Non-fragile Robust H_∞ Fuzzy Controller Design for a Class of Nonlinear Descriptor Systems with Time-Varying Delays in States

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Abstract. The controller fragility can cause the performance debasement of the closed-loop system due to small perturbations in the coefficients of the controller design, and is one of the most important factors to be considered during practical controller design. To take the controller fragility into consideration for a class of nonlinear time-delayed descriptor systems with norm-bounded time-varying uncertainties in the matrices of state, delayed state and control gain, we have proposed non-fragile robust H_∞ fuzzy control design via state feedback controllers in this paper. The nonlinear descriptor system is approximated by Takagi-Sugeno (T-S) fuzzy model. In combination of parallel-distributed compensation (PDC) scheme, sufficient conditions are derived for the existence of non-fragile robust H_∞ fuzzy controllers in terms of linear matrix inequalities (LMI). Finally, an example is given to demonstrate the use of the proposed controller design.

1 Introduction

Generally, the perturbations during the controller's implementation are quite difficult to avoid due to finite word length in digital systems, the imprecision inherent in analog systems, the need for additional tuning of parameters in the final controller implementation and other reasons. some examples in [1] had been presented to show that small perturbations in the coefficients of the controller designed by using robust H_2, H_∞, l_1 and μ approaches can destabilize the closed-loop control system. The authors therein had suggested to take into account uncertainties both in the controller structure and in the system structure. After [1], the research of non-fragile controller has been an active area during the past several years, see [2] and the references therein. However, the efforts therein were mainly focused on linear systems. The non-fragile controller for nonlinear system was discussed in [3]. The method therein needs positive-definite solution to a pair of coupled Hamilton-Jacobi inequalities, which are much complicated and only have solutions for a special kind of systems. Therefore, it's still an open problem to design non-fragile controller for nonlinear system.

Very recently, many effects [6,7] have been devoted to descriptor systems with time delay, because the descriptor system has a tighter representation for

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a wider class of systems for representing real independent parametric perturbations in comparison to traditional state-space representation [4]. Due to the difficulties of constructing Lyapunov function and the complexity of the existence and uniqueness of the solution, there still remain some difficulties in controlling the nonlinear descriptor systems with time delays. Recent studies [8,10,9] have shown Takagi-Sugeno (T-S) fuzzy model is a universal approximator of any smooth nonlinear systems having a first order that is differentiable. Therefore, it is meaningful to consider applying the fuzzy model to approximate the nonlinear descriptor system with time delays. To stabilize the nonlinear descriptor system with time delays, some researchers considered T-S fuzzy descriptor system with time delays [11,12].

Motivated by the aforementioned pioneering works, the goal of this paper is to propose non-fragile robust H_∞ fuzzy controller for a class of nonlinear descriptor systems with time-varying delays and norm-bounded uncertainties. First, the nonlinear descriptor system with time-varying delays is described by T-S fuzzy model. Then, the sufficient conditions for non-fragile robust H_∞ fuzzy controller are presented by use of PDC scheme. Finally, numerical example is given to illustrate the effectiveness of the controller design.

2 Problem Formulation and Some Preliminaries

We utilize T-S fuzzy system approximate the nonlinear time-delayed descriptor system with parametric uncertainties as follows

$$\begin{aligned}
&\text{Plant Rule } k: \\
&\qquad \text{IF } \vartheta_1(t) \text{ is } N_{k1} \text{ and, } \ldots, \text{ and } \vartheta_g \text{ is } N_{kg}, \\
&\qquad \text{THEN} \\
&\quad \text{Right-Hand-Side Plant Rule } i: \\
&\qquad \text{IF } \vartheta_1(t) \text{ is } J_{i1} \text{ and, } \ldots, \text{ and } \vartheta_g \text{ is } J_{ig}, \\
&\qquad \text{THEN } E_k\dot{x}(t) = (A_i + \varDelta A_i)\, x(t) + (A_{di} + \varDelta A_{di})\, x(t - \sigma(t)) \\
&\qquad\qquad\qquad + (B_i + \varDelta B_i)\, u(t) + B_{2i}\omega(t), \\
&\qquad\qquad z(t) = C_i x(t), \\
&\qquad\qquad x(t) = \phi(t),\ t \in [-\sigma_0, 0], \\
&\qquad \text{for } k = 1,\ 2,\ \cdots,\ r,\ i = 1,\ 2,\ \cdots,\ r_e.
\end{aligned} \tag{1}$$

where $\vartheta(t) = \{\vartheta_1(t), \vartheta_2(t), \cdots, \vartheta_g(t)\}$ denotes the variables of premise part, $A_i, A_{di} \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{m\times n}$ are known real constant matrices, and N_{kl} and J_{il} denote fuzzy sets, the real-valued functional $\sigma(t)$ is the time-varying delay in the state and satisfies $\sigma(t) \le \sigma_0$, σ_0 is a real positive constant representing the upper bound of the time-varying delay. It is further assumed that $\dot{\sigma}(t) \le \beta < 1$ and β is a known constant. $\phi(t)$ are continuous vector-valued initial functions, and r and r_e denote the number of IF-THEN rules. We introduce the following assumption on E_i and C_i.

Assumption 1. *Without loss of generality, it is assumed that* $E_1 = E_2 = \cdots = E_g = E$ *and* $C_1 = C_2 = \cdots = C_g = C$ *in the fuzzy representation of the nonlinear descriptor system (1). Obviously, we have* $N_{kl} = J_{il}$ *and* $r = r_e$.

The assumption 1 will simplify our following discussions much but without loss of generality. In (1), $\Delta A_i,$, $\Delta A_{di} \in \mathrm{IR}^{n\times n}$, $\Delta B_i(t) \in \mathrm{IR}^{m\times n}$, are the system's uncertainty matrices and satisfy Assumption 2.

Assumption 2. *Uncertainty matrices* ΔA_i, ΔB_i *and* ΔA_{di} *in system (1) take the following structures*

$$\begin{bmatrix} \Delta A_i & \Delta B_i & \Delta A_{di} \end{bmatrix} = D_i F_i(\nu) \begin{bmatrix} E_{1i} & E_{2i} & E_{di} \end{bmatrix}, \tag{2}$$

where D_i, E_{1i}, E_{di} *and* E_{2i} *are constant real matrices of appropriate dimensions, and* $F_i(t) \in \mathrm{IR}^{i\times j}$ *is unknown matrix-valued functions*

$$F_i^T(\nu)F_i(\nu) \le I, \tag{3}$$

where $\nu \in \Omega$, Ω *is a compact set in* IR. *and* I *is the identity matrix of appropriate dimensions.*

Based on Assumption 1, the final output of the T-S fuzzy model is inferred as follows, by using the fuzzy inference method with a singleton fuzzifier, product inference and center average defuzzifiers

$$\begin{aligned} E\dot{x}(t) = \sum_{i=1}^{r} h_i(\vartheta(t))[&(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-\sigma(t)) \\ &+ (B_i + \Delta B_i)u(t) + B_{2i}\omega(t)], \end{aligned} \tag{4}$$

where $h_i(\vartheta(t)) = w_i(\vartheta(t)) \Big/ \sum_{i=1}^{r} w_i(\vartheta(t))$, $w_i(\vartheta(t)) = \prod_{j=1}^{r} M_{ij}(\vartheta(t))$ and $J_{ij}(\vartheta(t))$ denotes the degree of membership of $z(t)$ on J_{ij}. It is assumed that the degree of membership satisfies $\sum_{i=1}^{r} w_i(\vartheta(t)) > 0, w_i(\vartheta(t)) \ge 0, i = 1,\ 2,\ \cdots,\ r$. Note that for all t, there exists $\sum_{i=1}^{r} h_i(\vartheta(t)) = 1, h_i(\vartheta(t)) \ge 0$.

For PDC scheme, non-fragile robust H_∞ fuzzy controller and the fuzzy model (1) possess the same premises. The resulting overall controller is nonlinear in general which is a fuzzy blending of each individual linear controller designed for each local linear model. Then, supposing that all the states are observable, the i-th controller rule can be expressed by

Controller Rule i :

IF $\vartheta_1(t)$ is N_{i1} and $\ldots$, and ϑ_g is N_{ig}, (5)

THEN $u(t) = (K_i + \Delta K_i)x(t)$, $i = 1,\ 2,\ \cdots,\ r$.

where $u(t)$ is the actually implemented local controller, K_i is the local nominal gain, ΔK_i represents drifting from the nominal solution. It has been proved that fuzzy logic controller in (5) is an approximator for any nonlinear state feedback controller [9]. The overall fuzzy controller can be represented as follows

$$u(t) = \sum_{i=1}^{r} h_i(\vartheta(t))(K_i + \Delta K_i)x(t). \tag{6}$$

Applying the controller (6) to the system (4) will result in the following closed-loop control system

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{r} h_i(\vartheta(t))\{[(A_i + \Delta A_i) + (B_i + \Delta B_i)(K_i + \Delta K_i)]x(t) \\ \qquad\qquad + (A_{di} + \Delta A_{di})x(t - \sigma(t)) + B_{2i}\omega(t)\}, \\ z(t) = Cx(t), \\ x(t) = \phi(t), \ t \in [-\sigma_0, 0], \end{cases} \tag{7}$$

In the following, we introduce some definitions and useful properties for the system (7).

Definition 1. *A pencil $sE - \Sigma_{i=1}^{r} h_i(\vartheta(t))A_i$ (or pair$(E - \Sigma_{i=1}^{r} h_i\ (\vartheta(t))A_i)$) is regular, if $det(sE - \Sigma_{i=1}^{r} h_i(\vartheta(t))A_i)$ is not identically zero;*

Fuzzy descriptor system (7) has no impulsive mode (or impulse free) if and only if $rank(E) = degdet(sE - \Sigma_{i=1}^{r} h_i(\vartheta(t))A_i)$.

Remark 1. The notations $det(\cdot)$, $rank(\cdot)$ and $deg(\cdot)$ denote determinant, rank and degree of a matrix, respectively. The property of regularity guarantees the existence and uniqueness of solution for any specified initial condition. The condition of impulse free ensures that singular system has no infinite poles.

Definition 2. *The closed-loop system 7 is asymptotically stable with disturbance attenuation γ, if the followings are fulfilled for time-varying delays and norm-bounded parametric uncertainties*

1). The closed-loop system (7) is asymptotically stable;

2). The closed-loop system guarantees, under zero initial conditions, $\|z(t)\|_2 \leq \gamma \|\omega(t)\|_2$, for all non-zero $\omega(t) \in L_2 [0, \infty)$.

The objective of this paper is to design non-fragile robust H_∞ controller in the presence of time-varying delays, parameter uncertainties of system and additive uncertainty of controller. Also the controller guarantees disturbance attenuation of the closed-loop system from $\omega(t)$ to $z(t)$.

3 Non-fragile Robust H_∞ Fuzzy Controller Design

Now we are in a position to present the main result in this paper. Firstly, stability conditions are presented for the systems (7) without external disturbances.

Theorem 1. *Consider the uncertain nonlinear system with time delays (7) and suppose that the disturbance inputs are zero for all the time. The closed-loop system (7) is asymptotically stable if there exist positive definite matrix P, and controller gains K_i satisfying such that*

$$PE^T = EP \geq 0, \begin{bmatrix} \Pi_1 & * \\ A_{di}^T P & \Lambda_1 \end{bmatrix} < 0, \begin{bmatrix} \Pi_2 & * \\ A_{di}^T P + A_{dj}^T P & \Lambda_2 \end{bmatrix} < 0, \qquad (8)$$

where

$$\begin{aligned}
\Pi_1 &= PA_i + PB_iK_i + A_i^T P + K_i^T B_i^T P + \frac{R_1}{1-\beta} + (\varepsilon_{1i} + \varepsilon_{3i} \cdot \varepsilon_{2i} \\
&\quad + \varepsilon_{4i}) PD_iD_i^T P + \varepsilon_{2i} PB_i (I - \varepsilon_{3i}(E_{2i}H_i)^T (E_{2i}H_i))^{-1} B_i^T P + \varepsilon_{1i}^{-1}(E_{1i} \\
&\quad + E_{2i}K_i)^T (E_{1i} + E_{2i}K_i) + \varepsilon_{2i}^{-1} E_{Ki}^T E_{Ki}, \\
\Pi_2 &= PA_i + PB_iK_j + A_i^T P + K_j^T B_i^T P + PA_j + PB_jK_i + A_j^T P \\
&\quad + K_i^T B_j^T P + \frac{2R_1}{1-\beta} + (\varepsilon_{1ij} + \varepsilon_{2ij} \cdot \varepsilon_{3ij} + \varepsilon_{2ij}) PB_i (I - \varepsilon_{3ij}^{-1} (E_{2i}H_j)^T \\
&\quad \times (E_{2i}H_j))^{-1} B_i^T P + \varepsilon_{5ij} \cdot \varepsilon_{6ij} + \varepsilon_{4ij} + \varepsilon_{4i}) PD_iD_i^T P + \varepsilon_{4j} PD_jD_j^T P \\
&\quad + \varepsilon_{1ij}^{-1} (E_{1i} + E_{2i}K_j)^T (E_{1i} + E_{2i}K_j) + \varepsilon_{2ij}^{-1} E_{Kj}^T E_{Kj} \\
&\quad + \varepsilon_{5ij} PB_j (I - \varepsilon_{6ij}^{-1} (E_{2j}H_i)^T (E_{2j}H_i))^{-1} B_j^T P + \varepsilon_{5ij}^{-1} E_{Ki}^T E_{Ki} \\
&\quad + \varepsilon_{4ij}^{-1} (E_{1j} + E_{2j}K_i)^T (E_{1j} + E_{2j}K_i), \\
\Lambda_1 &= \varepsilon_{4i}^{-1} E_{di}^T E_{di} - \frac{R_1}{1-\beta}, \Lambda_2 = \varepsilon_{4i}^{-1} E_{di}^T E_{di} + \varepsilon_{4j}^{-1} E_{dj}^T E_{dj} - \frac{2R_1}{1-\beta},
\end{aligned}$$

where $1 \leq i < j \leq r$, ε_{ci} $(1 \leq c \leq 4)$, ε_{dij} *(*$1 \leq d \leq 6$) *are arbitrary positive scalars, * denotes the transposed element in the symmetric position.*

Proof. Define the following functional candidate for the system (7) as follows

$$V(x(t)) = x^T(t)E^T Px(t) + \frac{1}{1-\beta} \int_{t-\sigma(t)}^{t} x^T(s) R_1 x(s) ds, \qquad (9)$$

where P is a time-invariant, symmetric positive definite matrix. Then, the time derivative of the Lyapunov candidate $V(x(t))$ is given by

$$\begin{aligned}
\frac{dV(x(t))}{dt} &= \dot{x}^T(t)E^T Px(t) + x^T(t)E^T P\dot{x}(t) + \frac{1}{1-\beta} x^T(t) R_1 x(t) \\
&\qquad - \frac{1-\sigma(t)}{1-\beta} x^T(t-\sigma(t)) R_1 x(t-\sigma(t)).
\end{aligned}$$

After some manipulations, the above formulae can be rewritten as follows

$$
\begin{aligned}
\frac{dV(x(t))}{dt} &= \sum_{i=1}^{r} h_i^2(\vartheta(t))x^T(t)(P((A_i+\Delta A_i)+(B_i+\Delta B_i)(K_i+\Delta K_i) \\
&+((A_i^T+\Delta A_i^T)+(K_i^T+\Delta K_i^T)(B_i^T+\Delta B_i^T))P)x(t)+\sum_{i<j}^{r} h_i(\vartheta(t))h_j(\vartheta(t)) \\
&\times(P(A_i+\Delta A_i)+(B_i+\Delta B_i)(K_j+\Delta K_j))+((A_i^T+\Delta A_i^T) \\
&+(K_j^T+\Delta K_j^T)(B_i^T+\Delta B_i^T))P+P((A_j+\Delta A_j)+(B_j+\Delta B_j)(K_i+\Delta K_i)) \\
&+((A_j^T+\Delta A_j^T)+(K_i^T+\Delta K_i^T)(B_j^T+\Delta B_j^T))P)x(t) \\
&+x^T(t)P(A_{di}+\Delta A_{di})x(t-\sigma(t))+x^T(t-\sigma(t))(A_{di}^T+\Delta A_{di}^T)Px(t)) \\
&+\frac{1}{1-\beta}x^T(t)R_1x(t)-\frac{1-\sigma(t)}{1-\beta}x^T(t-\sigma(t))R_1x(t-\sigma(t)).
\end{aligned}
$$

Applying Lemmas in [5] to the above formulae results in

$$
\frac{dV(x(t))}{dt} \le \Xi_1+\Xi_2, \tag{10}
$$

where

$$
\begin{aligned}
\Xi_1 &= \sum_{i=1}^{r} h_i^2(\vartheta(t))x^T(t)[PA_i+PB_iK_i+A_i^TP+K_i^TB_i^TP+\varepsilon_{1i}PD_iD_i^T+\varepsilon_{1i}^{-1} \\
&\times(E_{1i}+E_{2i}K_i)^T(E_{1i}+E_{2i}K_i)+\varepsilon_{2i}PB_i(I-\varepsilon_{3i}^{-1}(E_{2i}H_i)^T(E_{2i}H_i))^{-1}B_i^TP \\
&+\varepsilon_{3i}\cdot\varepsilon_{2i}PD_iD_i^TP+\varepsilon_{2i}^{-1}E_{Ki}^TE_{Ki}]x(t)+\varepsilon_{4i}^{-1}x^T(t-\sigma(t))E_{di}^TE_{di}x(t-\sigma(t)) \\
&+\varepsilon_{4i}x^TPD_iD_i^TPx(t)+x^T(t)PA_{di}x(t-\sigma(t))+x^T(t-\sigma(t))A_{di}^TPx(t) \\
&+\frac{1}{1-\beta}x^T(t)R_1x(t)-\frac{1}{1-\beta}x^T(t-\sigma(t))R_1x(t-\sigma(t))\}, \\
\Xi_2 &= \sum_{i<j}^{r} h_i(\vartheta(t))h_j(\vartheta(t))\{x^T(t)[PA_i+PB_iK_j+A_i^TP+K_j^TBP+\varepsilon_{1ij}^{-1}(E_{1i} \\
&+E_{2i}K_j)^T(E_{1i}+E_{2i}K_j)+\varepsilon_{1ij}PD_iD_i^TP+\varepsilon_{2ij}PB_i(I-\varepsilon_{3ij}^{-1}(E_{2i}H_j)^T \\
&\times(E_{2i}H_j))^{-1}B_i^TP+\varepsilon_{3ij}\cdot\varepsilon_{2ij}PD_iD_i^TP+\varepsilon_{2ij}^{-1}E_{Kj}^TE_{Kj}+PA_j+PB_jK_i \\
&+A_j^TP+K_i^TB_j^TP+\varepsilon_{4ij}PD_iD_i^TP+\varepsilon_{4ij}^{-1}(E_{1j}+E_{2j}K_i)^T(E_{1j}+E_{2j}K_i) \\
&+\varepsilon_{5ij}PB_j(I-\varepsilon_{6ij}^{-1}(E_{2j}H_i)^T(E_{2j}H_i))^{-1}B_j^TP+\varepsilon_{5ij}\cdot\varepsilon_{6ij}PD_jD_j^TP \\
&+\varepsilon_{5ij}^{-1}E_{Ki}^TE_{Ki})x(t)+\varepsilon_{4i}x^T(t)PD_iD_i^TPx(t)+\varepsilon_{4j}PD_jD_j^TPx(t) \\
&+\varepsilon_{4i}^{-1}x^T(t-\sigma(t))E_{di}^TE_{di}x(t-\sigma(t))+\varepsilon_{4j}^{-1}x^T(t-\sigma(t))E_{dj}^TE_{dj}x(t-\sigma(t)) \\
&+x^T(t)PA_{di}x(t-\sigma(t))E_{dj}^TE_{dj}x(t-d(t))+x^T(t-\sigma(t))A_{di}^TPx(t)+x^T(t \\
&-\sigma(t))A_{dj}^TPx(t)+\frac{2}{1-\beta}x^T(t)R_1x(t)-\frac{2}{1-\beta}x^T(t-\sigma(t))R_1x(t-\sigma(t)).
\end{aligned}
$$

From the properties of quadratic form, the above formulae will lead to

$$\frac{dV(x(t))}{dt} = \sum_{i=1}^{r} h_i^2(\vartheta(t)) \begin{bmatrix} x(t) \\ x(t-d(t)) \end{bmatrix}^T \begin{bmatrix} \Pi_1 & PA_{di} \\ A_{di}^T P & \Lambda_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d(t)) \end{bmatrix}$$

$$+ \sum_{i<j}^{r} h_i(\vartheta(t))h_j(\vartheta(t)) \begin{bmatrix} x^T(t) & x^T(t-d(t)) \end{bmatrix}$$

$$\times \begin{bmatrix} \Pi_2 & PA_{di} + PA_{dj} \\ A_{di}^T P + A_{dj}^T P & \Lambda_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d(t)) \end{bmatrix}.$$

So far, if inequalities (8) hold, there exists $dV(x(t))/dt < 0$, and the closed-loop control system (7) will asymptotically stable. This completes the proof.

Next, non-fragile robust H_∞ fuzzy controller is presented for the system (7) with external disturbances based on Theorem 1.

Theorem 2. *Consider uncertain nonlinear descriptor system with time-varying delays (7). (5) is non-fragile robust H_∞ fuzzy controller for the system (7), if there exist matrices M_i, symmetric positive definite matrix N, U such that*

$$NE^T = EN \geq 0, \begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 \\ * & \Omega_{22} & \Omega_{23} \\ * & * & -\varepsilon_{4i} I \end{bmatrix} < 0, \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & 0 \\ * & \Upsilon_{22} & \Upsilon_{23} \\ * & * & \Upsilon_{33} \end{bmatrix} < 0, \quad (11)$$

hold, where

$$\Omega_{11} = A_i N + B_i M_i + NA_i^T + M_i^T B_i^T + \frac{U}{1-\beta} + (\varepsilon_{1i} + \varepsilon_{3i} \cdot \varepsilon_{2i} + \varepsilon_{4i}) D_i D_i^T,$$
$$\Omega_{12} = \begin{bmatrix} A_{di} N & B_{2i} & NE_{1i}^T + M_i^T E_{2i}^T & NE_{Ki}^T & NC^T \end{bmatrix},$$
$$\Omega_{22} = -\text{diag}\{ \tfrac{U}{1-\beta}, \quad \gamma^2 I, \quad \varepsilon_{1i} I, \quad \varepsilon_{2i} I, \quad I \},$$
$$\Omega_{23}^T = \begin{bmatrix} E_{di} N & 0 & 0 & 0 & 0 \end{bmatrix};$$
$$\Upsilon_{11} = A_i N + B_i M_j + NA_i^T + M_j^T B_i^T + A_j N + B_j M_i + NA_j^T + M_i^T B_j^T$$
$$+ \frac{2U}{1-\beta} + (\varepsilon_{1ij} + \varepsilon_{2ij} \cdot \varepsilon_{3ij} + \varepsilon_{5ij} \cdot \varepsilon_{6ij} + \varepsilon_{4ij} + \varepsilon_{4i}) D_i D_i^T + \varepsilon_{4j} D_j D_j^T,$$
$$\Upsilon_{12} = \big[\ (A_{di} + A_{dj})N \quad B_{2i} + B_{2j} \quad NE_{1i}^T + M_j E_{2i} \quad NE_{1j}^T + M_i^T E_{2j}^T$$
$$NE_{Kj}^T \quad NE_{Ki}^T \quad NE \ \big],$$
$$\Upsilon_{22} = -\text{diag}\left\{ \frac{2U}{1-\beta}, 2\gamma^2 I, \varepsilon_{1ij} I, \varepsilon_{4ij} I, \varepsilon_{2ij} I, \varepsilon_{5ij} I, \frac{1}{2} I \right\},$$
$$\Upsilon_{23}^T = \begin{bmatrix} E_{di} N & 0 & 0 & 0 & 0 & 0 & 0 \\ E_{dj} N & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Upsilon_{33} = -\text{diag}\{\varepsilon_{4i} I, \varepsilon_{4j} I\}.$$

Proof. First, let

$$\Gamma = (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t - d(t)) + (B_i + \Delta B_i)(K_i + \Delta K_i)x(t),$$

then we have

$$\begin{aligned} J &= \int_0^\infty \{z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\}dt \\ &\le \int_0^\infty \{z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \frac{dV(x(t))}{dt}\}dt \\ &= \int_0^\infty \{\sum_{i<j}^r h_i(\vartheta(t))h_j(\vartheta(t))\xi^T(t)\Phi_2\xi(t) + \sum_{i=1}^r h_i^2(\vartheta(t))\xi^T(t)\Phi_1\xi(t)\}dt, \end{aligned}$$

where $\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-\sigma(t)) & \omega^T(t) \end{bmatrix}^T$, $\Pi_1 = \tilde{\Pi}_1 + C^TC$ and $\Pi_2 = \tilde{\Pi}_2 + 2C^TC$.

If there exist $\Phi_1 < 0$ and $\Phi_2 < 0$, then $J \le 0$, which implies that $\|z(t)\|_2 \le \gamma \|\omega(t)\|_2$, for any $\omega(t) \in L_2\,[0,\,\infty)$. The closed-loop system (7) is asymptotically stable with disturbance attenuation γ according to definition 2 in section 2. Then, to make the make the results solvable by convex optimization method, we multiply the resulting inequalities $\Phi_1 < 0$ and $\Phi_2 < 0$ with $\Theta = \mathrm{diag}(P^{-1}, P^{-1}, I)$ both left and right side, respectively. Introduce new variables $N = P^{-1}$, $M_i = K_iP^{-1}$ and $U = NR_1N$. Then, we obtain

$$\begin{bmatrix} \tilde{\chi}_{ii} & * & * \\ NA_{di}^T & \hat{\Lambda}_1 & * \\ B_{2i}^T & 0 & -\gamma^2 I \end{bmatrix} < 0, \begin{bmatrix} \tilde{\chi}_{ij} & * & * \\ N(A_{di}^T + A_{dj}^T) & \hat{\Lambda}_2 & * \\ B_{2i}^T + B_{2j}^T & 0 & -2\gamma^2 I \end{bmatrix} < 0, \qquad (12)$$

where

$$\begin{aligned} \tilde{\chi}_{ii} &= A_iN + B_iM_i + NA_i^T + M_i^TB_i^T + \frac{U}{1-\beta} + (\varepsilon_{1i} + \varepsilon_{3i}\cdot\varepsilon_{2i} \\ &\quad + \varepsilon_{4i})D_iD_i^T + \varepsilon_{2i}B_i(I - \varepsilon_{3i}(E_{2i}H_i)^T(E_{2i}H_i))^{-1}B_i^T + \varepsilon_{1i}^{-1}(E_{1i}N \\ &\quad + E_{2i}M_i)^T(E_{1i}N + E_{2i}M_i) + \varepsilon_{2i}^{-1}NE_{Ki}^TE_{Ki}N + NC^TCN, \\ \tilde{\chi}_{ij} &= A_iN + B_iM_j + NA_i^T + M_j^TB_i^T + A_jN + B_jM_i + NA_j^T \\ &\quad + M_i^TB_j^T + \frac{2U}{1+\beta} + (\varepsilon_{1ij} + \varepsilon_{2ij}\cdot\varepsilon_{3ij} + \varepsilon_{5ij}\cdot\varepsilon_{6ij} + \varepsilon_{4ij} \\ &\quad + \varepsilon_{4i})D_iD_i^T + \varepsilon_{4j}D_jD_j^T + \varepsilon_{2ij}B_i(I - \varepsilon_{3ij}^{-1}(E_{2i}H_j)^T(E_{2i}H_j))^{-1}B_i^T \\ &\quad + \varepsilon_{5ij}B_j(I - \varepsilon_{6ij}^{-1}(E_{2j}H_i)^T(E_{2j}H_i))^{-1}B_j^T + \varepsilon_{1ij}^{-1}(E_{1i}N + E_{2i}M_j)^T \\ &\quad \times (E_{1i}N + E_{2i}M_j) + \varepsilon_{4ij}^{-1}(\varepsilon_{1j}N + \varepsilon_{2j}M_i)^T(\varepsilon_{1j}N + \varepsilon_{2j}M_i) \\ &\quad + N(\varepsilon_{2ij}^{-1}E_{Ki}^TE_{Kj} + \varepsilon_{5ij}^{-1}E_{Ki}^TE_{Ki} + 2C^TC)N, \\ \hat{\Lambda}_1 &= \varepsilon_{4i}^{-1}NE_{di}^TE_{di}N - \frac{1}{1-\beta}NR_1N, \end{aligned}$$

$$\hat{A}_2 = \varepsilon_{4i}^{-1} N E_{di}^T E_{di} N + \varepsilon_{4j}^{-1} N E_{dj}^T E_{dj} N - \frac{2}{1-\beta} N R_1 N.$$

Then, multiply the resulting inequalities (12) with $\Theta = \mathrm{diag}(P^{-1}, P^{-1}, I)$ both left and right side, respectively. Introduce new variables $N = P^{-1}$, $M_i = K_i P^{-1}$ and $U = NR_1N$. However, the conditions are not jointly convex in M_is and N in Theorem 1. Therefore, Schur complement is applied to the obtained matrix inequalities. Then, the LMIs in 11 can be obtained. This completes the proof.

4 Numerical Example

To demonstrate the use of our method, we consider a nonlinear descriptor system with time-varying delays approximated by using the following IF-THEN fuzzy rules:

IF $x_1(t)$ is P, THEN
$$E\dot{x}(t) = (A_1 + \Delta A_1)x(t) + (A_{d1} + \Delta A_{d1})x(t - \sigma(t)) \; + (B_1 + \Delta B_1)u(t) + B_{11}\omega(t);$$
IF $x_1(t)$ is N, THEN
$$E\dot{x}(t) = (A_2 + \Delta A_2)x(t) + (A_{d2} + \Delta A_{d2})x(t - \sigma(t)) \; + (B_2 + \Delta B_2)u(t) + B_{11}\omega(t);$$

where the membership functions of ‘P’, ‘N’ are given as follows

$$M_1(x_1(t)) = 1 - \frac{1}{1 + \exp(-2x_1)}, \; M_1(x_1(t)) = 1 - M_1(x_1(t)) \qquad (13)$$

The uncertainties ΔA_i, ΔA_{di} and ΔB_i are assumed to have the form of (2). Then, the relevant matrices in the T-S fuzzy model are given as follows

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & 0.4 \\ 0 & -0.5 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.3 & -0.4 \\ 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, A_2 = \begin{bmatrix} -0.5 & 0 \\ 0.5 & -1 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.4 & 0 \\ 0.4 & 0.3 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, D_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, D_2 = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, E_{11} = E_{12} = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$E_{d1} = E_{d2} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, E_{21} = 0.3, E_{22} = 0.2, F_1(t) = F_2(t) = \sin(t),$$
$$H_1 = H_2 = 0.5, E_{K1} = E_{K2} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}, \quad \phi(t) = \begin{bmatrix} e^{t+1} & 0 \end{bmatrix}^T,$$

and $\sigma(t) = h \sin t$. In Theorem 2, we choose the scalar coefficients $\varepsilon_{ci} = \varepsilon_{dij} = 1$, $1 \leq c \leq 4$, $1 \leq d \leq 6$. By using Matlab LMI Control Toolbox, positive definite matrices P, R_1 and feedback gain K_is can be obtained as follows

$$P = \begin{bmatrix} 5.8860 & 3.0870 \\ 3.0870 & 3.1162 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 1.0412 & 0.7638 \\ 0.7638 & 1.5298 \end{bmatrix},$$
$$K_1 = [\text{-0.47855 -0.76740}], \quad K_2 = [\text{-0.97124 -1.1999}].$$

5 Conclusions

In this paper, non-fragile robust H_∞ fuzzy controller design has been addressed for a class of nonlinear descriptor systems with time-varying delays via fuzzy interpolation of a series of linear systems. The fuzzy controller is reduced to the solution of a set of LMIs, which make the design much more convenient. Furthermore, an example has demonstrated the use of the proposed fuzzy model-based controller.

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