# Agent-Based Simulation Versus Econometrics – from Macro- to Microscopic Approaches in Route Choice Simulation

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**Abstract.** Econometrics is nowadays an established approach to the discrete choice problem relying on statistical methods. It is used in several fields, e.g. route choice modelling, telecommunication analysis, etc. Despite its advantages, there are also some drawbacks. Thus, alternatives for modelling human choice are sought, which can reproduce overall system behavior and be valid at microscopic level.

In this paper, we propose an agent-based approach inspired in econometric techniques producing similar results on the macro level from microscopic behavior. This work aims to be a step forward on searching an alternative for econometrics.

## 1 Motivation

Discrete choice problems have to be solved by humans in several domains and contexts. Examples are route selection for driving to work or choosing a particular shop to go. Thus, it is highly interesting to find an appropriate modelling approach for analysing that decision making process. Econometrics discrete choice models form the currently most used approach for representing such process not only in the traffic domain. However, it is a macroscopic approach based on utility functions and rational decision making.

However using an agent-based approach relying on local information for reproducing particular macroscopic data, is by no way trivial. Due to the local population problem, it may be not possible to reproduce the results of macro models without extensive parameter calibration [1]. The aim of this paper is to present and discuss an agent-based approach that actually is able to produce similar results. This may form the basis for more elaborated agent-based route choice simulations.

The remainder of the paper is structured as follows. We will first introduce the basic concepts for econometric approach to discrete choice problem modelling. After discussing advantages and drawbacks, we introduce and comment an econometric inspired agent-based model. Following that we show a case study, where combined route and model choice decision is taken.Then the results are presented, that came from our agent-based model. By the end are shown a short comparison to related work, conclusion and a short outlook to future work.

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## 2 Discrete Choice Modelling

Discrete Choice means that participants must chose an option or alternative from a set where the taken option has the best utility for them. In that case, the chosen option of participant $n$ is given by the expression 1, where $i_n$ is the option taken by the $n$, $O$ is the set of options and $u(i', n)$ is the utility of alternative $i'$ for participant $n$.

$$i_n = argmax_{i' \in O}(u(i', n)) \tag{1}$$

Usually econometric models aim at describing and modelling the situation from a macroscopic point of view. That is made based on data sampling, i. e., a potentially large number of participants that have to select between options. It also assumes that every participant takes the alternative with maximum utility for him. However, the participants' function $u(i, n)$ is neither completely known nor formally expressible by a modeller. The partial observable knowledge about utility is generalised for all participants and expressed in $V_i$. It is formalised in equation 3 and further explaind on section 2.1.

To express everything that is unobservable or unknown, an error component $\varepsilon$ is introduced. That component actually represents a drawn from a particular statistical distribution, like Normal Distribution. Thus, the general utility expression is given by equation 2. There $U_i$ is the utility of alternative $i$ generalised for all participants. All errors made when generalising from the individual $u(i, n)$ are integrated into $\varepsilon$. The closer $U_i$ is to every $u(i, n)$, the better is the econometric model from a macroscopic point of view.

$$U_i = V_i + \varepsilon \tag{2}$$

$$V_i = \sum_{a \in Attributes} f(\beta_a, a_i) \tag{3}$$

Econometric approaches to the discrete choice problem differ in the way $V$ and $\varepsilon$ are characterised and in how the options are organised. Examples are Logit [2], Probit [3] and derived models form Generalised Extreme Value ([4] and [5]). Examples from the latter are Nested-Logit [6] and Mixed-Logit [7]. In case of Logit, the error distribution, or values for $\varepsilon$, is the logistic function and in case of Nested-Logit it is the extreme value distribution.

### 2.1 Estimating Econometric Models

Usually, the observable utility $V_i$ is computed based on an utility function. That expression combines all attributes of an option and corresponding scale/sensibility factor $\beta$. Mostly that expression means summation, like in equation 3.

The utility of a particular option is a function that has as parameters all weights $\beta_a$ and its corresponding attributes value $a$ for that option $i$. A simple example for that function would be $f(\beta_a, a_i) = \beta_a \cdot a_i$. Note that $\beta_a$ depends only on the attribute and not on the alternative. The set of all $\beta$s is the called weight set and here referred as $B$ and the attributes' set as $A$.

For actually building an econometric model, the following problems have to be solved in advance:

- Find the relevant set of attributes $A$;
- Model the function $f(\beta_a, a_i)$ and
- Chose an appropriate econometric approach, like Nested-Logit.

As result of a that process a probability expression, that depends on $B$ and $A$, for all alternatives is given. That set of functions express the altenatives' distribution. Actually, the expression $P(i)$ – probability of alternative $i$ – is almost solved but $B$ must be estimated.

The first problem, when using more sofisticated econometrics approaches like Mixed-Logit [7], can emerge. That problem is to solve $P(i)$. For more simple approaches, like Logit and Nested-Logit, that function is already derived but for others it must be derived. The deduction relies not only on the approach itself (Logit, Probit or Mixed-Logit) but also on alternatives organisation and on utility function's nature. And because of that an analytical solution may not be possible.

It was stated that $P(i)$ is a function of $B$ and that the later must be estimated. That means that the $B^*$ must be found. A $B^*$ express the better possible fine-tune of $P(i)$, or the better $P(i)$ can be approximated to participants real decisions. $B^*$ is usually searched using a maximum likelihood estimator. In a very abstract explanation, that estimator works evaluating a particular hypothesised weight set $B_h$. Then a quality measure is calculated with a *likelihood* or $log - likelihood$ function. They compare $P(i)$, using $B_h$, with the real share. After that, according to its quality – using $log - likelihood$ for instance – a new $B_{h+1}$ is proposed.

Here a very simple example of that process using a Logit[2] model is taken (chosen because of its simplicity). It is also assumed that $V_{ni} = B_h \cdot a_{ni}$, where $V_{ni}$ is the rational utility of alternative $i$ observed for participant $n$. That posted, a probability expression $P_{ni}$ (from [5]) for every participant $n$ and every alternative $i$ can be derived as shown on equation 4.

$$P_{ni} = \frac{e^{B_h \cdot a_{ni}}}{\sum_j e^{B_h \cdot a_{nj}}} \tag{4}$$

With equation 4 one can calculate the likelihood between the model and real data, using a *likelihood* or $log - likelihood$ function, comparing $P(i)$ with $P_{ni}$. Then an estimation (using a *Maximum Likelihood Extimator*) must be made on $B_h$ until it achieves $B^*$, where the *likelihood* can no longer be improved.

## 2.2 Advantages and Disadvantages

Econometric modelling approaches have several advantages that make them highly attractive for describing discrete choice behaviour. Once the optimal weight set $B^*$ is computed a function is given as result. More than that:

- The weight set $B^*$ forms a compact representation of the relevance of distinct attributes in relation to others. Thus, the model can be easily analysed in this direction.
- If attribute values are changed or the option set is manipulated, the effects on the participants distribution $P(i)$ can be immediately computed and thus predicted. It has been shown in different applications, that this prediction is possible [8]. Of course the preciseness of a prediction depends on how changes affect alternatives.

Once $B^*$ is computed, the model is complete and can be used in an elegant way. However, this elegance has costs the generalisation of agents' utility function. As the macro approach assumes homogeneous utility functions, the weights are the same for all participants. Therefore, it cannot take into account individual evaluation of attributes and options, and thus no individual behaviour.

Every participant is assumed to evaluate and select on its own, independently from the others decisions. Thus, direct relations between decisions are very hard to tackle.

The basic structure of the utility function is mostly very simple, namely a linear combination of weights and attribute values – although the computation of these values can be sophisticated. Theoretically the utility function itself may also contain complicated computations. We assume that the practical use is restricted because the function has to be evaluated very often during the already costly search for $B^*$.

Moreover, the participants are not seen as capable of adaptation or evolution. That means that, the final model is just able to describe the current state, i.e., the weights of the participants in the given situation. Therefore, the validity of the model depends on the situation's stability. For using the model in a different situation one has to rely on modeller's experience, to adapt the existing model.

## 3 From Econometrics to Agents

Here, the called agent-based econometric model is a multi-agent system, where agents use a decision making algorithm based on rational utility. The first difference is individualisation, i.e., the model is now microscopic. That means that, all participants are simulated, through its agent representation. Because the agents in the proposed model can adapt themselves, they achieve an "optimum" by an iterative process. Note that in this context, "optimum" means equivalence to original econometric macroscopic model results.

### 3.1 Agent-Based Econometric Model

In our first approach – devoted to reproducing macroscopic econometrics – we restricted agents' individualism. All values that concern resources' attributes are pre-calculated and all agents share the same values. For example, in traffic, the relevant attribute could be route segment cost, that is a function of segment's occupancy and capacity. In that case, all agents share the same segment cost function – a generalisation from econometrics.

Moreover, all information about the world is available to all agents. And also agents are neither allowed to communicate with each nor share information directly, i.e., they can just observe the others actions' effects.

The adaptation capability is restricted to weighting resources' attributes and taking an alternative based on that weights. To take a decision and to improve its own profit an agent can fine tune its decision making algorithm but not change it.

Those restrictions are necessary to stay as close as possible to the standard econometric approach for facilitating reproduction. Therefore, agents' weighting or decision making fine tune procedures mimic "optimum's" search, or searching for $B^*$. That "optimum" is achieved when system's equilibrium is achieved. It happens through the local "greedy" agent's maximisation decision making method.

### 3.2 Potential and Drawbacks

Some disadvantages from econometrics also appears on agent-based approach. Modelling problem is always present and for agents that means developing an agent's reasoning algorithm. Since the model must be computable it implies into simplifications or restrictions. But agents could lead to a better approximation, i.e., nearer to the real world behaviour.

Another problem is human behaviour modelling and here two characteristics are important: adaptation and evolution capabilities. Those characteristics are relevant to avoid agents remodelling in face of new situations. And due to the fact that the closer to human behaviour the more complex is a model to compute, computational complexity became an issue. By being a multi-agent systems (MAS), all participants are represented and have their own reasoning algorithm. Thus, the computational resources needed will be probably higher than those needed for econometrics. It also could implies in a longer stabilisation process.

The individualisation leads also to a lack of clarity on participants decision making process. With an econometric model the attributes relevance is evident, when not obvious. That can be made by glancing the final utility expression and that is not true for MAS. To evaluate attributes significance a statistical analysis over the agents is needed, if that analysis is possible.

On the other hand there are advantages in using agents. As all participants are present, so the utility function $u(i, n)$ is better approached. That leads to a lesser $\varepsilon$'s importance, so a better treatment to the unobservable/unknown utility's part. The unknown behaviour can be partially modelled by a heuristic, like reinforment-learning, which is not possible in econometrics. More than that, agents' heterogeneity can be easily modelled, like different social classes or sex (if that plays a role on the decision making observed behaviour).

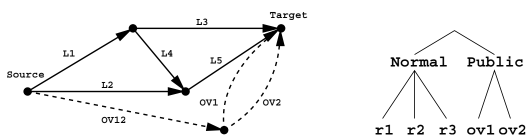
But the most significant difference is adaptation and evolution. When correctly modelled, it prolongs model's validity, in case of changes in alternatives' set. Using standard econometrics, a modeller has either to determine by hand how the distribution changes or re-estimate the complete model, when a new alternative is added or an existing one is removed. Using adaptation, this can be done without external interference or calibration. A user must just wait until the system is stabilised for the new situation. Usually a user's or modeller's direct manipulation on the model is an error prone procedure.

Other advantage from MAS is the absence of a special estimation method or a derived distribution function ($P(i)$). The optimum is achieved by an iterative agents' stabilisation process.

## 4 Application Example: Combined Route and Mode Choice

To assert that those concepts from section 3.1 are valid, a simple traffic network was adopted. It was originally analysed using econometrics, extracted from Vrtic's PhD thesis [9]. From there, the analysed data was taken for that comparing. The decision to use that example relies on its process' good documentation, simplified structure and availability of results. The problem itself is to identify the network use, or agents' distribution among the avaiable links.

A graphical representation of the network is on figure 1(a) In that network edges represent road segments and nodes crossings. There are special nodes identified on the figure with *Source* and *Target* that represent the origin and destination on that network. Edges with continuous lines represent the segments where private vehicles can circulate and segmented lines where the public transportation takes place.




(a) Network. (b) Nested Alternatives.

**Fig. 1.** Network Example

The agents' goal is to drive from *Source* to *Target* and 3000 agents were used for that. Equation 5 presents time/cost funtion – used to compute agents' penality (as higher the time as worst perform the agent). On equation 5 $t(s)$ is the time/cost for segment $s$, $t_0(s)$ a minimum cost, $a$ represents a dependency between occupation and travel time, $q(s)$ is the current occupancy, $L(s)$ capacity and $b$ a scale parameter. For the normal road segments, identified with the letter $L$, the parameters are $a = 3$ and $b = 8$ and for the public transportation, identified by $OV$, they are $a = 1$ and $b = 8$. Those parameters and function are extracted from [9] and its validity is not discussed in this text.

$$t(s) = t_0(s) \cdot \left(1 + a \cdot \left(\frac{q(s)}{L(s)}\right)^b\right) \tag{5}$$

The possible routes for that scenario are the following: $R = \{r1, r2, r3, ov1, ov2\}$, $r1 = L1 \rightarrow L3$, $r2 = L2 \rightarrow L5$, $r3 = L1 \rightarrow L4 \rightarrow L5$, $ov1 = OV12 \rightarrow OV1$ and $ov2 = OV12 \rightarrow OV2$.

All routes, identified with $r$, correspond to routes for normal traffic and with $ov$ for public transportation. That possibilities for route choice determine the transportation mode too. It can be more clear seen on figure 1(b).

### 4.1 Econometric Macroscopic Model

The results accomplished in [9] are summarised in table 1. Those results were achieved by different econometric approaches. Here it will be not covered details given in [9], but shortly explained procedure for results' generation.

To estimate an econometric model, already taken decisions are needed. But in that artificial scenario a modified method was adopted. There are two main steps, according to [9]: internal equilibrium and external equilibrium. The second is the standard econometric estimation with an end criterion. And the other is a stochastic equilibrium through the occupation's gradient.

In this case, the standard econometric estimation uses a utility equation that is derived from the simplified equation in 6, where $U_i$ is the utility for route $i$ and $t(i)$ the time/cost of route $i$.

$$U_i = \beta \cdot t(i) + \varepsilon \tag{6}$$

First, an initial segment distribution guess is taken.With that initial estimation an external equilibrium step is made and a new set $B_h$ is disposed. After that, a whole internal equilibrium process takes place. That process finds a local optimum through a stochastic gradient approximation method, it generates a new distribution.

By the end of internal equilibrium, an external equilibrium process is applied. From that process a new set $B_{h+1}$ is achieved and if the gradient between $B_{h+1}$ and $B_h$, $\nabla(B_t, B_{t+1})$, is less than a certain threshold then $B_h$ is the final estimation and therefore $B^*$. If that is not the case, another internal equilibrium routine is execute, but now with the values from $B_{t+1}$.

### 4.2 Microscopic Agent-Based Econometric Model

Our agent-based econometric model must achieve the same, or almost the same, results as those from econometrics. By following the instructions given on section 3.1 the time/cost function is homogenous and will not undergo adaptation. That leaves freedom just to apply heuristics on route re-evaluation willingness and on route choice algorithms.

To cope with that, the route re-evaluation willingness was modelled as a probability to change the current route. And the route choice algorithm was modelled as a weighted roulette, where the weights are assigned according to a heuristic.

Those two pares are responsible for making agents "greedy" and also providing some stability. The aim si to make agents act egoistic but also to be satisfied with a reasonable profit. Both algorithms are explained on following sub-sections and after that their parameters are discussed.

**Route Change Probability.** To determine agent's route change probability, the agent base its decision on the past experience with a tolerance. The algorithm is shown in equation 7. There, $P(a)$ is the probability to change route for agent $a$; $P_{min}$ and $P_{max}$ minimum and maximum probability; $\overline{t_n}$ mean time – based on agents own history – at step $n$; $k$ a fixed tolerance scale parameter; $t_{min}$ and $t_{max}$ global minimum and maximum time/cost and $t(a)$ current time/cost for agent $a$.

$$
\begin{aligned}
P(a) &= \begin{cases} P_{min}, & \text{if } \overline{t_n} \leq (\overline{t_{n-1}} + k \cdot \sigma_{t_n}) \\ p_b + p_a \cdot t(a), & \text{if } \overline{t_n} > (\overline{t_{n-1}} + k \cdot \sigma_{t_n}) \end{cases} \\
p_a &= \frac{P_{max} - P_{min}}{t_{max} - t_{min}} \\
p_b &= P_{min} - p_a \cdot t_{min}
\end{aligned} \tag{7}
$$

With the equation 7 an agent just increases its change probability $P(a)$ if its current mean time/cost is greater than last mean, with a threshold. The closer its current decision is to the global minimum more it tries to stay with that decision for the next step, in a linear way.

**Weighted Route Choice.** If an agent decides to change its route it will chose between all available routes according to a weighted roulette. Routes' weights are evaluated according to the equation 8, where $W_a$ is a set of weights for agent $a$; $w_{a,r}$ the weight of route $r$ for agent $a$; $w_{min}$ and $w_{max}$ minimum and maximum weights allowed; $w_{ref}$ weight for own time/cost and $t(r)$ time for route $r$.

$$
\begin{aligned}
W_a &= \{w_{a,r} \in W_a \wedge r \in R| \\
&\qquad w_{a,r} = \left| \begin{array}{ll} w_{max}, & \text{if } t(a) = t_{min} \\ w(a,r), & \text{if } t(a) > t_{min} \end{array} \right\} \\
w(a,r) &= w_{min} + e^{w_b(a) + w_a(a) \cdot t(r)} \\
w_a(a) &= \frac{ln\left(\frac{w_{ref} - w_{min}}{w_{max} - w_{min}}\right)}{t(a) - t_{min}} \\
w_b(a) &= ln(w_{max} - w_{min}) - w_a(a) \cdot t_{min} \\
&\qquad w_{min} < w_{ref} < w_{max}
\end{aligned} \tag{8}
$$

With that equation the fastest route will have the maximum weight and the others have an exponential decreasing weight according to its time/cost, but the current decision has a fixed weight $w_{ref}$. Through that algorithm agents try to maximise its own profit, by taking a faster route.

**Parameters' Influence.** The influence of parameters can be summarised into the following. As higher the history size as less the route will change, but a higher size gives a less efficient route distribution and, therefore, a higher total cost ($\sum_{a \in Agents} t(a)$), that express how bad are the decisions. $P_{min}$ and $P_{max}$ also interferes on stability, but with less effect on total cost. As higher $P_{min}$ and $P_{max}$ higher the route change, as expected. The parameter $k$ has the inverse effect on route change, as higher as more stable. It acts like a "satisfaction" parameter and can be expressed as the interference of system instability on agent's "willingness" to change its current route, a higher $k$ leads to a lesser sensibility to system's instability.

There are also weights – $w_{min}$, $w_{ref}$ and $w_{max}$ – that controls how strong it will prefer better routes. It can not be a binary function in terms of assigning $w_{max}$ to all better routes, because it greatly increase the route change instability.

# 5 Results

Analysis criteria will be explain here. First, the same results from econometrics must be achieved using the agent-base econometric model. That means that the model's quality is not its "distance" from global optimal distribution, but the "optimum" distribution from table 1. The measures used here to analyse the model were route occupation. So the modell was calibrated to reproduce the results from econometrics, assumed to represent current state and thus reality.

**Table 1.** Route results comparison

| | Econometrics | | | | MAS | MAS × Econometrics |
|---|---|---|---|---|---|---|
| Route | $\overline{q_r}$ | $\sigma_{q_r}$ | min | max | $q_r$ | $q_r - \overline{q_{r_e}}$ |
| r1 | 776.25 | 107.37 | 570 | 990 | 796 | 19.75 |
| r2 | 850.00 | 142.55 | 600 | 1080 | 876 | 26 |
| r3 | 418.75 | 107.43 | 120 | 570 | 462 | 43.25 |
| ov1 | 661,25 | 75.83 | 600 | 930 | 667 | 5.75 |
| ov2 | 307.50 | 174.54 | 0 | 600 | 199 | -108.5 |

## 5.1 General Simulation Results and Comparison

The first evaluation value is route total cost on figure 2. Through that, it is possible to detect equilibrium achievement and its type. There are different curves plotted on figure 2. Those curves refer to routes' occupation and total cost. There, is possible to see the agents behaviour and equilibrium process. Note that after 400 simulations steps the system is already stable.

Econometric's values were extracted from [9] (table 15, on page 100) where Probit, Nested-Logit, Cross-Nested, C-Logit, PS-Logit and Nested-C-Logit were used, and they are expressed on columns identified with *Econometrics* from table 1. Results that are in columns identified with *MAS* represent the agent approach

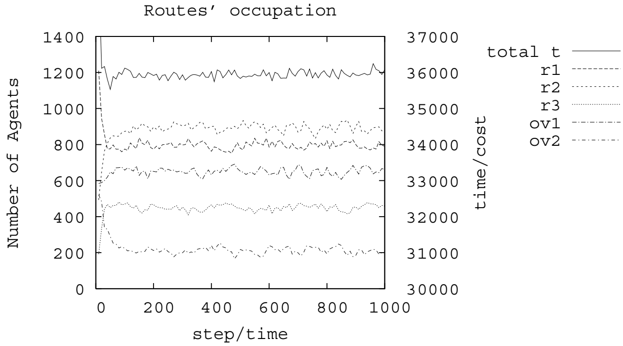



**Fig. 2.** Route occupation

**Table 2.** Agent-based econometric model's parameters

| Param. | hs. | $P_{min}$ | $P_{max}$ | $k$ | $w_{min}$ | $w_{ref}$ | $w_{max}$ |
|---|---|---|---|---|---|---|---|
| Value | 20 | 0.05 | 0.9 | 0.02 | 1 | 1.5 | 2 |
| eq. | 7 | 7 | 7 | 7 | 8 | 8 | 8 |

and were collected with the parameters shown on table 2, where *hs.* is history size and *eq.* is equation's reference.

The objective was neither to perform better nor worse than econometrics approach and rather to reproduce it. The results are on table 1, where MAS results differ less than one standard deviation from econometrics – refer to column $q_r - \overline{q_{r_e}}$ from table 1. And all values are on the limits established by econometrics – columns *min* and *max* on table 1.

On figure 2 it is clear that no absolut equilibrium is achieve, that would be characterised by flat lines, but rather a dynamic one. That can be seen as advantage or disadvantage. If one expect an absolut equilibrium and clear differences between the options, this model is not an option. But it can also be seen as sign of model's adaptadability where agents are always trying to find a new better route (according to criteria already discussed).

The MAS was modelled using SeSAm [10]. The parameters' setup shown in table 2 were found to be the most suitable for this example. But some fine-tuning on those leads to other results.

## 6 Related Work

That is not the first approach trying to tackle discrete choice problems using agents. In traffic engineering and simulation, there are several works using different agent technologies, like BDI in [11] and [12]; reinforced learning in [13] and [14]. In all of them, agents try to maximise their rewards with a monetary interpretation. Compared to this work, where agents minimise their costs (with a time interpretation), the approach is very similar.

Other approaches using Artificial Intelligence are also available but they are macroscopic and do not use agents, like Neural Networks in [15]. Also an individual layered fuzzy logic was used for an artificial scenario in [16]. A Lagrangian heuristic on a commuter scenario was used by [17] to schedule traffic demand. Our present work aims at finding a relation between agents and econometric simulation and is focused on a methodological level.

## 7 Conclusion and Future Work

We presented an econometric agent-based model and discussed how it performs. A simple example was analysed. The results from econometrics were compared with the econometric agent-based model simulation. Those results fit into the acceptable range, determined by the different econometric models. One can argue that the example is too simple, but the objective in this paper was to show that it is possible to get equivalent results using agent technology.

It is important to develop an agent-based alternative to econometrics. Agents are the natural paradigm for modelling and simulating humans, besides the fact that it is easier to model agent behavior than the whole system. Moreover, agent can adapt their parameters and behavior, a characteristic not present on econometrics.

Currently, we are extending the model for applying it to a complex real-world scenario, namely for developing a combined route and mode choice to Bern, Switzerland. We therefore apply also agent-based learning for generation of alternatives, etc. A first prototype already exists which dynamics are currently evaluated.

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