Novel Prediction Approach – Quantum-Minimum Adaptation to ANFIS Outputs and Nonlinear Generalized Autoregressive Conditional Heteroscedasticity

Bao Rong Chang*

Department of Computer Science and Information Engineering
National Taitung University, Taitung, Taiwan 950
phone: +886-89-318855 ext. 2607; fax: +886-89-350214
brchang@nttu.edu.tw

Abstract. Volatility clustering degrades the efficiency and effectiveness of time series prediction and gives rise to large residual errors. This is because volatility clustering suggests a time series where successive disturbances, even if uncorrelated, are yet serially dependent. To overcome volatility clustering problems, an adaptive neuro-fuzzy inference system (ANFIS) is combined with a nonlinear generalized autoregressive conditional heteroscedasticity (NGARCH) model that is adapted by quantum minimization (QM) so as to tackle the problem of time-varying conditional variance in residual errors. The proposed method significantly reduces large residual errors in forecasts because volatility clustering effects are regulated to trivial levels. Two experiments using real financial data series compare the proposed method and a number of well-known alternative methods. Results show that forecasting performance by the proposed method produces superior results, with good speed of computation. Goodness of fit of the proposed method is tested by Ljung-Box Q-test. It is concluded that the ANFIS/NGARCH composite model adapted by QM performs very well for improved predictive accuracy of irregular non-periodic short-term time series forecast and will be of interest to the science of statistical prediction of time series.

1 Introduction

In practice, predictions are obtained by extrapolating a value at the next time instant based on a prediction algorithm [1]. The autoregressive moving-average (ARMA) is a traditional method very suitable for forecasting regular periodic data like seasonal or cyclical time series [2]. On the other hand, ARMA does not work well on irregular or non-periodic data sequences such as international stock prices or future volume indices [3]. This is because ARMA lacks a learning mechanism and cannot tackle large fluctuations in a complex time series. In particular, the back-propagation neural

* Corresponding author.

L. Wang et al. (Eds.): FSKD 2006, LNAI 4223, pp. 908 – 918, 2006.

network (BPNN) [4] and radial basis function neural network (RBFNN) [5] has been successfully applied to time series forecasting but requires a large amount of pattern/target training data to capture the dynamics of the time series. An alternate predictor, the grey model [6], has been widely applied to non-periodic short-term forecasts and however commonly encounters the overshoot phenomenon [1] whereby huge residual errors emerge at the inflection points of a data sequence. The adaptive neuro-fuzzy inference system (ANFIS) [7] has been widely applied to random data sequences with highly irregular dynamics [8] [9], e.g. forecasting non-periodic short-term stock prices [1]. However, volatility clustering effects [10] in the data sequence prevent ANFIS from reaching desired levels of accuracy. Further, a revised version of GARCH called a nonlinear generalized autoregressive conditional heteroscedasticity (NGARCH) [11] was presented for resolving volatility clustering effects. To do so, an adaptation called quantum minimization (QM) [12] is applied to adapt the coefficients of a linear combination of ANFIS and NGARCH so that large residual error is compensated by NGARCH and near-optimal solutions can be obtained.

2 ANFIS/NGARCH Composite Model Resolving Volatility Clustering

2.1 NGARCH Resolving Volatility Clustering

ARMAX(r,m,Nx) [13] encompasses autoregressive (AR), moving-average (MA) and regression (X) models, in any combination, as expressed below

$$y_{armax}(t) = C^{armax} + \sum_{i=1}^{r} R_i^{armax} y(t-i) + e_{resid}(t) + \sum_{j=1}^{m} M_j^{armax} e_{resid}(t-j) + \sum_{k=1}^{N_x} \beta_k^{armax} \mathbf{X}(t,k) \quad (1)$$

where C^{armax} = a constant coefficient, R_i^{armax} = autoregressive coefficients, M_j^{armax} = moving average coefficients, $e_{resid}(t)$ = residuals, $y_{armax}(t)$ = responses, β_k^{armax} = regression coefficients, $\mathbf{X}$ = an explanatory regression matrix in which each column is a time series and $X(t,k)$ denotes a element at the t th row and k th column of input matrix.

NGARCH(p,q) [11] describes nonlinear time-varying conditional variances and Gaussian residuals $e_{resid}(t)$. Its mathematical formula is

$$\sigma_{ntvcv}^2(t) = K^{ng} + \sum_{i=1}^{p} G_i^{ng} \sigma_{ntvcv}^2(t-i) + \sum_{j=1}^{q} A_j^{ng} \sigma_{ntvcv}^2(t-j) \left[\frac{e_{resid}(t-j)}{\sqrt{\sigma_{ntvcv}^2(t-j)}} - C_j^{ng} \right]^2 \quad (2)$$

with constraints

$$\sum_{i=1}^{p} G_i^{ng} + \sum_{j=1}^{q} A_j^{ng} < 1, \; K^{ng} > 0, \; G_i^{ng} \geq 0, \; i = 1,\ldots,p, \; A_j^{ng} \geq 0, \; j = -1,\ldots,q$$

where K^{ng} = a constant coefficient, G_i^{ng} = linear-term coefficients, A_j^{ng} = nonlinear-term coefficients, C_j^{ng} = nonlinear-term thresholds, $\sigma_{ntvcv}^2(t)$ = a nonlinear time-varying conditional variance and $e_{resid}(t-j)$ = j-lag Gaussian distributed residual in ARMAX.

In the presence of conditional heteroscedasticity, this composite model can perform ARMAX and NGARCH separately over every period in a time series. For simplicity as employed in [14], it is possible to merge the outputs of ARMAX and NGARCH linearly to attain better results as

$$y_{Composite\ Model}(t) = f(y_{ARMAX}(t),\ y_{NGARCH}(t)) = Cf_1 \cdot y_{ARMAX}(t) + Cf_2 \cdot y_{NGARCH}(t) \tag{3}$$

where f is defined as a linear function of ARMAX and NGARCH outputs, $y_{ARMAX}(t)$ and $y_{NGARCH}(t)$. Cf_1 and Cf_2 in Eq. (3) are the coefficients of a linear combination of ARMAX and NGARCH outputs. The resulting residual $y_{NGARCH}(t)$ at time t is obtained from a product of $\sqrt{\sigma_{ntvcv}^2(t)}$ in Eq. (2) and a normalized random number $randn(1)$ where $0 \le randn(1) \le 1$.

2.2 ANFIS Coordinated with NGARCH to Improve Regression

ARMAX cannot fit data sequences very well for irregular or non-periodic time series due to the lack of a dynamic learning mechanism. So, we propose an improved approach, i.e. to replace ARMAX with ANFIS for the conditional mean component of composite model because ANFIS has its own self-adaptive learning ability to fit irregular or non-periodic time series. This proposed composite model is rewritten as ANFIS/NGARCH. Formulation of the linear combination [14] is expressed as

$$y_{Proposed\ Composite\ Model}(t) = g(y_{ANFIS}(t),\ y_{NGARCH}(t)) = Coef_1 \cdot y_{ANFIS}(t) + Coef_2 \cdot y_{NGARCH}(t) \tag{4}$$

where g is defined as a linear function of the ANFIS and NGARCH outputs, respectively, $y_{ANFIS}(t)$ and $y_{NGARCH}(t)$, while $Coef_1$ and $Coef_2$ are respectively the coefficients of the linear combination of the ANFIS and NGARCH outputs.

A novel adaptation mechanism, called quantum minimization (QM) [12], is presented in the next section and will be exploited to search for optimal or near-optimal coefficients $Coef_1$ and $Coef_2$ in Eq. (4).

3 Quantum Minimization Adapting ANFIS/NGARCH

3.1 Quantum Exponential Searching Algorithm

As reported in [15], we assume in this section that the number t of solutions is known and that it is not zero. Let $A = \{i \mid F(i) = 1\}$ and $B = \{i \mid F(i) = 0\}$.

Step 1: For any real numbers k and l such that $tk^2 + (N-t)l^2 = 1$, redefine

$$|\Psi(k,l)\rangle = \sum_{i\in A} k\,|i\rangle + \sum_{i\in B} l\,|i\rangle\ .$$

A straightforward analysis of Grover's algorithm shows that one iteration transforms $|\Psi(k,l)\rangle$ into

$$\left|\Psi\left(\frac{N-2t}{N}k+\frac{2(N-t)}{N}l,\frac{N-2t}{N}l-\frac{2t}{N}k\right)\right\rangle.$$

Step 2: This gives rise to a recurrence similar to the iteration transforms in Grover's algorithm [16], whose solution is that the state $|\Psi(k_j,l_j)\rangle$ after j iterations is given by

$$k_j=\frac{1}{\sqrt{t}}\sin((2j+1)\theta)\ \ and\ \ l_j=\frac{1}{\sqrt{N-t}}\cos((2j+1)\theta)\ .$$

where the angle θ is so that $\sin^2\theta = t/N$ and $0<\theta\le\pi/2$.

3.2 Quantum Minimum Searching Algorithm

We second give the minimum searching algorithm [12] in which the minimum searching problem is to find the index i such that $T[i]$ is minimum where $T[0,...,N-1]$ is to be an unsorted table of N items, each holding a value from an ordered set.

Step 1: Choose threshold index $0\le i\le N-1$ uniformly at random.

Step 2: Repeat the following stages (2a and 2b) and interrupt it when the total running time is more than $22.5\sqrt{N}+1.4\lg^2 N$. Then go to stage (2c).

(a) Initialize the memory as $\sum_j \frac{1}{\sqrt{N}}|j\rangle|i\rangle$. Mark every item j for which $T[j]<T[i]$.

(b) Apply the quantum exponential searching algorithm [15].

(c) Observe the first register: let i' be the outcome. If $T[i']<T[i]$, then set threshold index i to i' .

Step 3: Return i

This process is repeated until the probability that the threshold index selects the minimum is sufficiently large.

3.3. QM-AFNG Forecasting Based on Signal Deviation

Single-step-look-ahead prediction, as shown in Fig. 1 and Fig. 2, can be arranged by adding the most recent predicted signal deviation $\delta\hat{o}(k+1)$ of Eq. (5) to the observed current output $o(k)$.

$$\delta\hat{o}(k+1)=h(o(k),o(k-1),...,o(k-s),\delta o(k),\delta o(k-1),...,\delta o(k-s)) \tag{5}$$

$$\hat{o}(k+1)=o(k)+\delta\hat{o}(k+1) \tag{6}$$

Based on the QM-AFNG structure, one can form the function p of the ANFIS output, $\delta\hat{o}_{anfis}(k+1)$, and the square-root of NGARCH's output, $\hat{\sigma}_{\delta\hat{o}}(k+1)$, as presented below and shown in Fig. 1.

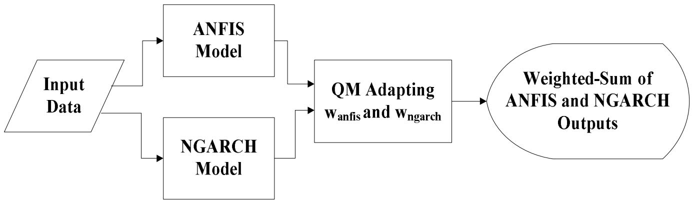

Fig. 1. Diagram of QM adapting ANFIS/NGARCH outputs (denoted as QM-AFNG)

$$\delta\hat{o}_{qm-afng}(k+1) = p(\delta\hat{o}_{anfis}(k+1),\ \hat{\sigma}_{\delta o}(k+1)) \tag{7}$$

A weighted-average function is assumed to combine both $\delta\hat{o}_{anfis}(k+1)$ and $\hat{\sigma}_{\delta o}(k+1)$ to attain a near-optimal result $\delta\hat{o}_{qm-afng}(k+1)$.

$$\begin{aligned} \delta\hat{o}_{qm-afng}(k+1) &= w_{anfis}\cdot\delta\hat{o}_{anfis}(k+1) + w_{ngarch}\cdot\hat{\sigma}_{\delta o}(k+1) \\ s.t. \qquad & w_{anfis} + w_{ngarch} = 1 \end{aligned} \tag{8}$$

Here, the linear combination of two nonlinear functions, $\delta\hat{o}_{anfis}(k+1)$ and $\hat{\sigma}_{\delta o}(k+1)$, can also optimally approximate an unknown nonlinear target $\delta\hat{o}_{qm-afng}(k+1)$. Let $W_{afng} = [w_{anfis}\ w_{ngarch}]^T$ denote a weight-vector of w_{anfis} and w_{ngarch}. A digital cost-function (DCF) [17] is defined as

$$DCF = \frac{\left\|W_{afng}\right\|^2}{2} + Const.\sum_{k=0}^{l-1}\left\|y(k+1) - y(k) - o(k) - \delta\hat{o}_{qm-afng}(k+1)\right\|^2, \tag{9}$$

$Const.$: a constant,

which can be used for measuring the accuracy when the respected cost is minimized. Quantum minimization mentioned above is employed for adapting the appropriate weights, w_{anfis} and w_{ngarch}, for the forecast $\delta\hat{o}_{anfis}(k+1)$ and $\hat{\sigma}_{\delta o}(k+1)$ as per Eq. (8), respectively. Quantum minimization gives an order of computational cost as $O(\sqrt{N})$.

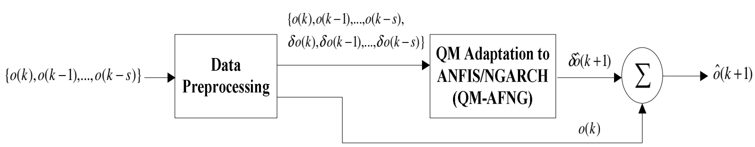

Fig. 2. Prediction using QM-AFNG system

4 Experimental Results and Discussions

In order to justify reasonable accuracy for a time series forecast, four well-known criteria [18] are commonly utilized. The terminology of these criteria is indicated as: (a) mean absolute deviation (MAD); (b) mean absolute percent error (MAPE); (c) mean squared error (MSE); (d) Theil'U inequality coefficient (Theil'U).

$$Theil'U = \sqrt{\frac{MSE}{MS}} = \sqrt{\frac{\sum_{t=1}^{l}\left(y_{t_c+t} - \hat{y}_{t_c+t}\right)^2 \Big/ l}{\sum_{t=1}^{l} y_{t_c+t}^2 \Big/ l}} \tag{10}$$

where l = the number of periods in forecasting, t_c = the current period, y_{t_c+t} = a desired value at the $t_c + t$ th period and $\hat{y}_{t_c+t}$ = a predicted value at the $t_c + t$ th period.

As shown in Figs. 3 to 8, with a sliding widow size of 7 data points, the forecasting abilities of our proposed method and several alternative methods are compared in experiments The alternative methods used are grey model (GM), auto-regressive moving-average (ARMA), back-propagation neural network (BPNN), ARMA/NGARCH composite model (ARMAXNG), adaptive neuro-fuzzy inference system (ANFIS), and the ANFIS/NGARCH composite model adapted by quantum minimization (QM-AFNG). Single-step-look-ahead prediction methodology is employed in all experiments. In single-step-look-ahead design, a small number of the most recent observed data are collected as a sliding window (i.e. data queue) for modeling an intermediate predictor to predict the next period output. Once the next period's sampled datum is obtained, we drop a datum at the bottom of the data queue and add the most recent sampled datum into the data queue at the top position, thereby forming the new data queue used for the next prediction. This process continues until the task is terminated. To simplify comparison of the tested methods as plotted curves, only the three most representatives are shown in the figures. Thus GM, ARMA and the proposed QM-AFNG are illustrated in Figs. 3 to 8, where "$\bullet$" represents the sequential output of GM prediction, "$\circ$" represents the sequential output of ARMA prediction and "$-*-$" represents the sequential output of QM-AFNG prediction. All six methods, however, are compared for goodness-of-fit in Tables 1 to 8.

First, the forecast of international stock price indices of four markets (New York Dow-Jones Industrials Index, London FTSE-100 Index, Tokyo Nikkei Index, and Taipei Taiex Index) [19] are shown in Figs. 3 to 6. In addition, this study shows performance evaluation based on (a) mean absolute deviation (MAD), (b) mean absolute percent error (MAPE) ×100, (c) mean squared error (MSE) (unit=10^5), and (d) Theil'U inequality coefficient (Theil'U) between the actual sampled values and the predicted results of international stock price monthly indices over 48 months from Jan. 2002 to Dec. 2005. Forecasting performance of all six methods is summarized in Tables 1 to 4, showing QM-AFNG obtains the best prediction results. The goodness of fit of QM-AFNG prediction modeling for the four markets is tested by Ljung-Box Q-test [20] with p-values of 0.5082, 0.3239, 0.4751 and 0.3702, where each p-value is greater than the level of significance (0.05).

Second, Figs 7 and 8 show the comparative forecasts of the equity volume index futures and options over 24 months (Jan. 2001 to Dec. 2002) as quoted from the London International Financial Futures and Options Exchange (LIFFE) [21]. Performance evaluation is again made on the basis of MAD, MAPE, MSE, and Theil'U between the actual and predicted values. Tables 5 to 8 summarize prediction performance of our alternative methods and shows that QM-AFNG achieves superior results. The goodness of fit of QM-AFNG prediction modeling for futures and options is also tested by Ljung-Box Q-test with p-values of 0.2677 and 0.1523, in which each p-value is greater than level of significance (0.05).

Table 1. The comparison between different prediction models based on Mean Absolute Deviation (MAD) on international stock price monthly indices

Methods	Mean Absolute Deviation				
	New York D.J. Industrials Index	London FTSE-100 Index	Tokyo Nikkei Index	Taipei TAIEX Index	Average
GM	340.5970	153.8277	477.2157	355.1361	331.6941
ARMA	339.7215	153.7628	439.8190	321.1152	313.6046
BPNN	279.1350	134.5064	453.7069	277.5879	286.2341
ARMAXNG	320.7695	152.3504	437.0319	317.9291	307.0202
ANFIS	284.5725	145.3118	441.5919	296.1719	291.9120
QM-AFNG	274.8238	125.3910	430.0475	269.3103	274.8932

Table 2. The comparison between different prediction models based on Mean Absolute Percent Error (MAPE) on international stock price monthly indices

Methods	Mean Absolute Percent Error (unit=10^{-2})				
	New York D.J. Industrials Index	London FTSE-100 Index	Tokyo Nikkei Index	Taipei TAIEX Index	Average
GM	3.65	3.54	4.49	6.49	4.54
ARMA	3.61	3.53	4.14	5.81	4.27
BPNN	2.98	3.06	4.19	5.05	3.82
ARMAXNG	3.52	3.50	4.12	5.77	4.23
ANFIS	3.06	3.31	4.13	5.40	3.98
QM-AFNG	2.83	2.97	4.05	4.93	3.70

Table 3. The comparison between different prediction models based on Mean Squared Error (MSE) on international stock price monthly indices

Methods	Mean Squared Error (unit=10^5)				
	New York D.J. Industrials Index	London FTSE-100 Index	Tokyo Nikkei Index	Taipei TAIEX Index	Average
GM	1.9582	4.0063	3.2209	1.7472	2.7332
ARMA	1.8230	3.8832	2.9384	1.4737	2.5296
BPNN	1.2652	3.0656	3.0189	1.0461	2.0990
ARMAXNG	1.8170	3.8527	2.9193	1.4772	2.5166
ANFIS	1.3550	3.8494	2.8912	1.1683	2.3160
QM-AFNG	1.1784	2.9536	2.7689	1.0113	1.9781

Table 4. The comparison between different prediction models based on Theil'U Inequality Coefficient (Theil'U) on international stock price monthly indices

Methods	Theil'U Inequality Coefficient				
	New York D.J. Industrials Index	London FTSE-100 Index	Tokyo Nikkei Index	Taipei TAIEX Index	Average
GM	0.0435	0.0414	0.0501	0.0721	0.0518
ARMA	0.0420	0.0408	0.0479	0.0662	0.0492
BPNN	0.0349	0.0362	0.0485	0.0558	0.0439
ARMAXNG	0.0409	0.0406	0.0477	0.0663	0.0489
ANFIS	0.0362	0.0411	0.0475	0.0590	0.0460
QM-AFNG	0.0331	0.0351	0.0463	0.0545	0.0423

Table 5. The comparison between different prediction models based on Mean Absolute Deviation (MAD) on futures and options volumes monthly indices of equity products

Methods	Mean Absolute Deviation		
	Futures Index of Equity Products	Options Index of Equity Products	Average
GM	0.2607	0.0957	0.1782
ARMA	0.1935	0.1198	0.1567
BPNN	0.1022	0.0746	0.0884
ARMAXNG	0.1803	0.0722	0.1263
ANFIS	0.0851	0.0713	0.0782
QM-AFNG	0.0704	0.0668	0.0686

Table 6. The comparison between different prediction models based on Mean Absolute Percent Error (MAPE) on futures and options volumes monthly indices of equity products

Methods	Mean Absolute Percent Error		
	Futures Index of Equity Products	Options Index of Equity Products	Average
GM	0.0441	0.0168	0.0305
ARMA	0.0328	0.0210	0.0269
BPNN	0.0172	0.0131	0.0152
ARMAXNG	0.0305	0.0127	0.0216
ANFIS	0.0144	0.0125	0.0135
QM-AFNG	0.0132	0.0109	0.0121

Table 7. The comparison between different prediction models based on Mean Squared Error (MSE) on futures and options volumes monthly indices of equity products

Methods	Mean Squared Error		
	Futures Index of Equity Products	Options Index of Equity Products	Average
GM	0.0945	0.0138	0.0542
ARMA	0.0547	0.0114	0.0331
BPNN	0.0196	0.0087	0.0142
ARMAXNG	0.0507	0.0096	0.0302
ANFIS	0.0112	0.0092	0.0102
QM-AFNG	0.0093	0.0071	0.0082

Table 8. The comparison between different prediction models based on Theil'U Inequality Coefficient (Theil'U) on futures and options volumes monthly indices of equity products

Methods	Theil'U Inequality Coefficient		
	Futures Index of Equity Products	Options Index of Equity Products	Average
GM	0.0483	0.0191	0.0337
ARMA	0.0303	0.0332	0.0318
BPNN	0.0220	0.0152	0.0186
ARMAXNG	0.0368	0.0183	0.0276
ANFIS	0.0166	0.0155	0.0161
QM-AFNG	0.0152	0.0137	0.0145

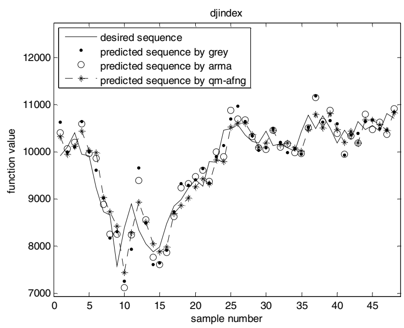

Fig. 3. Forecasts of monthly New York D.J. industry index

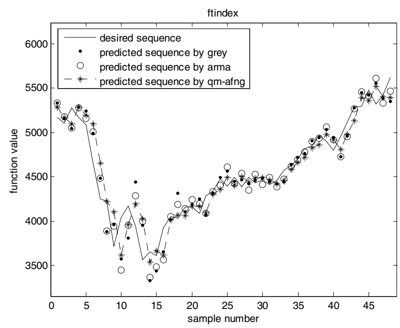

Fig. 4. Forecasts of monthly London FTSE-100 index

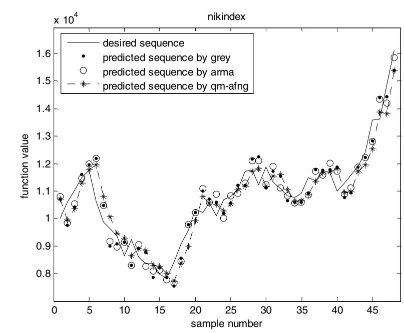

Fig. 5. Forecasts of monthly Tokyo Nikkei index

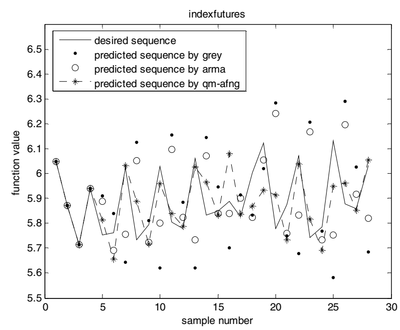

Fig. 7. Forecasts of monthly equity volume index futures

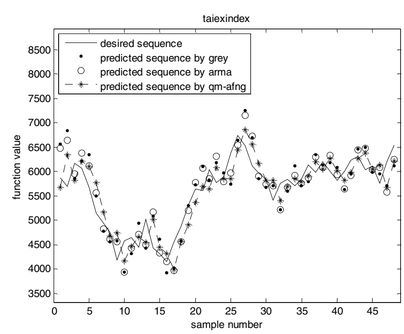

Fig. 6. Forecasts of monthly Taipei Taiex index

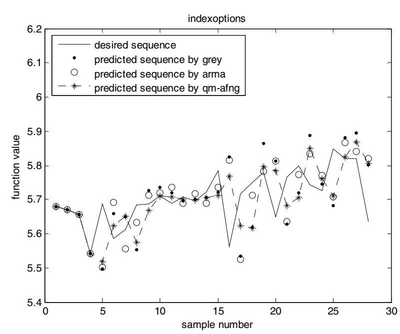

Fig. 8. Forecasts of monthly equity volume index options

5 Concluding Remarks

This study has proposed a method that incorporates a nonlinear generalized autoregressive conditional heteroscedasticity (NGARCH) into an ANFIS approach so as to correct the crucial problem of time-varying conditional variance in residual errors. In this manner, large residual error is significantly reduced because the effect of volatility clustering is regulated to a trivial level. Experimental comparison of a range of systems shows that the ANFIS/NGARCH composite model adapted by QM provides superior prediction accuracy and good computation speed for irregular non-periodic short-term time series forecast.

Acknowledgements

This work is fully supported by the National Science Council, Taiwan, Republic of China, under grant number **NSC 94-2218-E-143-001**.

References

1. Chang, B. R.: Advanced Hybrid Prediction Algorithm for Non-Periodic Short-Term Forecasting. International Journal of Fuzzy System. 5 3 (2003) 151-160
2. Box, G. E. P., Jenkins, G. M., Reinsel, G. C.: Time Series Analysis: Forecasting & Control. Prentice-Hall, New Jersey (1994)
3. Chang, B. R.: Tuneable Free Parameters C and Epsilon-Tube in Support Vector Regression Grey Prediction Model -SVRGM(1,1|C,ε) Approach. In Proc. IEEE International Conference on Systems, Man, and Cybernetics. (2004) 2431-2437
4. Castillo, O., Melin, P.: Simulation and Forecasting Complex Economic Time Series Using Neural Network and Fuzzy Logic. In Proc. International Joint Conference on Neural Network, (2001) 1805-1810
5. Thomson, R., Hodgman, T. C., Yang, Z. R., Doyle, A. K.: Characterizing Proteolytic Cleavage Site Activity Using Bio-Basis Function Neural Networks. Bioinformatics. 19 14 (2003) 1741–1747.
6. Chang, B. R.: Hybrid BPNN-Weighted Grey-CLMS Forecasting. Journal of Information Science and Engineering. 21 1 (2005) 209-221
7. Jang, J.-S. R.: ANFIS: Adaptive-Network-based Fuzzy Inference Systems. IEEE Transactions on Systems, Man, and Cybernetics. 23 3 (1993) 665-685
8. Wang, J.-S.: An Efficient Recurrent Neuro-Fuzzy System for Identification and Control of Dynamic Systems. In Proc. IEEE International Conference on Systems, Man, and Cybernetics. (2003) tracking #: 146
9. Neter, J., Wasserman W., and Kutner M.H.: Applied Linear Statistical Models. 2nd Ed. Homewood, IL: Irwin, (1985)
10. Bellerslve, T.: Generalized Autoregressive Conditional Heteroscedasticity. Journal of Econometrics. 31 (1986) 307-327
11. Gourieroux, C.: ARCH Models and Financial Applications, Springer-Verlag, New York (1997)
12. Durr, C., Hoyer, P.: A Quantum Algorithm for Finding the Minimum. (1996) http://arxiv.org/abs/quant-ph/9607014

13. Hamilton, J. D.: Time Series Analysis. Princeton University Press, New Jersey (1994)
14. Pshenichnyj, B. N., Wilson, S. S.: The Linearization Method for Constrained Optimization. Springer, New York (1994)
15. Boyer, M., Brassard, G., Hoyer, P., Tapp, A.: Tight Bounds on Quantum Searching. Fortschritte Der Physik (1998)
16. Grover, L. K.: A Fast Quantum Mechanical Algorithm for Database Search. In Proc.28th Ann. ACM Symp, Theory of Comp., ACM Press (1996) 212-219
17. Anguita, D., Ridella, S., Rivieccio F., Zunino R.: Training Support Vector Machines: a Quantum- Computing Perspective. In Proc. IEEE IJCNN. (2003) 1587-1592
18. Diebold, F. X.: Elements of Forecasting. South-Western, Cincinnati (1998)
19. FIBV FOCUS MONTHLY STATISTICS, International Stock Price Index (2005)
20. Ljung, G. M., Box, G. E. P.: On a Measure of Lack of Fit in Time Series Models. Biometrika. 65 (1978) 67-72
21. London International Financial Futures and Options Exchange (LIFFE). http://www.liffe.com/, 2002