A New Fuzzy MADM Method: Fuzzy RBF Neural Network Model

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Abstract. An RBF neural network model with fuzzy triangular numbers as inputs is set up to solve fuzzy multi-attribute decision making (MADM) problems. The model can determine the weights of attributes automatically so that weights are more objectively and accurately distributed. In this model, decision maker's specific preferences are considered in the determination of weights. It is simple, and can give objective results while taking into decision maker's subjective intensions. A numerical example is given to illustrate the method.

1 Introduction

Weight determination methods in MADM include subjective and objective ones. The former can fully reflect decision makers' subjective intensions, but at the expense of objectivtiy. The latter can yield objective results, but without fully reflecting decision makers' intensions. Since decision makers' preferences for uncertainty: risk-loving, risk-averse or risk-neutral, have an effect on decision results [1], how to combine both subjective and objective information to make results both objective and reflect decision makers' subjective intensions is of theoretical and practical importance [2, 3, 4].

Fuzzy decision making deals with decision making under fuzzy environments [5, 6, 7]. Prevalent fuzzy decision methods are too complicated since they need huge number of calculations. We put forward a fuzzy neural network model which uses triangular fuzzy numbers as inputs, and whose feature is that weights are allocated more objectively and accurately. It has a strong self-learning ability and can also reflect the influence of the decision-maker's preferences for uncertainty on decision results.

2 Neural Network Model with Fuzzy Inputs

Fuzzy RBF neural network method for fuzzy MADM involves a four-leveled network, with the input level being composed of initial uncertain signal sources, the second level being the input revision level which adjusts inputs after considering the decision-maker's specific preferences for uncertainty, the third level being hidden levels, and the fourth level being the output level.

Suppose a decision making problem has M fuzzy attributes and m crisp attributes, then there are $(M+m)$ input neurons. Further suppose there are N hidden units, in the hidden levels, see Fig. 1. We use standardized triangular fuzzy numbers $x_j = (x_{j1}, x_{j2}, x_{j3})$ as inputs of the neural network [1, 2], the output of the network is:

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$$y_k = \sum_{i=1}^{N} w_i \varphi(X_k, t_i) = \sum_{i=1}^{N} w_i \exp\left(-\frac{1}{2\sigma_i^2} \sum_{p=1}^{M+m} (t_{kp} - t_{ip}) \right) \tag{1}$$

where w_i represent the weights of the output level, $\varphi(X_k, X_i)$ represent the incentive output functions of the hidden levels, which generally are Gauss functions, and $t_i=(t_{i1}, t_{i2},\ldots, t_{i,M+m})$ is the centre of the Gauss functions, and σ_i^2 the variance.

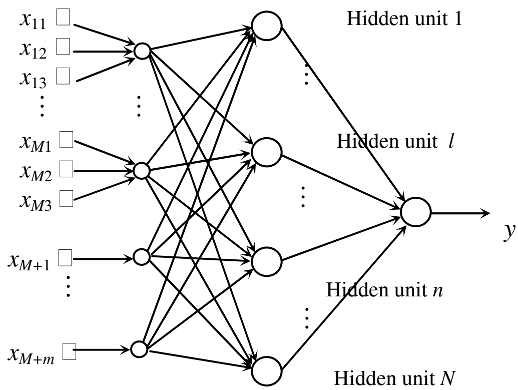

Fig. 1. Fuzzy RBF neural network

We adopt the monitored centre-selection algorithm. The specific learning steps are: Define the objective function to be: $E = \frac{1}{2}\sum_{k=1}^{N} e_k^2 = \frac{1}{2}\sum_{k=1}^{N}\left[d_k - y_k\right]^2$

where d_k represent the expected output of network samples.

The learning of the network is the solving of freedom parameters, t_i, w_i, Σ_i^{-1}, and β_j to minimize the objective function. For weights of the output level, w_i, there is,

$$w_i(n+1) = w_i(n) - \eta_1 \sum_{k=1}^{N} e_k(n)\varphi(X_k, Z_i) \tag{2}$$

$$t_i(n+1) = t_i(n) - \eta_2 2w_i(n) \sum_{k=1}^{N} e_k(n)\varphi'(X_k, t_i(n))\Sigma_i^{-1}(n)(X_k - t_i(n) \tag{3}$$

where $\Sigma_i^{-1} = -\frac{1}{2\sigma_{ii}^2}$. So, for the freedom parameters of the hidden levels, there is,

$$\Sigma_i^{-1}(n+1) = \Sigma_i^{-1}(n) + \eta_3 w_i(n) \sum_{k=1}^{N} e_k(n)\varphi'(X_k, t_i(n))Q_{ki}(n) \tag{4}$$

where $Q_{ki}(n) = (X_k - t_i(n))(X_k - t_i(n))^{\mathrm{T}}$, and for weights of the input levels, there is

$$\beta_j(n+1) = \beta_j(n) - \eta_4 - 2\sum_{k=1}^{N} e_k(n)\varphi'(X_k, t_i(n))\Sigma_i^{-1}(n)(X_k - t_i(n))S \tag{5}$$

$$S_{kj} = \int [\mu(x_{kj}^L) - \mu(0^L)]dx - \int [\mu(x_{kj}^R) - \mu(0^R)]dx$$

$$\mu(x_j^L) = \sup_{x_j^L = y+z, z \le 0} \mu(y) \quad \mu(x_j^R) = \sup_{x_j^R = y+z, z \ge 0} \mu(y)$$

where β_j represents the coefficient of the decision maker's preference for uncertainty, or the decision maker's uncertainty preference weight for the *j*-th attribute.
and with $\eta_1, \eta_2, \eta_3, \eta_4$ being the learning rate.

3 Fuzzy RBF Neural Network MADM Method and Its Application

If there are K alternatives, with M fuzzy attributes and m crisp attributes, then, the decision matrix is: $C = \{c_{ij}\}, i = 1,..., K; j = 1,..., M + m.$ The corresponding evaluation results, or output samples, are: $D = (d_1, d_2,..., d_k)$. To take into account the decision-maker's specific preferences, positive and negative ideal solutions are introduced. The attribute scales of the ideal and negative ideal solutions respectively are (use benefit type scales as examples): $c_j^+ = \sup \max_i \{c_{ij}\}$, $c_j^- = \inf \min_i \{c_{ij}\}$. Let the expected output of the positive and negative ideal solutions be 0.95 and 0.05 respectively.

Suppose a firm has four new product development alternatives, A_1, A_2, A_3, and A_4, which are to be evaluated from eight aspects: production cost, operational cost, performance,noise,maintenance, reliability, flexibility and safety.The firm makes decisions according to the overall market performances of 15 similar products in the market,whose attribute indices and overall market performances are shown in Table1.

Table 1. Attribute scaless and overall market performances of similar products in the market

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	IS	NIS
Production cost ($)	42	20	35	40	30	63	64	84	35	75	49	44	80	41	57	20	48
Operational cost ($)	64	52	47	50	55	65	40	60	40	41	68	35	31	45	68	35	65
Noise (db)	35	70	65	40	55	79	40	54	88	50	79	90	46	42	53	19	70
Function	VG	A	A	G	RG	G	RG	RG	RG	A	G	RG	G	VG	RG	1	0.4
Maintenance	RB	RB	RG	G	G	RG	RG	VG	G	RG	A	G	RG	RG	A	0.9	0.25
Reliability	VG	RG	G	G	G	VB	VG	A	RG	G	G	A	A	VG	RG	1	0.4
Flexibility	RB	G	G	RG	A	G	G	VG	G	RG	G	VG	RG	A	A	1	0.4
Safety	RG	A	RG	G	VG	RG	RG	RG	G	VG	VG	A	A	VG	G	1	0.4
Overall performances	0.78	0.56	0.73	0.92	0.8	0.57	0.87	0.82	0.76	0.69	0.76	0.73	0.74	0.87	0.58	0.95	0.05

(IS , NIS represent the ideal solution and Negative ideal solution respectively).

Table 2. Transformation rules for fuzzy linguistic words[8,9]

Order	Linguistic words	Corresponding triangular fuzzy numbers
1	Very good (VG)	(0.85,0.95,1.00)
2	Good (G)	(0.70,0.80,0.90)
3	Relatively good (RG)	(0.55,0.65,0.75)
4	Average (A)	(0.40,0.50,0.60)
5	Relatively bad (RB)	(0.25,0.35,0.45)
6	Bad (B)	(0.10,0.20,0.30)
7	Very bad (VB)	(0.00,0.05,0.15)

In Table 1, the former 3 are crisp attributes and the latter 5 are fuzzy attributes. For the fuzzy attributes, we can transform them into fuzzy numbers according to Table 2.

Having the network trained, input the data in Table 3 into the network, we will get the outputs (see Table 3).

Table 3. Alternative index values of the product being developed

	Production cost ($)	Operational cost ($)	Noise (db)	Function	Maintenance	Reliability	Flexibility	Safety	Overall performances
A_1	45	35	25	G	A	G	G	RG	0.83
A_2	25	50	60	RG	A	G	A	VG	0.72
A_3	35	45	50	A	G	A	VG	RG	0.75
A_4	48	65	19	RG	RB	RG	A	G	0.71

The ordering of the alternatives are: $A_1 \succ A_3 \succ A_2 \succ A_4$.

4 Conclusion

This Paper set up a RBF neural network model with fuzzy triangular numbers as inputs to solve MADM problems. The model can automatically give weights distributed objectively and accurately . It also has a great self-learning ability so that calculations are greatly reduced and simplified. Further, decision maker's specific preferences for uncertainty are considered in the determination of weights.

References

1. Li, R.-J.: Fuzzy Multi-attribute Decision Making Theory and Its applications. Beijing: Science Press (2002).
2. Song, R.: Multi-attribute Decision Making Method Based on Wavelet Neural Networks. Computer Engineering and Applications, Vol. 35 (2000) 46-48.
3. Qiu, C, Liu, Y.: Multi-attribute Decision Making Based on Artificial Neural Network. Journal of Beijing Science and Engineering University, Vol. 20 (2000) 65-68.
4. Hdgan, M. T., Demuth, H. B., Beale, M.: Neural Network Design. PWS Publishing Company (1996).
5. Song, G., Zou, P.: Weight Determination in Multi-attribute Decision Making. Systems Engineering, Vol. 19 (2001) 84-89.
6. Fuller, R., Carlsson, C.: Fuzzy Multiple Criteria Decision Making: Recent Development. Fuzzy sets and Fuzzy systems, Vol. 78 (1996) 139-153.
7. L, Jie, Hou, Z.: Weight Determination Method Based on a Combination of AHP, Delphi method and Neural Network. Systems Engineering Theories and Practices, Vol. 20 (2001) 59-63.
8. Herrera, F., Herrera E.: Linguistic decision analysis: steps for solving decision problems under linguistic information. Fuzzy Sets and Systems, Vol. 115 (2000) 67-82.
9. Cheng, C.-H., Lin, Y.: Evaluating the best main battle tank using fuzzy decision theory with linguistic criteria evaluation. European Journal of Operational Research, Vol. 142 (2002) 174-186.