PWM Fuzzy Controller for Nonlinear Systems

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Abstract. In this paper, we develop an intelligent digitally redesigned PAM fuzzy controller for nonlinear systems. Takagi-Sugeno fuzzy model is used to model the nonlinear systems and a continuous-time fuzzy-model-based controller is designed based on extended parallel-distributed-compensation method. The digital controllers are determined from existing analogue controllers. The proposed method provides an accurate and effective method for digital control of continuous-time nonlinear systems and enables us to efficiently implement a digital controller via pre-determined continuous-time TS fuzzy-model-based controller. We have applied the proposed method to the balancing problem of the inverted pendulum to show the effectiveness and feasibility of the method.

1 Introduction

Fuzzy logic control is one of most useful approaches for control of complex and ill-defined nonlinear systems. The main drawback of fuzzy logic control is the empirical design procedure, which are based on trial-and-error process. Therefore, recent trend in fuzzy logic control is to develop systematic method to design the fuzzy logic controller. The studies on the systematic design of fuzzy logic controller have largely been devoted to two approaches. One is based on soft-computing method [1]. This approach utilizes neural network theory and genetic algorithm, etc.. This method is quite efficient since the most appropriate and optimal fuzzy logic controllers can be designed without the aids of human experts. However, it may suffer difficulties in determining the overall stability of the system. The other is based on the well-established conventional linear system theory [2-3]. One popular method is the TS fuzzy model or dynamic fuzzy- model-based control theory, which combines the fuzzy inference rule with the local linear state model [2-3]. This kind of method has been widely used in the control of nonlinear systems since it is easy to incorporate the mathematical analysis developed in the linear control theory.

At the same time, we have been witnessed rapid development of flexible, low-cost microprocessors in the electronics fields. Therefore, it is desirable to implement the recent advanced controller in digital. There are three digital design approaches for digital control systems. The first approach, called the direct digital design approach, is to convert an analogue plant to a digital plant and then find the digital controller. The

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second is, which is named the digital redesign approach, is to find the analogue controller and then carry out the digital redesign. The other is to directly design a digital controller for the analogue plant, which is still under development [5-7]. In general, there exist two types of digital controllers: the pulse-amplitude-modulation (PAM) controller and the pulse-width-modulation (PWM) controller. The PAM controller, which is commonly used in digital control, produces a series of piecewise-constant continuous pulses having variable amplitude and variable or fixed width [5].

In this paper, we develop an intelligent digitally redesigned PAM fuzzy controller for digital control of continuous-time nonlinear systems. We first apply the digital redesign technique to each local linear model. By the proposed method, the conventional digital redesign method developed in linear system field can be then easily applied and extended to the control of nonlinear systems.

2 Fuzzy-Model-Based Controller

Consider a nonlinear dynamic system in the canonical form

$$x^{(n)}(t) = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}(t) \tag{1}$$

where, the scalar $x^{(n)}$ is the output state variable of interest, the vector $\mathbf{u}$ is the system control input, and $\mathbf{x} = \begin{bmatrix} x & \dot{x} & \dots & x^{(n-1)} \end{bmatrix}^T$ is the state vector. In (1), the nonlinear function $\mathbf{f}(\mathbf{x})$ is a known nonlinear continuous function of $\mathbf{x}$, and the control gain $\mathbf{g}(\mathbf{x})$ is a known nonlinear continuous and locally invertible function of $\mathbf{x}$. This nonlinear system can be approximated by the TS fuzzy model. More specifically, the ith rule of the TS fuzzy model in the continuous time case is formulated in the following form:

$$\begin{aligned} &\text{Plant Rule } i\text{: IF } x(t) \text{ is } F_1^i \text{ and ... and } x^{(n-1)}(t) \text{ is } F_n^i, \\ &\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i \mathbf{x}_c(t) + \mathbf{B}_i \mathbf{u}_c(t) \quad (i = 1, 2, ..., q) \end{aligned} \tag{2}$$

while the consequent part in the discrete-time case is represented by $\mathbf{x}_d(t+1) = \mathbf{F}_i \mathbf{x}_d(t) + \mathbf{G}_i \mathbf{u}_d(t)$ in (2).

Here, $\mathrm{F}_\mathrm{j}^\mathrm{i}$ $(j = 1, \dots, n)$ is the fuzzy set, q is the number of rules, $\mathbf{x}_\mathrm{c}(\mathrm{t}) \in \Re^\mathrm{n}$ is the state vector, $\mathbf{u}_\mathrm{c}(\mathrm{t}) \in \Re^\mathrm{m}$ is the input vector, $\mathbf{A}_i \in \Re^{n\times n}$ and $\mathbf{B}_i \in \Re^{n\times m}$, $x_1(t)$, …, $x_n(t)$ are the premise variables (which are the system states) and $(\mathbf{A}_i, \mathbf{B}_i)$ in the continuous-time case and $(\mathbf{F}_i, \mathbf{G}_i)$ in discrete-time case denote the ith local model of the fuzzy system, respectively. Subscript ‘c’ and ‘d’ represent continuous-time and discrete-time case, respectively. Using the center of gravity defuzzification, product inference, and single fuzzifier, the final output of the overall fuzzy system is given by

$$\dot{\mathbf{x}}_\mathrm{c}(\mathrm{t}) = \sum_{\mathrm{i}=1}^{\mathrm{q}} \mu_\mathrm{i}(\mathbf{x}_\mathrm{c}(\mathrm{t}))(\mathbf{A}_\mathrm{i}\mathbf{x}_\mathrm{c}(\mathrm{t}) + \mathbf{B}_\mathrm{i}\mathbf{u}_\mathrm{c}(\mathrm{t})) \tag{3}$$

where

$$w_i(\mathbf{x}(t)) = \prod_{j=1}^{n} F_j^i(x^{(j-1)}(t)), \quad \mu_i(\mathbf{x}(t)) = \frac{w_i(\mathbf{x}(t))}{\sum_{i=1}^{q} w_i(\mathbf{x}(t))}$$

Using the same premise as (2), the EPDC fuzzy controller in continuous-time model has the following rule structure:

$$\begin{aligned} &\text{Controller Rule } i\text{: IF } x(t) \text{ is } F_1^i \text{ and ... and } x^{(n-1)}(t) \text{ is } F_n^i, \\ &\text{THEN } \mathrm{u(t)} = -\mathbf{K}_{\mathrm{i}}\mathbf{x}(\mathrm{t}) + \mathbf{E}_{\mathrm{i}}(\mathrm{t})\mathbf{r}(\mathrm{t}) \quad (i = 1, 2, ..., q) \end{aligned} \tag{4}$$

where $\mathbf{K}_i = [k_1^i, \cdots, k_n^i]$ and $\mathbf{E}_i = [e_1^i \ \cdots \ e_n^i]$ are the feedback and feedforward gain vectors in ith subspace, respectively. $\mathbf{r}$(t) is the reference input.
The overall closed-loop fuzzy system becomes

$$\dot{\mathbf{x}} = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(\mathbf{x})\mu_j(\mathbf{x})((\mathbf{A}_i - \mathbf{B}_i\mathbf{K}_j)\mathbf{x}(t) + \mathbf{B}_i\mathbf{E}_j\mathbf{r}(t)) \tag{5}$$

3 PAM Fuzzy Controller

In general, when applying the dual-rate sampling method to a dynamic system, the fast sampling rate is used for the system parameter identification without losing the information of the dynamic system, and the slow sampling rate is used for the computation of advanced controllers in real time.

For the implementation of a digital control law, it needs to find a digital control law from the obtained optimal analogue control law. This can be carried out using the digital redesign technique, which is called the generalized digital redesign. This technique matches the continuous-time closed-loop state $\mathbf{x}_c(t)$ with the discrete-time closed-loop state $\mathbf{x}_d(t)$ at $t = kT_s$, where T_s is the slow sampling time. We can expect that this method consider the system responses only at the slow-rate sampling time, $t = k_s T_s$. If the slow sampling time T_s is not sufficiently small, then the control law cannot capture the system's behavior during the slow sampling time T_s. Thus, it needs to consider the system's behavior during the slow-rate sampling and reflect it to the control law. We adopt a new digital redesign method that considers the inter-sampling behaviors [6].

Subscript i representing ith subspace is omitted to avoid the complexity. Consider a controllable and observable analogue plant represented by

$$\dot{\mathbf{x}}_c(t) = \mathbf{A}\mathbf{x}_c(t) + \mathbf{B}\mathbf{u}_c(t), \quad \mathbf{x}_c(0) = \mathbf{x}_{c0} \tag{6}$$

$$\mathbf{y}_c(t) = \mathbf{C}\mathbf{x}_c(t) + \mathbf{D}\mathbf{u}_c(t) \tag{7}$$

The optimal state-feedback control law, which minimizes the performance index

$$J = \int_0^{\infty} \{[\mathbf{x}_c(t) - \mathbf{r}(t)]^T \mathbf{Q}[\mathbf{x}_c(t) - \mathbf{r}(t)] + \mathbf{u}_c(t)\mathbf{R}\mathbf{u}_c(t)\}dt \tag{8}$$

with $\mathbf{Q} \geq 0, \mathbf{R} > 0$ is

$$\mathbf{u}_c(t) = -\mathbf{K}_c \mathbf{x}_c(t) + \mathbf{E}_c \mathbf{r}(t) \tag{9}$$

where $\mathbf{r}(t)$ is a reference input vector and

$$\mathbf{K}_c = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \tag{10}$$

$$\mathbf{E}_c = -\mathbf{R}^{-1}\mathbf{B}^T(\mathbf{A} - \mathbf{B}\mathbf{K}_c)^{-T}\mathbf{Q} \tag{11}$$

where $\mathbf{P}$ is the solution to

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = 0 \tag{12}$$

The analogue control law $\mathbf{u}_\mathrm{c}(\mathrm{t})$ in (7) can be approximated as

$$\mathbf{u}_\mathrm{c}(\mathrm{t}) \cong \mathrm{W}_{\mathrm{k_f}} \mathbf{\Phi}_{\mathrm{k_f}}(\mathrm{t}) = \mathbf{u}_{\mathrm{dk_f}}(\mathrm{k_f T_f}) \tag{13}$$

for $\mathbf{r}(t) = \mathbf{r}(k_f T_f)$ with $k_f T_f \leq t < k_f T_f + T_f$, where

$$\begin{aligned} W_{k_f} &= \frac{1}{T_f}\int_{k_f T_f}^{K_f T_f + T_f} u_c(t)dt \\ &= -\frac{\mathbf{K}_c}{T_f}\int_{k_f T_f}^{K_f T_f + T_f} \mathbf{x}_c(t)dt + \mathbf{E}_c\mathbf{r}(k_f T_f) \end{aligned} \tag{14}$$

where $\mathbf{\Phi}_{k_f}(t)$ is orthonormal series and T_f is the fast rate sampling time.

Consider and the closed-loop of the analogue system in (5) represented with piecewise constant input by

$$\dot{\mathbf{x}}_c(t) = \mathbf{A}_c\mathbf{x}_c(t) + \mathbf{E}_c\mathbf{r}(k_f T_f) \tag{15}$$

where $\mathbf{A}_c = \mathbf{A} - \mathbf{B}\mathbf{K}_c$.

The corresponding digital system is

$$\mathbf{x}_c(k_f T_f + T_f) = \mathbf{G}_{cN}\mathbf{x}_c(k_f T_f) + \mathbf{H}_{cN}\mathbf{E}_c\mathbf{r}(k_f T_f) \tag{16}$$

where $\mathbf{G}_{cN} = e^{\mathbf{A}_c \mathbf{T}_f}$, $\mathbf{H}_{cN} = [\mathbf{G}_{cN} - \mathbf{I}_n]\mathbf{A}_c^{-1}\mathbf{B}$.

Consider the digital system with a piecewise constant input $\mathbf{u}_{\mathrm{dk}}(\mathrm{k_f T_f})$ as

$$\mathbf{x}_\mathrm{d}(\mathrm{K_f T_f} + \mathrm{T_f}) = \mathbf{G}_\mathrm{N}\mathbf{x}_\mathrm{d}(\mathrm{k_f T_f}) + \mathbf{H}_\mathrm{N}\mathbf{u}_{\mathrm{dk}}(\mathrm{k_f T_f}) \tag{17}$$

where $\mathbf{G}_N = e^{\mathbf{A}T_f}$, $\mathbf{H}_N = [\mathbf{G}_N - \mathbf{I}_n]\mathbf{A}^{-1}\mathbf{B}$. Let the desired digitally redesigned control law be

$$\mathbf{u}_{\mathrm{dk}}(\mathrm{k_f T_f}) = -\mathbf{K}_{\mathrm{dk}}\mathbf{x}_{\mathrm{d}}(\mathrm{k_f T_f}) + \mathbf{E}_{\mathrm{dk}}\mathbf{r}(\mathrm{k_f T_f}) \tag{18}$$

Its digital closed-loop system becomes

$$\mathbf{x}_d(k_f T_f + T_f) = \hat{\mathbf{G}}_{cN}\mathbf{x}_d(k_f T_f) + \hat{\mathbf{H}}_{cN}\mathbf{r}(k_f T_f) \tag{19}$$

where $\hat{\mathbf{G}}_{cN} = \mathbf{G}_N - \mathbf{H}_N\mathbf{K}_{dk}$, $\hat{\mathbf{H}}_{cN} = \mathbf{H}_N\mathbf{E}_{dk}$.

The integration of the analogue closed-loop system in (13) is represented by

$$\begin{aligned} &\int_{k_f T_f}^{k_f T_f + T_f} \mathbf{x}_c(t)dt \\ &\quad = \mathbf{A}_c^{-1}\left[\mathbf{x}_c(k_f T_f + T_f) - \mathbf{x}_c(k_f T_f) - T_f\mathbf{B}\mathbf{E}_c\mathbf{r}(k_f T_f)\right] \end{aligned} \tag{20}$$

Substituting (16) into (14), (20), and its result, $\mathbf{u}_{\mathrm{c}}(\mathrm{t})$, into (8). Matching the resulting system's state with (17), we have

$$\mathbf{G}_N - \mathbf{H}_N\mathbf{K}_{dk} = \mathbf{G}_N - \mathbf{H}_N\mathbf{K}_c(\mathbf{A}_c T_f)^{-1}(\mathbf{G}_{cN} - \mathbf{I}_n) \tag{21}$$

$$\mathbf{H}_N\mathbf{E}_{dk} = \mathbf{H}_N\left[\mathbf{I}_m + (\mathbf{K}_c - \mathbf{K}_{dk})\mathbf{A}_c^{-1}\mathbf{B}\right]\mathbf{E}_c \tag{22}$$

The solutions to (21) and (22) become

$$\begin{aligned} \mathbf{K}_{dk} &= \mathbf{K}_c(\mathbf{A}_c\mathbf{T}_f)^{-1}(\mathbf{G}_{cN} - \mathbf{I}_n) \\ \mathbf{E}_{dk} &= \left[\mathbf{I}_m + (\mathbf{K}_c - \mathbf{K}_{dk})\mathbf{A}_c^{-1}\mathbf{B}\right]\mathbf{E}_c \end{aligned} \tag{23}$$

To improve the performance during the slow rate sampling time, it is desired to find the digitally redesigned control law that matches both the digital closed-loop state with the analogue closed-loop state, and has the slow sampling rate time. The slowly sampled digital system with the digitally redesigned fast rate sampling time control law $\overline{\mathbf{u}}_d^{(N)}$ can be described as

$$\begin{aligned} \mathbf{x}_d(k_s T_s + T_s) &= \mathbf{G}_N^N\mathbf{x}_d(k_s T_s) + \overline{\mathbf{H}}_N^{(N)}\overline{\mathbf{u}}_d^{(N)}(k_s T_s) \\ &= \mathbf{G}_N^N\mathbf{x}_d(k_s T_s) + \sum_{i=1}^{N}\overline{\mathbf{H}}_i\mathbf{u}_{di}(k_s T_s + (i-1)T_f) \end{aligned} \tag{24}$$

where,

$$\mathbf{G}_N^N = (\mathbf{G}_N)^N \tag{25}$$

$$\begin{aligned}\overline{\mathbf{H}}_N^{(N)} &= \begin{bmatrix}\overline{\mathbf{H}}_1 & \overline{\mathbf{H}}_2 & \cdots & \overline{\mathbf{H}}_{N-1} & \overline{\mathbf{H}}\end{bmatrix} \\ &= \begin{bmatrix}\mathbf{G}_N^{N-1}\mathbf{H}_N & \mathbf{G}_N^{N-2}\mathbf{H}_N & \cdots & \mathbf{G}_N\mathbf{H}_N & \mathbf{H}_N\end{bmatrix}\end{aligned} \tag{26}$$

$$\begin{aligned}\overline{\mathbf{u}}_d^{(N)}(k_sT_s) &= \big[\overline{u}_{d1}^T(k_sT_s) \quad \overline{u}_{d2}^T(k_sT_s) \quad \cdots \\ &= \qquad \cdots \quad \overline{u}_{dN}^T(k_sT_s)\big]^T \\ &= \big[\overline{u}_{d1}^T(k_sT_s) \quad \overline{u}_{d2}^T(k_sT_s+T_f) \quad \cdots \\ &\qquad \cdots \quad \overline{u}_{dN}^T(k_sT_s+(N-1)T_f)\big]^T \\ &= -\overline{\mathbf{K}}_d^{(N)}\mathbf{x}_d(k_sT_s) + \overline{\mathbf{E}}_d^{(N)}\mathbf{r}(k_sT_s)\end{aligned} \tag{27}$$

Considering the digital system in (19) with the reference input $\mathbf{r}(t) = \mathbf{r}(k_sT_s)$ for $k_sT_s \le t < k_sT_s + T_s$, the closed-loop state $\mathbf{x}_d$ can be represented at $t = k_fT_f + (i-1)T_f$ and $k_fT_f = k_sT_s$ by

$$\begin{aligned}\mathbf{x}_d(k_sT_s+(i-1)T_f) &= \mathbf{G}_{cN}^{i-1}\mathbf{x}_d(k_sT_s) \\ &\quad + \sum_{j=1}^{i-1}\hat{\mathbf{G}}_{cN}^{i-1-j}\hat{\mathbf{H}}_{cN}\mathbf{r}(k_sT_s)\end{aligned} \tag{28}$$

and by definition of (27), the digital control law with the dual rate sampling time can be written as

$$\begin{aligned}u_{di}(k_sT_s+(i-1)T_f) &= -\mathbf{K}_{dk}\mathbf{x}_d(k_sT_s+(i-1)T_f) \\ &\quad + \mathbf{E}_{dk}\mathbf{r}(k_sT_s)\end{aligned} \tag{29}$$

Substituting (28) into (29), its result is

$$\mathbf{x}_\mathrm{d}(\mathrm{K_f T_f + T_f}) = \mathbf{G}_\mathrm{N}\mathbf{x}_\mathrm{d}(\mathrm{k_f T_f}) + \mathbf{H}_\mathrm{N}\mathbf{u}_\mathrm{dk}(\mathrm{k_f T_f}) \tag{30}$$

$$\begin{aligned}\overline{\mathbf{E}}_d^{(N)} &= \begin{bmatrix}\overline{\mathbf{E}}_{d1}^T & \overline{\mathbf{E}}_{d2}^T & \cdots & \overline{\mathbf{E}}_{dN}^T\end{bmatrix} \\ &= \Bigg[\mathbf{E}_{dk}^T \quad (\mathbf{E}_{dk} - \mathbf{K}_{dk}\hat{\mathbf{H}}_{cN})^T \quad \cdots \\ &\qquad \cdots \quad \left(\mathbf{E}_{dk} - \sum_{j=1}^{N-1}\mathbf{K}_{dk}\hat{\mathbf{G}}_{cN}^{N-1-j}\hat{\mathbf{H}}_{cN}\right)^T\Bigg]^T\end{aligned} \tag{31}$$

4 Intelligent Digitally Redesigned Controller

The overall closed-loop system is obtained from the feedback interconnection of the nonlinear system (1) and the controller (5), resulting in the following equation:

$$\mathbf{x}^{(n)}(t) = \mathbf{F}_o(\mathbf{x}(t)) + \mathbf{G}_o(\mathbf{x}(t))\mathbf{r}(t) \tag{32}$$

where $\mathbf{F}_o(\mathbf{x}(t)) = \mathbf{f}(\mathbf{x}(t)) - \mathbf{g}(\mathbf{x}(t))\mathbf{K}(\mu)\mathbf{x}(t)$, $\mathbf{G}_o(\mathbf{x}(t)) = \mathbf{g}(\mathbf{x}(t))\mathbf{E}(\mu)$.

The following theorem is our stability result for the equilibrium state:

Theorem 1. Consider the following nonlinear system:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) - \mathbf{K}(\mu)\mathbf{x}(t) + \mathbf{G}_o(\mathbf{x}(t), \mathbf{E}(\mu))\mathbf{r}(t) \tag{33}$$

where $\mathbf{r}(t)$ is a given reference signal, and $\mathbf{K}(\mu)$ and $\mathbf{E}(\mu)$ are control gain matrices with parameters $\mu \in \Re^q$, which can also be functions of $\mathbf{x}(t)$ and t in general. If $\mathbf{K}(\mu)$ and $\mathbf{E}(\mu)$ are designed such that

(i) the matrix $[\mathbf{A} - \mathbf{K}(\mu)]$ is stable uniformly for all $\mathbf{x} \in \Re^n$,

(ii) $\int_0^\infty \|\mathbf{G}_o(\mathbf{x}(t), \mathbf{E}(\mu)\mathbf{r}(t)\| \, dt < \infty$, or

(ii)' $\int_0^\infty \|\mathbf{G}_o(\mathbf{x}(t), \mathbf{E}(\mu)\mathbf{r}(t)\| \, dt \le \|\mathbf{C}(\mathbf{x}(t,t)\| \, \|\mathbf{x}(t)\|$ with $\int_0^\infty \|\mathbf{C}(\mathbf{x}(t), t)\| \, dt < \infty$,

where $\|\cdot\|$ is the Euclidean norm, then the controlled system (5) is stable in the sense of Lyapunov.

Proof. See [2].

Corollary 1. In the nonlinear control system (1) with a fuzzy controller (5), namely,

$$\mathbf{x}^{(n)}(t) = \mathbf{F}_o(\mathbf{x}(t), \mathrm{K}(\mu)) + \mathbf{G}_o(\mathbf{x}(t), \mathrm{E}(\mu))\,\mathbf{r}(t) \tag{35}$$

where,

$$\mathbf{F}_o(\mathbf{x}(t), \mathbf{K}(\mu)) = \mathbf{f}(\mathbf{x}(t)) - \mathbf{g}(\mathbf{x}(t))\,\mathbf{K}(\mu)\,\mathbf{x}(t), \quad \mathbf{G}_o(\mathbf{x}(t), \mathbf{E}(\mu)) = \mathbf{g}(\mathbf{x}(t))\,\mathbf{E}(\mu),$$

if the TS fuzzy model

$$\mathbf{x}^{(n)}(t) = [\mathbf{A} - \mathbf{B}\mathbf{K}(\mu)]\,\mathbf{x}(t) \tag{36}$$

is designed such that it can uniformly approximate the given uncontrolled system (35), namely, $\|\mathbf{F}_o(\mathbf{x}(t), \mathbf{K}(\mu)) - [\mathbf{A} - \mathbf{B}\mathbf{K}(\mu)]\|$ can be arbitrary small, and if the control gains $\mathbf{K}(\mu)$ and $\mathbf{E}(\mu)$ are designed such that the two conditions (i) and (ii) (or (ii)') of Theorem 1 are satisfied, then the fuzzy control system (35) is stable in the sense of Lyapunov.

Proof. See [2].

5 Simulation

To illustrate the proposed method, let us consider the problem of balancing of an inverted pendulum on a cart. The dynamic equation is in [9]

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{g\sin(x_1) - amlx_2{}^2\sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)}$$

where x_1 is the angle of the pendulum in radians from vertical axis, x_2 is the angular velocity, $g = 9.8m/s^2$ is the acceleration by the gravity with the mass of the pendulum $m = 2.0kg$ and the mass of the cart $M = 8.0kg$, $a = m + M$, $2l = 1.0m$ is the length of pendulum, and u is the force applied to the cart.

An approximated TS fuzzy model is as follows [9]:

Rule 1: IF x_1 is about 0 THEN $\dot{x} = A_1x + B_1u$

Rule 2: IF x_1 is about $\pm\pi$ THEN $\dot{x} = A_2x + B_2u$.

where

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ \dfrac{g}{4l/3 - aml} & 0 \end{bmatrix}, \qquad \mathbf{B}_1 = \begin{bmatrix} 0 \\ -\dfrac{a}{4l/3 - aml} \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ \dfrac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 0 \\ -\dfrac{a\beta}{4l/3 - aml\beta^2} \end{bmatrix}$$

and $\beta = \cos(88^\circ)$. The membership function for Rules is shown in Fig. 1.

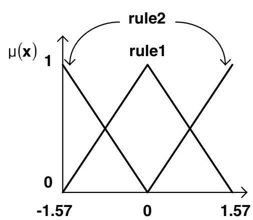

Fig. 1. Membership Functions

We choose the slow-rate sampling period T_s as 0.02 and the fast-rate sampling period T_f as 0.01, thus $N = T_s/T_f$ is 2. The initial conditions are $\mathrm{x}_1 = 43^\circ$ (0.7505rad) and $\mathrm{x}_2 = 0$. In order to check the stability of the global fuzzy control system, Based on LMI [3], we found the common positive definite matrix $\mathbf{P}_c$ to be

$$\mathbf{P}_c = \begin{bmatrix} 0.4250 & 0.0189 \\ 0.0189 & 0.0084 \end{bmatrix}$$

The other conditions are also satisfied. Therefore, the overall continuous-time fuzzy system is stable in the sense of Lyapunov. Figure 2–4 show the comparisons of the position angle $x_1(t)$, the angular velocity $x_2(t)$, and the control input of this example by the proposed method and the original analogue controller, respectively. As seen in these results, the proposed scheme is successful for digital control of nonlinear system.

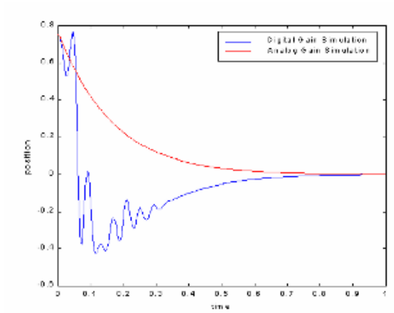

Fig. 2. Position angle $x_1(t)$

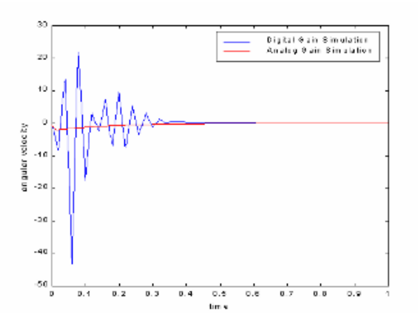

Fig. 3. Angular velocity $x_2(t)$

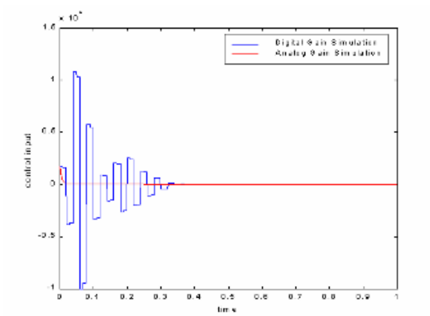

Fig. 4. Control input

6 Conclusions

In this paper, we have proposed the digitally redesigned PAM fuzzy-model-based controller design method for a nonlinear system. We represent the nonlinear system as a TS fuzzy-model-based system, and the EPDC technique is then utilized to design a fuzzy-model-based controller. In analogue control law design, the optimal regional pole assignment technique is adopted and extended with some new stability conditions to construct multiple local linear systems. The PAM digital redesign method is carried out to obtain the digital control law for control of each local analogue system, where a new state matching has been developed. The inverted pendulum balancing simulation has shown that the proposed method is very effective in controlling a nonlinear system.

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