A Novel Fourier Descriptor for Shape Retrieval

Bin Wang[1] and Chaojian Shi[1,2]

[1] Department of Computer Science and Engineering, Fudan University, Shanghai, 200433, P.R. China
[2] Merchant Marine College, Shanghai Maritime University, Shanghai, 200135, P.R. China
wangbin.cs@fudan.edu.cn, cjshi@shmtu.edu.cn

Abstract. A novel Fourier descriptor (FD), which is derived from chord-length functions (CLF) obtained through equal-arc-length partitions of a contour, is proposed. The proposed FD is tested on a standard shape database and experimental results show that it outperforms the existing FDs which are derived from other shape signatures.

1 Introduction

Shape-based image retrieval is a hot topic in image processing and pattern recognition. Its applications can be found in many areas, such as meteorology, medicine, space exploration, manufacturing, entertainment, education, law enforcement and defense. Shape retrieval includes three primary issues: shape description, shape similarity measure and shape indexing. Among them, shape description is the most important issue.

Fourier descriptor (FD) [1,2] is one of the widely used shape descriptors. In general, the FD is obtained by applying a Fourier transform on a shape signature. A shape signature is any 1D function representing 2D areas or boundaries. Since different shape signature will lead to different FD, the performance of FD method is affected by the shape signature. Till now, many shape signatures, such as complex coordinates, centroid distance, tangent angle, curvature, cumulative angle and so on, have been proposed for deriving FD. Zhang et al.[3] compared six different FDs which are derived from different shape signatures. They claim that the FD derived from centroid distance signature is significantly better than those derived from the other shape signatures.

To further improve the performance of FD, we develop a novel shape signature, chord-length functions (CLF), which is obtained by partitioning the contour into arcs of the same length. The advantage of CLF is that it can capture both the global and local shape features. Therefore, CLF can characterize the shape more accurately. Experimental results show that our method can achieve higher retrieval performance than the existing FDs.

2 Brief Review of Fourier Descriptor

A contour C can be denoted as an ordered sequence of N coordinate points, $C = \{\lambda_t = (x(t), y(t)), t = 0, 1, \ldots, N-1\}$, where C is closed, i.e. $\lambda_{i+N} = \lambda_i$. Suppose

L. Wang et al. (Eds.): FSKD 2006, LNAI 4223, pp. 822–825, 2006.

that $r(t)$ is a shape signature derived from contour C. One dimensional Fourier transform is then applied on $r(t)$ to obtain the Fourier transform coefficients

$$U(n) = \frac{1}{N} \sum_{t=0}^{N-1} r(t) exp(\frac{-j2\pi nt}{N}),\ n = 0, 1, \ldots, N-1. \tag{1}$$

Since the signal, $r(t)$, is real, there are only $N/2$ different frequencies in the result of Fourier transformation (magnitude of the frequency response is symmetric) and the feature vector is formed by the first $N/2$ coefficients corresponding to the low frequency components. Scale invariance is achieved by dividing the magnitude values of FD, rotation invariance is achieved by taking only the magnitude values of the FD. The invariant feature vector used to describe the shape consists of the first $N/2$ coefficients, that is,

$$f = \left[\frac{|U(1)|}{|U(0)|}, \frac{|U(2)|}{|U(0)|}, \ldots, \frac{|U(N/2)|}{|U(0)|} \right]^T \tag{2}$$

3 The Proposed Fourier Descriptor

3.1 Chord-Length Function (CLF)

Let $L = \sum_{i=0}^{N-1} d(\lambda_i, \lambda_{i+1})$ be the perimeter of the contour C, where $d(\lambda_i, \lambda_{i+1})$ is Euclidean distance between points λ_i and λ_{i+1}, Let us start from a point $\lambda_i \in C$ and follow the contour anti-clockwise to equally divide it into k sections $\widehat{\lambda_i s_1}, \widehat{s_1 s_2}, \ldots, \widehat{s_{k-1} \lambda_i}$, and obtain $k-1$ chords $\overline{\lambda_i s_1}, \overline{\lambda_i s_2}, \ldots, \overline{\lambda_i s_{k-1}}$, where s_j is the jth division point and $k > 1$ is a pre-specified parameter. We now have $k-1$ chord lengths $L_1^{(i)}, L_2^{(i)}, \ldots, L_{k-1}^{(i)}$, where $L_j^{(i)}$ is the length of the chord $\overline{\lambda_i s_j}$, i.e. the Euclidean distance between the points λ_i and s_j.

As point λ_i moves along the contour, the chord lengths $L_j^{(i)}, j = 1, \ldots, k-1$, vary accordingly. In other words, $L_j^{(i)}$ are functions of λ_i. Without loss of generality, we specify λ_0 as the reference point. Then each point λ_i can be uniquely identified with the length $l_i \in [0, L]$ of arc $\widehat{\lambda_0 \lambda_i}$. Therefore each chord length $L_j^{(i)}$ can be considered as a function of arc length l_i. Then we obtain a set of chord length functions $\Phi = \{L_1, L_2, \ldots, L_{k-1}\}$. Since $L_j(l) = L_{k-j}(l + j \cdot L/k), j = 1, 2, \ldots, k-1$, only half of the set of chord-length functions are needed for description shape, i.e. $\Phi_h = \{L_1, L_2, \ldots, L_{k/2}\}$.

From the definition of the chord-length functions, we can see that the value of function L_j is the length of the chord corresponding to the arc whose length is $j \cdot L/k$. Different level chords which correspond to arcs with different length are used to characterize the shape and in these chord-length functions, both the global feature and local feature can be reflected. Therefore, CLF descriptor is superior to the existing shape signatures such as centroid distance, curvature function and so on. It should be pointed out that k is the only parameter of CLF. The larger the k is, the smaller the partitions will be and the more details of the boundary will be described. So if we expect higher accuracy in shape distinction, k will be set larger.

3.2 The FD Using CLF for Shape's Difference Measure

We have proposed a shape description CLF $\Phi_h = \{L_1, L_2, \ldots, L_{k/2}\}$. One dimensional Fourier transformation is then applied on each chord-length function L_i, and an invariant feature vector which is similar with Eq. 2 is obtained as follows

$$f_i = [\mu_1^{(i)}, \mu_2^{(i)}, \ldots, \mu_{N/2}^{(i)}]^T \tag{3}$$

The set of feature vectors $\{f_1, f_2, \ldots, f_{k/2}\}$ which is invariant to translation, scaling and rotation is then used to describe the shape.

Suppose $F^{(A)} = \{f_1^{(A)}, f_2^{(A)}, \ldots, f_{k/2}^{(A)}\}$ and $F^{(B)} = \{f_1^{(B)}, f_2^{(B)}, \ldots, f_{k/2}^{(B)}\}$ are FDs of shape A and shape B derived from CLF. The difference between shape A and shape B is then defined as follows

$$d(A, B) = (\sum_{i=1}^{k/2} \mid f_i^{(A)} - f_i^{(B)} \mid^2)^{1/2} \tag{4}$$

where $| \cdot |$ denotes the Euclidean distance between two feature vectors.

4 Experimental Results and Discussions

To evaluate the retrieval performance of the proposed FD method, we use a standard shape database, MPEG-7 Part B [4], which consists 1400 images: 70 shape

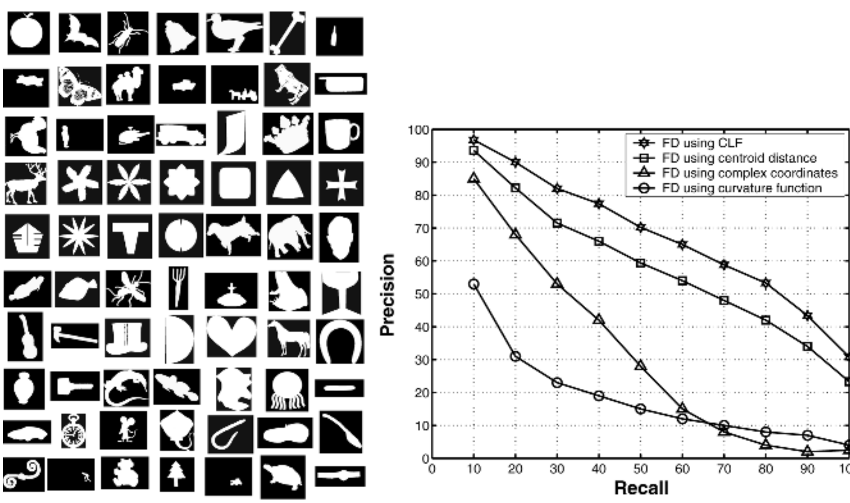

Fig. 1. Left: MPEG-7 Part B, 70 categories with 20 shapes each. Right: Precision-Recall plot of the proposed FD and other three classical FDs.

categories as shown in Fig. 1(Left), 20 images per category. Three widely used FDs which are derived from centroid distance signature, complex coordinates and curvature function, respectively, are selected for comparison.

The commonly used retrieval performance measurement, precision and recall [5] are adopted as evaluation of the query results. The precision P and recall R are calculated as $P = r/n$ and $R = r/m$, where r is the number of retrieved relevant shapes, n is the total number of retrieved shapes and m is the total number of relevant shapes in the whole database. For each shape in the database, take it as a query to match all the shapes in the database. The precision at each level of recall is recorded. The final precision of the retrieval for a certain FD is the average precision of all the queries in the query set. The resulting precision and recall for different FDs are plotted in Fig. 1(Right), where the parameter for CLF is set to $k = 8$.

From the precision and recall plot, we can see that the proposed FD achieves higher precision at each level of recall than the other three FDs.

5 Conclusion

We have presented a FD which is derived from a novel shape signature: chord-length functions (CLF). CLF is obtained through equal-arc-length partitions of a contour. The proposed FD has been tested on a standard shape database and the experimental results show that it outperforms other existing FDs.

Acknowledgement

The research work in this paper is partially sponsored by Shanghai Leading Academic Discipline Project, T0603.

References

1. Chellappa, R., Bagdazian, R.: Fourier Coding of Image Boundaries. IEEE Tans. Pattern Anal. Mach. Intell. **6(1)** (1984) 102-105
2. Wallace, T. P., Wintz, P. A.: An Efficient Three-Dimensional Aircraft Recognition Algorithm Using Normalized Fourier Descriptors. Computer Graphics and Image Processing. **13** (1980) 99-126
3. Zhang, D., Lu, G.: Study and Evaluation of Different Fourier Methods for Image Retrieval. Image and Vision Computing. **23** (2005) 33-49
4. Latecki, L. J., Lakamper, R., Eckhardt, U.: Shape Descriptors for Non-Rigid Shapes with A Single Closed Contour. IEEE Conf. on Comp. Vis. and Patt. Recog. (2000) 424-429
5. Bimbo, A. Del.: Visual Information Retrieval. Morgan Kaufmann Publishers. Inc, San Francisco. CA. (1999) 56-57