $\Omega = \{u_\omega = \sum_{i=1}^{r} \lambda_i \Phi_{\omega i} K_i x, \text{and } \Phi_{\omega i} \geq \Phi_{\Omega i}, i = 1, 2, \cdots, m\}$ is called an admissible set of actuator fault. Namely, symbol $\Phi_{\Omega i}, i = 1, 2, \cdots, m$ in set Ω describes the worst status of the scaling factor $\Phi_{\omega i}, i = 1, 2, \cdots m$. Once the scaling factor extent become smaller than $\Phi_{\Omega i}$, the reliable controller can not work properly anymore .

Remark 1. It is obvious that when $\Phi_{\omega i} = \Phi_\omega, i = 1, 2, \cdots, r$, and $\delta_\omega(j)$ takes only the values of 0 and 1, the actuator failure model is just the same as that in [6, 7] and the references cited therein.From this point,the problem to be solved here is more general.

For the case of $u_\omega \in \Omega$, the closed-loop system is given by

$$E\dot{x} = \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i \lambda_j \left[(A_i + \Delta A_i) x + (A_{1i} + \Delta A_{1i}) x_\tau + (B_i + \Delta B_i) \Phi_{\omega j} K_j x \right] \ . \tag{4}$$

Theorem 1. *Consider system (4), if there exist nonsingular matrices $P, S > 0, K_i, X_{ij}$, where $X_{ii} = X_{ii}^T, X_{ij} = X_{ji}^T, i \neq j, (i, j = 1, 2, \cdots, r)$, such that*

$$E^T P = P^T E \geq 0 \ , \tag{5}$$

$$\Psi_{ii} = \begin{bmatrix} \Theta_{ii} & P^T (A_{1i} + \Delta A_{1i}) \\ * & -S \end{bmatrix} < X_{ii} \ , \tag{6}$$

$$\Psi_{ij} = \begin{bmatrix} \Theta_{ij} + \Theta_{ji} & P^T (A_{1i} + \Delta A_{1i} + A_{1j} + \Delta A_{1j}) \\ * & -2S \end{bmatrix} \leq X_{ij} + X_{ij}^T \ , \ i < j, \tag{7}$$

$$X = (X_{ij})_{r \times r} = \begin{bmatrix} X_{11} & \cdots & X_{1r} \\ \vdots & \ddots & \vdots \\ X_{r1} & \cdots & X_{rr} \end{bmatrix} < 0 \ . \tag{8}$$

Where $\Theta_{ij} = P^T[(A_i + \Delta A_i) + (B_i + \Delta B_i)\Phi_{\omega j} K_j] + [(A_i + \Delta A_i) + (B_i + \Delta B_i)\Phi_{\omega j} K_j]^T P + \frac{1}{1-\beta} S$ and "$*$" denotes the transposed elements in the symmetric positions. Then the resultant closed-loop system (4) is asymptotically stable for any $u_\omega \in \Omega$.

Proof. Construct a Lyapunov function as

$$V(t) = x^T E^T P x + \frac{1}{1-\beta}\int_{t-\tau(t)}^{t} x^T(s) S x(s)\, ds \ ,$$

where $E^T P = P^T E \geq 0, S > 0$. Differentiating $V(t)$ along the trajectory of system (4) gives

$$\dot{V} \leq \sum_{i=1}^{r} \lambda_i^2 \xi^T \Psi_{ii} \xi + \sum_{i=1}^{r}\sum_{i<j}^{r} \lambda_i \lambda_j \xi^T \Psi_{ij} \xi \leq \begin{bmatrix} \lambda_1 \xi \\ \vdots \\ \lambda_r \xi \end{bmatrix}^T \begin{bmatrix} X_{11} & \cdots & X_{1r} \\ \vdots & \dots & \vdots \\ X_{r1} & \cdots & X_{rr} \end{bmatrix} \begin{bmatrix} \lambda_1 \xi \\ \vdots \\ \lambda_r \xi \end{bmatrix} < 0 \ ,$$

where $\xi = \begin{bmatrix} x \\ x_\tau \end{bmatrix}$, This completes the proof. □

Remark 2. It is worthwhile to be pointed out that for a system with large dimension, more than two subsystems are often activated at the same time. So, the interactions of subsystems are taken into account in Theorem 1. The method to do this is to introduce the relaxation matrix X into Theorem 1, which was firstly utilized in [13] and improved by [14] .

We will give out the scheme of how to design the reliable controller. The main idea is to convert conditions in Theorem 1 into LMI conditions.

Theorem 2. *Consider system (4), if there exist $\varepsilon_i > 0, \varepsilon_{ij} > 0$, nonsingular matrix X, $Y > 0, Z_{ij}$, where $Z_{ii} = Z_{ii}^T, Z_{ij} = Z_{ji}^T, i \neq j,\ i,j = 1,2,\cdots,r$ such that the following LMIs are satisfied:*

$$X^T E^T = EX \geq 0 \ , \tag{9}$$

$$\begin{bmatrix} \begin{matrix} A_i X + X^T A_i^T - B_i \Phi_{\Omega i} B_i^T \\ -Z_{ii1} + \varepsilon_i M M^T \end{matrix} & A_{1i} Y - Z_{ii2} & 0 & X^T E_i^T & X^T \\ * & -Y - Z_{ii3} & 0 & Y E_{1i}^T & 0 \\ * & * & -I & E_{bi}^T & 0 \\ * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & -(1-\beta) Y \end{bmatrix} < 0 \ , \tag{10}$$

$$\begin{bmatrix} \Omega & (A_{1i} + A_{1j}) Y - Z_{ij2} - Z_{ij3}^T & 0 & X^T (E_i + E_j)^T & X^T \\ * & -2Y - Z_{ij4} - Z_{ij4}^T & 0 & Y (E_{1i} + E_{1j})^T & 0 \\ * & * & -I & [E_{bi} \ \ E_{bj}]^T & 0 \\ * & * & * & -\varepsilon_{ij} I & 0 \\ * & * & * & * & -\frac{(1-\beta)}{2} Y \end{bmatrix} < 0 \ , \tag{11}$$

$$\begin{bmatrix} Z_{11} & \cdots & Z_{1r} \\ \vdots & \ddots & \vdots \\ Z_{r1} & \cdots & Z_{rr} \end{bmatrix} < 0 \ . \tag{12}$$

Where $\Omega = (A_i + A_j) X + X^T (A_i + A_j)^T + 2(B_i - B_j)(B_i - B_j)^T - B_i\Phi_{\Omega j}B_i^T - B_j\Phi_{\Omega i}B_j^T - Z_{ij1} - Z_{ij1}^T + \varepsilon_{ij}MM^T$. *$Z_{ij}$s are partitioned as* $Z_{ii} = \begin{bmatrix} Z_{ii1} & Z_{ii2} \\ * & Z_{ii3} \end{bmatrix}$,$Z_{ij} = \begin{bmatrix} Z_{ij1} & Z_{ij2} \\ Z_{ij3} & Z_{ij4} \end{bmatrix}$. *Then, the control gains are given by* $K_i = -B_i^T X^{-1}, i = 1, 2, \cdots, r$. *and the resultant closed-loop system (4) is asymptotically stable for any* $u_\omega \in \Omega$.

Proof. The proof is omitted because of the limited space. □

4 Observer-Based Reliable Control for T–S Model

In many cases, states are unknown or partly detected. Therefore, it is needed to estimate states. If the controller is designed with the effect of time-delay, the delay must be known exactly. But it is usually impossible to know the delay exactly. Here, we manage to design the state-observer not affected by the delay.

We consider system (2) without uncertainties. Construct the fuzzy observer

$$\begin{aligned} E\dot{\hat{x}} &= \sum_{i=1}^{r} \lambda_i \left[(A_i\hat{x} + B_i u) + L_i (y - \hat{y}) \right] , \\ \hat{y} &= \sum_{i=1}^{r} \lambda_i C_i \hat{x} . \end{aligned} \tag{13}$$

where L_i is the observer gain. Define the estimation error as $e = x - \hat{x}$, then

$$E\dot{e} = E\dot{x} - E\dot{\hat{x}} = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i\lambda_j [\,(A_i - L_iC_j)\,e + A_{1i}x_\tau] .$$

Consider the following fuzzy controller $u = \sum_{i=1}^{r} \lambda_i K_i \hat{x}$. For any actuator failures $u_\omega \in \Omega$, the system can be expressed as follows:

$$\bar{E}\dot{\bar{x}} = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i\lambda_j [\bar{A}_{ij}\bar{x} + \bar{A}_{1i}\bar{x}_\tau + \bar{B}_i\Phi_{\omega j}\bar{K}_j\bar{x}] , \tag{14}$$

where $\bar{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}$, $\bar{A}_{ij} = \begin{bmatrix} A_i & 0 \\ 0 & A_i - L_iC_j \end{bmatrix}$, $\bar{A}_{1i} = \begin{bmatrix} A_{1i} & 0 \\ A_{1i} & 0 \end{bmatrix}$, $\bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$, $\bar{K}_j = \begin{bmatrix} K_j & -K_j \end{bmatrix}$, $\bar{x} = \begin{bmatrix} x \\ e \end{bmatrix}$.

Theorem 3. *Consider system (14), if in Step 1 there exist nonsingular matrix* X_1, P_2, $Y_1 > 0$, $Y_2 > 0$, M_i, $\bar{Z}_{ii11} < 0$, $\tilde{Z}_{ii13} < 0$, $\bar{Z}_{ii31} < 0$, $\bar{Z}_{ii33} < 0$, $\bar{Z}_{ii21}$, $\tilde{Z}_{ii24}$, $\bar{Z}_{ij11}$, $\tilde{Z}_{ij14}$, $\bar{Z}_{ij21}$,, $\tilde{Z}_{ij24}$, $\bar{Z}_{ij31}^T$, $\tilde{Z}_{ij34}^T$, $\bar{Z}_{ij41}$, $\bar{Z}_{ij44}$, *such that*

$$X_1^T E^T = EX_1 \geq 0 \ , \quad E^T P_2 = P_2^T E \geq 0 \ , \tag{15}$$

$$\begin{bmatrix} \begin{array}{c} A_iX_1 + X_1^TA_i^T \\ -2B_i\Phi_{\Omega i}B_i^T - \bar{Z}_{ii11} \end{array} & A_{1i}Y_1 - \bar{Z}_{ii21} & X_1^T \\ * & -Y_1 - \bar{Z}_{ii31} & 0 \\ * & * & -(1-\beta)Y_1 \end{bmatrix} < 0 \ , \tag{16}$$

$$\begin{bmatrix} \begin{array}{l} (A_i + A_j) X_1 + X_1^T (A_i + A_j)^T \\ +2 (B_i - B_j)(B_i - B_j)^T \\ -B_i \Phi_{\Omega j} B_i^T - B_j \Phi_{\Omega i} B_j^T \\ -\bar{Z}_{ij11} - \bar{Z}_{ij11}^T \end{array} & (A_{1i} + A_{1j}) Y_1 - \bar{Z}_{ij21} - \bar{Z}_{ij31}^T & X_1^T \\ * & -2Y_1 - \bar{Z}_{ij41} - \bar{Z}_{ij41}^T & 0 \\ * & * & -\frac{(1-\beta)}{2} Y_1 \end{bmatrix} < 0 \tag{17}$$

$$\begin{bmatrix} \begin{array}{l} P_2^T A_i - M_i C_i \\ + \left(P_2^T A_i - M_i C_i\right)^T - \tilde{Z}_{ii13} \end{array} & -\tilde{Z}_{ii24} & I \\ * & -Y_2 - \bar{Z}_{ii33} & 0 \\ * & * & -(1-\beta) Y_2 \end{bmatrix} < 0 \ , \tag{18}$$

$$\begin{bmatrix} \begin{array}{l} P_2^T (A_i + A_j) - (M_i C_j + M_j C_i) \\ + (A_i + A_j)^T P_2 - (M_i C_j + M_j C_i)^T \\ -\tilde{Z}_{ij14} - \tilde{Z}_{ij14}^T \end{array} & -\tilde{Z}_{ij24} - \tilde{Z}_{ij34}^T & I \\ * & -2Y_2 - \bar{Z}_{ij44} - \bar{Z}_{ij44}^T & 0 \\ * & * & -\frac{(1-\beta)}{2} Y_2 \end{bmatrix} < 0 \ . \tag{19}$$

By computation, $\bar{Z}_{ii13} = P_2^{-T} \tilde{Z}_{ii13} P_2^{-1}, \bar{Z}_{ii24} = P_2^{-T} \tilde{Z}_{ii24}$. $\bar{Z}_{ij14} = P_2^{-T} \tilde{Z}_{ij14} P_2^{-1}$, $\bar{Z}_{ij24} = P_2^{-T} \tilde{Z}_{ij24}, \bar{Z}_{ij34} = P_2^{-T} \tilde{Z}_{ij34}^T$. *And (after solving the LMIs in Step 1) in Step 2 there exist matrices* $\bar{Z}_{ii12}$, $\bar{Z}_{ii22}$, $\bar{Z}_{ii23}$, $\bar{Z}_{ii32}$, $\bar{Z}_{ij12}$, $\bar{Z}_{ij13}$, $\bar{Z}_{ij22}$, $\bar{Z}_{ij23}$, $\bar{Z}_{ij32}$, $\bar{Z}_{ij33}$, $\bar{Z}_{ij42}$, $\bar{Z}_{ij43}$, *satisfying the following LMIs:*

$$Z = [\bar{Z}_{ij}]_{r \times r} = \begin{bmatrix} \bar{Z}_{11} & \cdots & \bar{Z}_{1r} \\ \vdots & \ddots & \vdots \\ \bar{Z}_{r1} & \cdots & \bar{Z}_{rr} \end{bmatrix} < 0 \ , \tag{20}$$

where

$$\bar{Z}_{ii} = \begin{bmatrix} \begin{bmatrix} \bar{Z}_{ii11} & \bar{Z}_{ii12} \\ * & \bar{Z}_{ii13} \end{bmatrix} & \begin{bmatrix} \bar{Z}_{ii21} & \bar{Z}_{ii22} \\ \bar{Z}_{ii23} & \bar{Z}_{ii24} \end{bmatrix} \\ \begin{matrix} * & * \\ * & * \end{matrix} & \begin{bmatrix} \bar{Z}_{ii31} & \bar{Z}_{ii32} \\ * & \bar{Z}_{ii33} \end{bmatrix} \end{bmatrix}, \bar{Z}_{ij} = \begin{bmatrix} \begin{bmatrix} \bar{Z}_{ij11} & \bar{Z}_{ij12} \\ \bar{Z}_{ij13} & \bar{Z}_{ij14} \end{bmatrix} & \begin{bmatrix} \bar{Z}_{ij21} & \bar{Z}_{ij22} \\ \bar{Z}_{ij23} & \bar{Z}_{ij24} \end{bmatrix} \\ \begin{bmatrix} \bar{Z}_{ij31} & \bar{Z}_{ij32} \\ \bar{Z}_{ij33} & \bar{Z}_{ij34} \end{bmatrix} & \begin{bmatrix} \bar{Z}_{ij41} & \bar{Z}_{ij42} \\ \bar{Z}_{ij43} & \bar{Z}_{ij44} \end{bmatrix} \end{bmatrix}.$$

Then control gains are $K_i = -B_i^T X_1^{-1}$, *observer gains are* $L_i = P_2^{-T} M_i$, $i = 1, 2, \cdots, r$. *and the system (14) is asymptotically stable for any* $u_\omega \in \Omega$.

Proof. The proof is omitted because of the limited space. □

Remark 3. It is involved to find the state observer of time delay systems. [15] provided a one-step approach to design observer-based controller for T-S fuzzy system without time delay. However, for a T-S fuzzy system with time-delay, how to obtain a controller with less conservatism in one-step is still open.

5 Numerical Examples

In this section, two examples are employed to illustrate the validity and the effectiveness of the approaches proposed in this paper.

Example 1. Consider the following fuzzy model:

$$R_1 : \text{IF } x_1 \text{ is } P_1, \text{THEN} \qquad R_2 : \text{IF } x_1 \text{ is } P_2, \text{THEN}$$
$$E\dot{x} = (A_1 + \Delta A_1)x + \qquad E\dot{x} = (A_2 + \Delta A_2)x +$$
$$(A_{11} + \Delta A_{11})x_\tau + B_1 u, \qquad (A_{12} + \Delta A_{12})x_\tau + B_2 u.$$

Where the membership function of $'P_1 ',' P_2 '$ are given as following the effectiveness of the method $w_1(x_1) = 1 - \frac{1}{1+e^{-2x_1}}, w_2(x_1) = 1 - w_1(x_1)$. And

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} 2 & 1 \\ -2 & -0.5 \end{bmatrix}, A_{11} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.1 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.5 \end{bmatrix}, B_1 = \begin{bmatrix} 3 \\ 0.1 \end{bmatrix}, B_2 = \begin{bmatrix} 5 \\ 0 \end{bmatrix}, M = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, F(t) = \sin(t),$$

$$E_1 = E_2 = [0.1 \ \ 0], E_{11} = [1 \ \ 0], E_{12} = [0.1 \ \ 0], E_{b1} = 0.1, E_{b2} = 0.$$

Let $\tau(t) = 1 + 0.5\sin t$. Consider $\Phi_{\omega 1} = 0.3 * I, \Phi_{\omega 2} = 0.01 * I$, if utilizing the method proposed in [16] to solve the problem, it is no way to design the controller, because the acurator failure was not taken into account in [16]. However, from Theorem 2, we get $P = \begin{bmatrix} 0.2842 & 0 \\ 0.3674 & 0.0281 \end{bmatrix}$, $S = \begin{bmatrix} 0.0063 & 0 \\ 0 & 0.0001 \end{bmatrix}$, state feedback gains are $K_1 = [-0.8893 \ \ -0.0028], K_2 = [-1.4210 \ \ 0]$.

Example 2. Let us consider the following T-S fuzzy system:

$$R_1 : \text{IF } x_1 \text{ is } N_1, \text{THEN} \qquad R_2 : \text{IF } x_1 \text{ is } N_2, \text{THEN}$$
$$E\dot{x} = A_1 x + A_{11} x_\tau + B_1 u, \qquad E\dot{x} = A_2 x + A_{12} x_\tau + B_2 u,$$
$$y = C_1 x. \qquad y = C_2 x.$$

Where the membership function of $'N_1 ',' N_2 '$ are given as following the effectiveness of the method $w_1(x_1) = 1 - \frac{1}{1+e^{-2x_1}}$, $w_2(x_1) = 1 - w_1(x_1)$. And

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}, A_2 = \begin{bmatrix} -2 & 2 \\ 1 & 2 \end{bmatrix}, A_{11} = A_{12} = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, B_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, C_1 = [1 \ \ 2], C_2 = [0.5 \ \ 1].$$

Firstly, assume that $\Phi_{\Omega 1} = \Phi_{\Omega 2} = I$, that is, there is no any actuator failures occurring. Solving the LMIs (15)–(20) for a standard fuzzy controller produces

$$K_1 = [0.0731 \ \ 0.2175], \qquad K_2 = [0.0094 \ \ 0.2175],$$
$$L_1 = [0.9198 \ \ -0.1160]^T, \ L_2 = [1.0598 \ \ -0.1325]^T.$$

Assume the admissible set of actuator failures is given by

$$\Omega = \left\{ u_\omega = \sum_{i=1}^{2} \lambda_i \Phi_{\omega i} K_i x, \text{and } \Phi_{\omega 1} \geq 0.2 * I, \Phi_{\omega 2} \geq 0.3 * I \right\}.$$

Solving the LMIs (15)–(20) for a reliable fuzzy controller, we get

$$\begin{array}{ll} K_1 = [0.2320 \quad 0.7093], & K_2 = [0.0154 \quad 0.7093], \\ L_1 = [3.1607 \quad -0.3990]^T, & L_2 = [3.6418 \quad -0.4559]^T. \end{array}$$

Simulations were carried out for the delay $\tau(t) = 2$ and the initial conditions $\begin{bmatrix} x_1(0) & x_2(0) \end{bmatrix} = \begin{bmatrix} -1 & 0.5 \end{bmatrix}$. The responses of both design schemes of standard control design and reliable control design for the case without actuator failures are shown in Fig.1. It is obvious that both standard controller and reliable controller guarantee the asymptotic stabilities of the closed-loop system. When actuator failures with $\Phi_{\Omega 1} = 0.2 * I, \Phi_{\Omega 2} = 0.3 * I$ occurred, the state responses for the two cases are shown in Fig.2. It is observed that when actuator failures occur, the closed-loop system with the standard fuzzy controller is not even asymptotically stable, while the closed-loop system using the reliable fuzzy controller still operates well and maintains an acceptable level of performance.

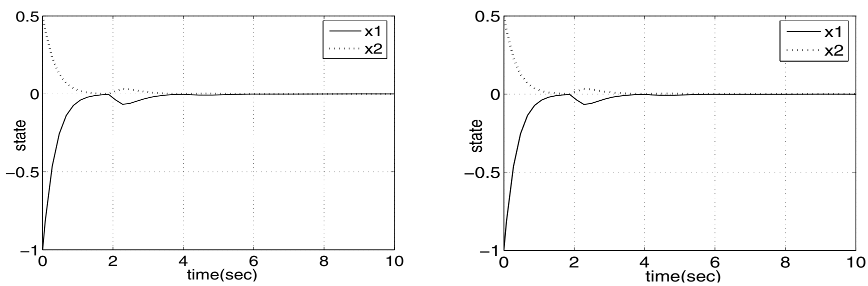

Fig. 1. standard control without failure and reliable control without failure

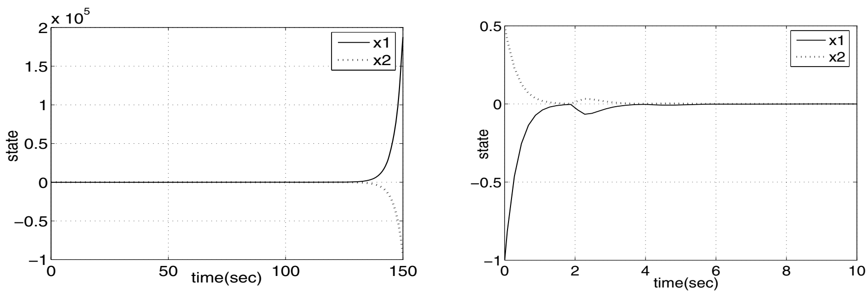

Fig. 2. standard control with failure and reliable control with failure

6 Conclusion

In this paper, we have considered the reliable fuzzy control design problem for nonlinear uncertain descriptor systems with state-delay. The effect among the subsystems are taken account of sufficiently. The less conservative reliable controller design schemes via state feedback and estimated state feedback are proposed.One of the future research topics is to extend the results developed in the present paper to the nonlinear systems, with more complex faults of actuator and sensor, to make the systems achieve appropriate performance.

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