Multiple Models Fuzzy Decoupling Controller for a Nonlinear System*

Xin Wang[1,2], Hui Yang[2], and Bing Wang[3]

[1] Center of Electrical & Electronic Technology, Shanghai Jiao Tong University, Shanghai, P.R. China, 200030
wangxin26@sjtu.edu.cn
[2] School of Electrical & Electronic Engineering, East China Jiaotong University, Jiangxi, P.R. China, 330013
[3] School of Information Engineering, Shandong University at Weihai, 264209

Abstract. For a discrete-time nonlinear MIMO system, a multiple models fuzzy decoupling controller is designed. At each equilibrium point, the system is expanded into linear and nonlinear terms. The linear term is identified using one FLSs, while nonlinear term using other FLSs, which compose one system model. Then, all models got at all equilibrium points compose the multiple models set. At each instant, the best model is chosen out according to the switching index. Accordingly, the nonlinear term of the best model is viewed as measurable disturbance and eliminated using the feedforward strategy.

1 Introduction

In recent years, for nonlinear MIMO systems, few works have been observed. Ansari *et al.* simplified a nonlinear system into a linear system by using Taylor's expansion at the equilibrium point and controlled it using a linear adaptive decoupling controller accordingly [1]. However, for a system with strong nonlinearity, it can not get good performance. In [2], an exact linear system can be produced utilizing a feedback linearization approach. But accurate information must be known precisely. Furthermore, a variable structure controller with sliding mode was proposed [3], which requires the system an affine system. Although the design methods above can realize nonlinear decoupling control, there were too many assumptions required. To solve this problem, intelligent decoupling controller was proposed. A hierarchical fuzzy sliding mode controller was designed for SIMO nonlinear system [4]. In [5], two NNs were needed to identify the linear and nonlinear terms expanded using Taylor's formula at the origin. Unfortunately, when the equilibrium point was far from the origin, the system lost its stability.

In this paper, a Multiple Models Fuzzy Decoupling Controller (MMFDC) is designed. Utilizing Taylor's formula, the linear term is the first-order derivative of the

* This work is supported by National Natural Science Foundation (No. 60504010, 50474020) and Shanghai Jiao Tong University Research Foundation.

L. Wang et al. (Eds.): FSKD 2006, LNAI 4223, pp. 860 – 863, 2006.

system and identified using Fuzzy Logic Systems (FLSs) while the nonlinear term using other FLSs, which compose one system model. All models compose the multiple models set. At each instant, one best model is chosen out. The nonlinear term of the above model is viewed as measurable disturbance and eliminated.

2 Description of the System

The system is a nonlinear MIMO system of the form $\boldsymbol{y}(t+1)=\boldsymbol{f}[\boldsymbol{y}(t),\cdots,\boldsymbol{u}(t),\cdots]$, where $\boldsymbol{u}(t)$, $\boldsymbol{y}(t)$ are $n\times 1$ input, output vectors respectively and $\boldsymbol{f}[\cdot]$ is a vector-based nonlinear function which is continuously differentiable and Lipshitz.

Suppose that $(\boldsymbol{u}_1,\boldsymbol{y}_1),\cdots(\boldsymbol{u}_l,\boldsymbol{y}_l),\cdots(\boldsymbol{u}_m,\boldsymbol{y}_m)$ are m equilibrium points. At each equilibrium point $(\boldsymbol{u}_l,\boldsymbol{y}_l)$, using Taylor's formula, it obtains

$$\begin{aligned}\boldsymbol{y}(t+1)=\boldsymbol{y}_l+\sum_{n_1=1}^{n_a}\boldsymbol{f}'_{n_1}\Big|_{\substack{u=u_l\\ y=y_l}}\cdot[\boldsymbol{y}(t-n_a+n_1)-\boldsymbol{y}_l]+\sum_{n_2=0}^{n_b}\boldsymbol{f}'_{n_2}\Big|_{\substack{u=u_l\\ y=y_l}}\cdot[\boldsymbol{u}(t-n_b+n_2)-\boldsymbol{u}_l]\\ +\boldsymbol{o}[\boldsymbol{x}(t)],\end{aligned}\tag{1}$$

where $\boldsymbol{f}'_{n_1}=\dfrac{\partial \boldsymbol{f}}{\partial \boldsymbol{y}(t-n_a+n_1)}$, $\boldsymbol{f}'_{n_2}=\dfrac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}(t-n_b+n_2)}$, $\boldsymbol{x}(t)=[\boldsymbol{y}(t)-\boldsymbol{y}_l,\cdots;\boldsymbol{u}(t)-\boldsymbol{u}_l,\cdots]$, $\boldsymbol{o}[\boldsymbol{x}(t)]$ satisfies $\lim\limits_{\|\boldsymbol{x}(t)\|\to 0}\dfrac{\|\boldsymbol{o}[\boldsymbol{x}(t)]\|}{\|\boldsymbol{x}(t)\|}=0$, where $\|\cdot\|$ is the Euclidean norm operator.

Define $\bar{\boldsymbol{y}}(t)=\boldsymbol{y}(t)-\boldsymbol{y}_l$, $\bar{\boldsymbol{u}}(t)=\boldsymbol{u}(t)-\boldsymbol{u}_l$, $\boldsymbol{v}(t)=\boldsymbol{o}[\boldsymbol{x}(t)]$, $\boldsymbol{A}^l_{n_1}=(-1)\cdot\boldsymbol{f}'_{n_1}\Big|_{\substack{u=u_l\\ y=y_l}}$, $\boldsymbol{B}^l_{n_2}=\boldsymbol{f}'_{n_2}\Big|_{\substack{u=u_l\\ y=y_l}}$, $\boldsymbol{A}^l(z^{-1})=\boldsymbol{I}+\cdots+\boldsymbol{A}^l_{n_a}z^{-n_a}$, $\boldsymbol{B}^l(z^{-1})=\boldsymbol{B}^l_0+\cdots+\boldsymbol{B}^l_{n_b}z^{-n_b}$, the system (1) is

$$\boldsymbol{A}^l(z^{-1})\bar{\boldsymbol{y}}(t+1)=\boldsymbol{B}^l(z^{-1})\bar{\boldsymbol{u}}(t)+\boldsymbol{v}(t)\,.\tag{2}$$

3 Design of MMFDC

At each equilibrium point $(\boldsymbol{u}_l,\boldsymbol{y}_l)$, one group of FLSs is utilized to approximate the system's first-order derivative offline. So $\hat{\boldsymbol{A}}^l(z^{-1})$ and $\hat{\boldsymbol{B}}^l(z^{-1})$ are obtained. The other is employed to approximate the nonlinear term $\boldsymbol{v}(t)$ online.

For the multiple models set, the switching index is chosen as $J_l=\|\boldsymbol{e}^l(t)\|^2=\|\boldsymbol{y}(t)-\boldsymbol{y}^l(t)\|^2$, where $\boldsymbol{e}^l(t)$ is the output error between the real system and the model *l*. $\boldsymbol{y}^l(t)$ is the output of the model *l*. Let $j=\arg\min(J_l)$ correspond to the model whose output error is minimum, then it is chosen as the best.

For the system (2), like the conventional controller design, the cost function is as $J_c=\|\boldsymbol{P}(z^{-1})\bar{\boldsymbol{y}}(t+k)-\boldsymbol{R}(z^{-1})\boldsymbol{w}(t)+\boldsymbol{Q}(z^{-1})\bar{\boldsymbol{u}}(t)+\boldsymbol{S}(z^{-1})\boldsymbol{v}(t)\|^2$, where $\boldsymbol{w}(t)$ is the known reference signal, $\boldsymbol{P}(z^{-1}),\boldsymbol{Q}(z^{-1}),\boldsymbol{R}(z^{-1})$, $\boldsymbol{S}(z^{-1})$ are weighting polynomial matrices respectively. Introduce $\boldsymbol{P}(z^{-1})=\boldsymbol{F}(z^{-1})\boldsymbol{A}(z^{-1})+z^{-1}\boldsymbol{G}(z^{-1})$, the control law is

$$\boldsymbol{G}(z^{-1})\overline{\boldsymbol{y}}(t)+[\boldsymbol{F}(z^{-1})\boldsymbol{B}(z^{-1})+\boldsymbol{Q}(z^{-1})]\overline{\boldsymbol{u}}(t)+[\boldsymbol{F}(z^{-1})+\boldsymbol{S}(z^{-1})]\boldsymbol{v}(t)=\boldsymbol{R}\boldsymbol{w}(t)\,. \tag{3}$$

Combing (3) with (2), the closed loop system equation is obtained as follows

$$\begin{aligned}\left[\boldsymbol{P}(z^{-1})+\boldsymbol{Q}(z^{-1})\boldsymbol{B}^{-1}(z^{-1})\boldsymbol{A}(z^{-1})\right]\overline{\boldsymbol{y}}(t+1)=&\ \boldsymbol{R}(z^{-1})\boldsymbol{w}(t)\\ &+\left[\boldsymbol{Q}(z^{-1})\boldsymbol{B}(z^{-1})^{-1}-\boldsymbol{S}(z^{-1})\right]\boldsymbol{v}(t)\,.\end{aligned} \tag{4}$$

To eliminate the nonlinear form and the interactions of the system exactly, let $\boldsymbol{Q}(z^{-1})=\boldsymbol{R}_1\boldsymbol{B}(z^{-1})$, $\boldsymbol{S}(z^{-1})=\boldsymbol{R}_1$, $\boldsymbol{P}(z^{-1})+\boldsymbol{R}_1\boldsymbol{A}(z^{-1})=\boldsymbol{T}(z^{-1})$, $\boldsymbol{R}(z^{-1})=\boldsymbol{T}(1)$, where $\boldsymbol{T}(z^{-1})$ is a stable diagonal polynomial matrix decided by the designer and $\boldsymbol{R}_1$ is a constant matrix. Then the closed loop system is derived as $\boldsymbol{T}(z^{-1})\boldsymbol{y}(t+k)=\boldsymbol{T}(1)\boldsymbol{w}(t)$. By the choice of weighting polynomial matrixes, it not only decouples the system dynamically but also places poles arbitrarily.

4 Simulation Studies

A discrete-time nonlinear multivariable system is described as follows

$$\begin{cases} y_1(t+1)=\dfrac{-0.2y_1(t)}{1+y_1^2(t)}+\sin[u_1(t)]-0.5\sin[u_1(t-1)]+1.5u_2(t)+0.2u_2(t-1)\\ y_2(t+1)=0.6y_2(t)+0.2u_1(t)+1.3u_1(t-1)+u_2(t)+u_2^2(t)+\dfrac{1.5u_2(t-1)}{1+u_2^2(t-1)}\end{cases} \tag{5}$$

which is the same as the simulation example in [5]. The known reference signal $\boldsymbol{w}$ is set to be a time-varying signal. When $t=0$, w_1 equals to 0 and when t is 40, 80, 120, 160, 200, it changed into 0.05, 0.15, 0.25, 0.35, 0.45 respectively.

In Fig.1 and 2, the system (5) is expanded only at the original point (0,0) and a Fuzzy Decoupling Controller (FDC) is used. In Fig.3 and 4, the system is expanded at six equilibrium points, *i.e.* $[0,0]^{\mathrm{T}}$, $[0.1,0]^{\mathrm{T}}$, $[0.2,0]^{\mathrm{T}}$, $[0.3,0]^{\mathrm{T}}$, $[0.4,0]^{\mathrm{T}}$.

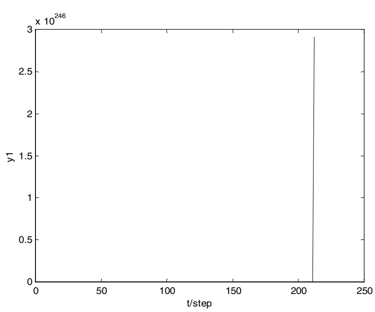

Fig. 1. The output $y_1(t)$ of FDC

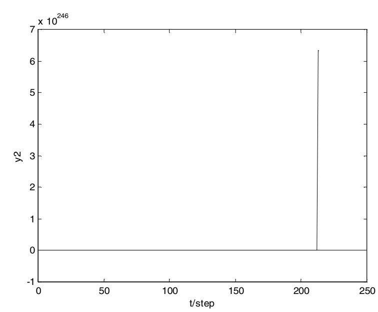

Fig. 2. The output $y_2(t)$ of FDC

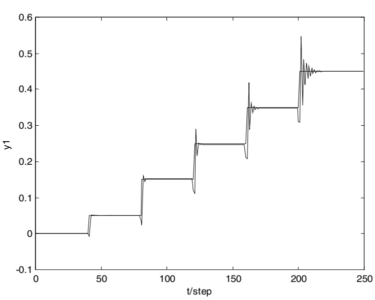

Fig. 3. The output $y_1(t)$ of MMFDC

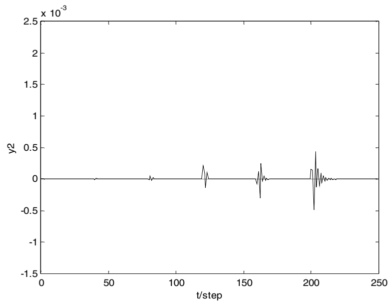

Fig. 4. The output $y_2(t)$ of MMFDC

Note that the equilibrium points are far away from the set points. The results show that although the same FDC method is adopted, the system using FDC loses its stability (see Fig.1 and 2), while the system using MMFDC not only gets the good performance but also has good decoupling result (see Fig.3 and 4).

5 Conclusion

A MMFDC is designed to control the discrete-time nonlinear multivariable system. At each equilibrium point, one group of FLSs is used offline to identify the linear term of the nonlinear system and the other is trained online to identify the nonlinear term. The multiple models set is composed of all models, which are got from all equilibrium points. According to the switching index, the best model is chosen as the system model. The nonlinear term is viewed as measurable disturbance and eliminated using feedforward strategy. The simulation example shows that the effectiveness of the controller proposed.

References

1. Ansari R.M., Tade M.O.: Nonlinear Model-based Process Control: Applications in Petroleum Refining. Springer, London (2000)
2. Wang W.J., Wang C.C.: Composite Adaptive Position Controller for Induction Motor Using Feedback Linearization, IEE Proceedings D Control Theory and Applications, 45 (1998) 25–32
3. Wai R.J., Liu W.K.: Nonlinear Decoupled Control for Linear Induction Motor Servo-Drive Using The Sliding-Mode Technique. IEE Proceedings D Control Theory and Applications, 148 (2001) 217–231
4. Lin C.M., Mon Y.J.: Decoupling Control by Hierarchical Fuzzy Sliding-Mode Controller. IEEE Transactions on Control Systems Techonology. 4 (2005) 593–598
5. Yue H., Chai T.Y.: Adaptive Decoupling Control of Multivariable Nonlinear Non-Minimum Phase Systems Using Neural Networks. Proceedings of the American Control Conference. (1998) 513–514