# Improvement of Grey Relation Analysis and Its Application on Power Quality Disturbances Identification

Ganyun Lv<sup>1</sup>, Xiushan Cai<sup>1</sup>, and Yuanyuan Jin<sup>2</sup>

<sup>1</sup> Department of Information Science and Engineering, Zhejiang Normal University, 299 Beishan Road, 321004 Jinhua, China {Ganyun\_lv, XSCai}@yahoo.com http://www.zjnu.cn/index.html
<sup>2</sup> Nanjing Micro One Electronics Inc., 11F Huaxin Building, 9 Guanjiaqiao 210005 Nanjin, China jinyy@microne.com.cn

**Abstract.** An improved grey relation analysis method was brought forward, based on concept of group relation index and relation index cube. The definition and calculating process was given out in this paper. In contrast to traditional grey relation analysis, the improved grey relation analysis had two advantages over traditional grey relation analysis: A) Greatly strengthened the veracity and reliability of relation analysis; B) Having a much broader range of its application. The improved method was applied to an application of power quality (PQ) disturbance identification in power system. The test result of the application has shows that the improved method has a much better effect than traditional grey relation analysis. The improved method can also be applied to many other applications in a wide range.

## **1** Introduction

Grey system theory [1, 2] is proposed in 1980's, as a tool for considering with uncertainty in extensive application, such as linear planning, forecasting, system control and identification. Grey relation analysis is an important part of grey system theory. It has been widely used in many applications [3, 4, 5, 6, 7] in recent years. It has even been applied in facing recognition combining with other statistical method [3]. Study of grey relation analysis is going on. Recently an optimal grey relation analysis was brought out in paper [6].

Power quality (PQ) has been an important concern for utility, facility and consulting engineers in recent years. It is necessary to analyze PQ problem and provide reasonable compensating measurements to improve the PQ. Detection and identification of PQ disturbances is an important task in the work. Researches have been adopted many technologies, such as neural networks [8], genetic net [9], expert system [10] and fast match [11], to identify these disturbances. These technologies have a common shortcoming of complex computation, and thus are difficult to be used in real time application. Many PQ disturbances, such as harmonics, transient pulse, low-frequency oscillation, white noise and so on, change in extent and frequency. These PQ disturbances can be taken as variable of grey system. In this paper, we presented an improved grey relation analysis for identification, and realized identification of the PQ disturbances simply and effectively.

### 2 Improvement of Grey Relation Analysis

Grey relation analysis is an important part of grey system theory. Here, theory of traditional grey relation analysis was introduced first, and then improved grey relation analysis was given out.

#### 2.1 Review of Traditional Grey Relation Analysis

Assuming that there have two groups of sequences, one group is the reference sequences, and the other group is the comparative sequences. The reference sequences are  $y_1, y_2, y_3, \dots, y_m$ , and the comparative sequences are  $x_1, x_2, x_3, \dots, x_n$ . They are

$$x_i = \{x_i(1), x_i(2), x_i(3), \cdots, x_i(N)\} \quad (i = 1, 2, 3, \cdots, n) .$$
(1)

$$y_{j} = \left\{ y_{j}(1), y_{j}(2), y_{j}(3), \cdots, y_{j}(N) \right\} \quad (j = 1, 2, 3, \cdots, m) .$$
<sup>(2)</sup>

Where  $x_i(k)$  and  $y_i(k)$  are the k-th characteristic component of  $x_i$  and  $y_i$  respectively.

Then the grey relevant coefficient is defined in traditional grey relation analysis as follow

$$\xi_{ij}(k) = \frac{\min_{k} \min_{k} |y_{j}(k) - x_{i}(k)| + 0.5 \max_{i} \max_{k} |y_{j}(k) - x_{i}(k)|}{|y_{j}(k) - x_{i}(k)| + 0.5 \max_{i} \max_{k} |y_{j}(k) - x_{i}(k)|}$$
(3)

In the above equation, the environmental coefficient is set to 0.5.

Then the grey relation value is to be

$$\xi_{ij} = (\xi_{ij}(1), \xi_{ij}(2), \xi_{ij}(3), \cdots \xi_{nj}(N)) \quad .$$
(4)

Finally the grey relevant index between the reference sequence  $y_j$  and the comparative sequence  $x_i$  is defined as follow

$$r_{ij} = \frac{1}{N} \sum_{k=1}^{N} \xi_{ij}(k) \quad .$$
(5)

Where  $r_{ij}$  represents the relevant degree between the reference sequence  $y_j$  and the comparative sequence  $x_i$ . These  $r_{ij}$  £<sup>"i=1,2,...,n;</sup> j=1,2,...,m£<sup>©</sup> composed a matrix named Relation Index Matrix, shown as follow

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} & \cdots & r_{1N} \\ r_{21} & r_{22} & r_{23} & \cdots & r_{2N} \\ r_{31} & r_{32} & r_{33} & \cdots & r_{3N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m2} & r_{m3} & \cdots & r_{mN} \end{bmatrix} .$$
(6)

The traditional relation analysis is widely used in many applications. However, it relies much on the correctness of reference array. When there are relatively strong characteristic of disperse and random in the reference sequences, the precision and reliability of relation analysis will greatly be worsened. That will be shown in the latter part of the paper.

#### 2.2 Improved Grey Relation Analysis

Assuming that there have m\*b reference sequences, as following,

$$y_{1}^{1}, y_{1}^{2}, y_{1}^{3}, \dots, y_{1}^{b};$$

$$y_{2}^{1}, y_{2}^{2}, y_{2}^{3}, \dots, y_{2}^{b};$$

$$y_{3}^{1}, y_{3}^{2}, y_{3}^{3}, \dots, y_{3}^{b}; .$$

$$\dots, \dots, \dots, \dots, \dots;$$

$$y_{m}^{1}, y_{m}^{2}, y_{m}^{3}, \dots, y_{m}^{b};$$
(7)

Where  $\{y_i^1, y_i^2, y_i^3, \dots, y_i^b;\}$   $(i=1,2,3,\dots,m)$  belong to the same group of reference sequence  $y_i$ . They are relevant each other, for example, they are the same type of PQ disturbances. On the other hand, they are independent in some degree. The comparative sequences are the same as equation (1).

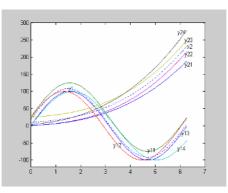


Fig. 1. The sketch map of reference sequences and comparative sequences

The relation sketch of the reference sequences and comparative sequences are shown in Fig.1, in which there have two groups of reference sequences  $(y_1, y_2)$  and two comparative sequences  $(x_1, x_2)$ .

With the assumption above, the Relation Index Cube is defined as follow:

$$V_{r} = \begin{bmatrix} r_{11}^{b} & r_{12}^{b} & \cdots & r_{1n}^{b} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{11}^{2} & r_{12}^{2} & \cdots & r_{1n}^{2} & \vdots & \vdots \\ r_{11}^{1} & r_{12}^{1} & \cdots & r_{1n}^{1} \\ r_{21}^{2} & r_{22}^{2} & \cdots & r_{2n}^{1} \\ \vdots & \vdots & \ddots \\ \cdots & \cdots & r_{ij}^{1} & \cdots \\ r_{m1}^{1} & r_{m2}^{1} & \cdots & r_{mn}^{1} \end{bmatrix} \begin{bmatrix} r_{2n}^{2} & \cdots & r_{mn}^{b} \\ \vdots & \ddots \\ r_{mn}^{2} & \cdots \\ r_{mn}^{2} & \cdots$$

Where  $r_{ij}^{w}$  is the individual relevant coefficient between individual reference sequence  $y_{i}^{w}$  in group w and comparative sequence  $x_{j}$ . It can be obtained from the following equation.

$$r_{ij}^{w} = \frac{1}{N} \sum_{k=1}^{N} \xi_{ij}^{w}(k) .$$
(9)

Where the individual grey relevant coefficient  $\xi_{ij}^{w}(k)$  is defined as :

$$\xi_{ij}^{w}(k) = \frac{\min_{k} \min_{k} \left| y_{i}^{w}(k) - x_{j}(k) \right| + 0.5 \max_{j} \max_{k} \left| y_{i}^{w}(k) - x_{j}(k) \right|}{\left| y_{i}^{w}(k) - x_{j}(k) \right| + 0.5 \max_{j} \max_{k} \left| y_{i}^{w}(k) - x_{j}(k) \right|}$$
(10)

With the Relation Index Cube, the Group Relation Index matrix  $R_{group}$  is defined at last as follow,

$$R_{group} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & r_{ij} & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}.$$
 (11)

Where the Group Relation Index  $\tilde{r}_{ij}$  is defined as follow

$$\tilde{r}_{ij} = \left(\sum_{w=1}^{b} (r_{ij}^w)^p\right)^{1/p}, \qquad p = 1, 2, .., \infty \quad .$$
(12)

It represents the relevant degree of the *i*-th comparative sequence and the j-th group reference sequences. The Group Relation Index matrix  $R_{group}$  can represent the

relationship between the comparative sequences and these groups of reference sequences as a whole.

In contrast to traditional grey relation analysis, the improved grey relation analysis has two following advantages at least. First, it has a much broader range of its application than traditional grey relation analysis because of its lower request of reference sequences data. The improved grey relation analysis can be applied in case of which the reference sequences are strong disperse. Second, it greatly strengthened the veracity and reliability of relation analysis. The efficiency of the improved grey relation analysis relies not on one individual reference sequence but on a group of reference sequences. The total effect of the improved method is much better than that of traditional grey relation analysis.

# **3** Application of Improved Grey Relation Analysis on Identifying PQ Disturbance

There have many kinds of PQ disturbances in power system, such as harmonics, voltage fluctuations, frequency deviation, sags, swells, over-voltage, under-voltage, transient pulse, low-frequency oscillation, high-frequency oscillation, and white noise. Among them, harmonics, transient pulse, low-frequency oscillation and white noise are similar and most difficult to identify. Many technologies are often relatively complex. This paper gives a new method based on the improved grey relation analysis.

The basic principle of identifying PQ disturbances with improved grey relation analysis is shown as Fig.2.

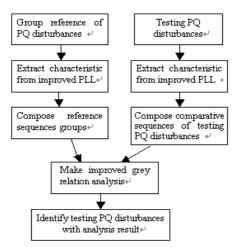


Fig. 2. Identification of PQ disturbances with improved grey relation analysis

Step of identifying PQ disturbances with the improved grey relation analysis is shown as below.

#### 3.1 Construct Reference Signal of PQ Disturbance

In this article, the reference and testing PQ disturbances are generated from power system simulation.

The PQ disturbance is detected through an improved PLL system first, described as Fig. 3.

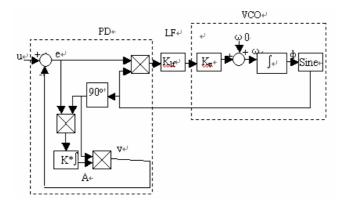


Fig. 3. Block diagram of the improved PLL system

The improved PLL system is a stable phase feedback control system, which could exact out the basic frequency component of the input signal without phase error. It also produces many useful outputs. The output y(t) is not only coherent with but also synchronized with the basic frequency component. The filter also produces instantaneous extent A(t), phase  $\phi(t)$  and frequency  $\omega(t)$  of the basic frequency component, and the error signal e(t) which represents deviation of input signal from the output signal. With output y(t) and error signal e(t), the filter can exact out the ideal component of input and the total disturbance. With outputs these outputs, the filter can detect out most of PQ disturbance. As an adaptive system, the PLL system is robust with respect to small change of basic frequency of input signal. Because of these characteristics, the PLL system is suited for detecting the PQ disturbance.

#### **3.2 Extract the Features**

With the output A(t),  $\omega(t)$  and e(t) of PLL system, the characteristic data of the PQ disturbances, including reference signal and detected signal, is obtained. Following features are extracted from PLL output.

1) Time percents of  $\Delta f \ge 1\%$ , where  $\Delta f = 100\% * \frac{f - f_0}{f_0}$ . 2) Time percents of  $\Delta A \ge 10\%$ , where  $\Delta A = 100\% * \frac{A - A_0}{A_0}$ .

- 3) Alternating times of  $|A \widetilde{A}| \ge 1\%$  a, where  $\widetilde{A} = \frac{1}{N} \sum A_n$ .
- 4) Time percents of  $\left| \frac{e(t)}{A_0} \right| \ge 2\%$ .

5) Time percents of  $\left|\frac{e(t)}{A_0}\right| \ge 2\%$ ,  $\left|\frac{e(t)}{A_0}\right| \ge 4\%$  in 1/4 circle after the disturbance hap-

pened.

6) Time percents of 
$$4\% \ge \frac{e(t)}{A_0} \ge 2\%$$
,  $8\% \ge \frac{e(t)}{A_0} \ge 4\%$ ,  $\frac{e(t)}{A_0} \ge 8\%$ ,

 $-2\% \ge \frac{e(t)}{A_0} \ge -4\%$ ,  $-4\% \ge \frac{e(t)}{A_0} \ge -8\%$  and  $\frac{e(t)}{A_0} \le -8\%$  in one circle after the dis-

turbance happened.

7) Time percents of 
$$\left|\frac{e(t)}{A_0}\right| \ge 4\%$$
,  $\left|\frac{e(t)}{A_0}\right| \ge 2\%$ ,  $\left|\frac{e(t)}{A_0}\right| \ge 1\%$ ,  $\left|\frac{e(t)}{A_0}\right| \ge 0.5\%$ ,

 $\left|\frac{e(t)}{A_0}\right| \ge 0.25\%$ , in one circle after the disturbance happened.

8) The four maximum and minimum value of e(t) and its relevant time.

With these features, the PQ disturbances, including harmonics, transient pulse, low-frequency oscillation and noise could be identified. Five samples of reference signal, harmonics, transient pulse, low-frequency oscillation and noise are produced in-order. Five testing signal of harmonics, transient pulse, low-frequency oscillation and noise are produced by the same order.

Five groups of reference sequences obtained from reference signal are shown as following

$y_{1}^{1}$	$y_{1}^{2}$	$y_{1}^{3}$	$y_{1}^{4}$	$y_1^5;$	
$y_{2}^{1}$	$y_{2}^{2}$	$y_{2}^{3}$	$y_{2}^{4}$	$y_2^5;$	(13)
$y_{3}^{1}$	$y_{3}^{2}$	$y_{3}^{3}$	$y_{3}^{4}$	$y_3^5;$	(13)
$y_{4}^{1}$	$y_{4}^{2}$	$y_{4}^{3}$	$y_{4}^{4}$	$y_4^5;$	

Where  $\begin{pmatrix} y_1^j & y_2^j & y_3^j & y_4^j \end{pmatrix}$ , j = 1,2,3,4,5 is one group of reference sequence, and  $\begin{pmatrix} y_1^1 & y_i^2 & y_i^3 & y_i^4 & y_i^5 \end{pmatrix}$  belongs to the same type of reference signal. When i=1, it belongs to harmonics; when i=2, it belongs to transient pulse; when i=3, it belongs to low-frequency oscillation; when i=4, it belongs to noise.

Five comparative sequences, including 20 test samples, are gotten from test signals. Each comparative sequences likes as

$$x_1 \quad x_2 \quad x_3 \quad x_4 \quad .$$
 (14)

Where  $x_i = \{x_i(1), x_i(2), x_i(3), \dots, x_i(N)\}$   $(i = 1, 2, 3, \dots, n)$ .  $x_i(k)$  is one component of features. In order to better display of analysis, each comparative sequences is ar-

range as:  $x_1$  is a comparative sequences of harmonics,  $x_2$  is of transient pulse,  $x_3$  is low-frequency oscillation and  $x_4$  is of noise.

#### 3.3 Improved Grey Relation Analysis

Set  $p = \infty$  and p = 1 in equation (13). Two kinds of Group Relation Index matrixes are obtained in relation analysis. The results of first test are shown as equation (15) and (16):

$$R_{group}\_max = \begin{bmatrix} 27.3914 & 25.1022 & 25.1639 & 25.3455 \\ 25.3305 & 30.4376 & 28.6053 & 25.2721 \\ 22.9171 & 25.3877 & 30.3629 & 23.8423 \\ 24.0167 & 27.2698 & 27.6393 & 30.7330 \end{bmatrix}, p = \infty$$
(15)  
$$R_{group}\_mean = \begin{bmatrix} 24.9689 & 24.6388 & 24.1959 & 24.9496 \\ 24.2590 & 29.4873 & 26.3457 & 24.7464 \\ 21.5191 & 24.5295 & 27.8664 & 23.7829 \\ 22.1396 & 24.8720 & 26.2100 & 28.8592 \end{bmatrix}, p = 1$$
(16)

The big value of grey relation means the strong relationship. Here we consider the largest relation index as belonging relation. Shown as equation (15) and (16), both two kinds of improved grey relation analysis methods can identify all these testing PQ disturbances correctly in this test. From first line of  $R_{group}$ \_max, the test sample  $x_1$  could be identified as the harmonics correctly. From second line of  $R_{group}$ \_max, the test sample  $x_2$  could be identified as the transient pulse correctly. From third line of  $R_{group}$ \_max, the test sample  $x_3$  could be identified as the low-frequency oscillation correctly. From last line of  $R_{group}$ \_max, the test sample  $x_4$  could be identified as the noise correctly. The same results could be obtained in four other tests.

Take each  $y_1^j \quad y_2^j \quad y_3^j \quad y_4^j$ , j = 1,2,3,4,5 as the reference sequences of one samples of reference signal for traditional grey relation analysis. Five times of relation analysis are made. Corresponding to five reference sequences, five Relation Index Matrixes are obtained as follow:

$$R1 = \begin{bmatrix} 24.5356 & 25.1022 & 25.3487 & 25.0411 \\ 25.1514 & 28.4135 & 25.6192 & 24.3341 \\ 22.0076 & 24.0814 & 28.7282 & 23.8423 \\ 21.8743 & 27.2698 & 25.6919 & 28.4574 \end{bmatrix}.$$

$$R2 = \begin{bmatrix} 25.4070 & 24.1566 & 24.0336 & 25.1549 \\ 23.7961 & 30.4376 & 28.6053 & 24.7687 \\ 21.7011 & 25.3877 & 30.3629 & 23.6083 \\ 22.5359 & 24.2295 & 27.3562 & 28.1090 \end{bmatrix}.$$
(17)

$$R3 = \begin{bmatrix} 24.9753 & 24.5170 & 25.1103 & 24.1655 \\ 25.3305 & 28.9150 & 24.8934 & 25.0231 \\ 22.9171 & 24.8408 & 24.4128 & 23.7956 \\ 23.2509 & 23.2702 & 26.0330 & 30.7330 \end{bmatrix} .$$
(19)  
$$R4 = \begin{bmatrix} 27.3914 & 24.9755 & 24.3228 & 25.3455 \\ 24.9985 & 30.0277 & 24.1683 & 25.2721 \\ 22.6025 & 24.2628 & 25.7878 & 23.8260 \\ 24.0167 & 24.5391 & 24.3297 & 28.5390 \end{bmatrix} .$$
(20)  
$$R5 = \begin{bmatrix} 22.5352 & 24.4425 & 28.1639 & 25.0411 \\ 22.0182 & 29.6428 & 28.4423 & 24.3341 \\ 18.3674 & 24.4746 & 29.6404 & 23.8423 \\ 19.0202 & 25.0516 & 27.6393 & 28.4574 \end{bmatrix} .$$
(21)

As shown above, some error results are produced in R1 and R3 because of unmatching between reference signal and test signal. It has been proved that identification with the improved grey relation analysis has better satisfaction with towards dispersed reference signals than that with tradition grey relation analysis. The correct ratio of identification in improved grey relation analysis is much higher than tradition relation analysis method.

The improved grey relation analysis is an exploration method for PQ disturbances identification, it is worth of explore for the future.

## 4 Conclusion

The main purpose of article is to investigate an improvement of grey relation analysis based on concept of Group Relation Index. Each kind signal of the reference sequences is only one sample in traditional grey relation analysis, while each kind signal of the reference sequences is a group sample in improved grey relation analysis. The improved method has the following properties:

- A) Has a much broader range of its application.
- B) Greatly strengthened the veracity and reliability of relation analysis.

The verification of the improved relation analysis is asserted through one application on identifying PQ disturbances. It has been proved that identification with the improved grey relation analysis has higher satisfaction with towards dispersed reference signals than that with traditional grey relation analysis. The improved grey relation analysis is a progress of traditional grey relation analysis, and it could be widely applied in many other applications for the future.

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