Coupling Bayesian Networks with GIS-Based Cellular Automata for Modeling Land Use Change

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Abstract. Complex systems theory and Cellular Automata (CA) are widely used in geospatial modeling. However, existing models have been limited by challenges such as handling of multiple datasets, parameter definition and the calibration procedures in the modeling process. Bayesian network (BN) formalisms provide an alternative method to address the drawbacks of these existing models. This study proposes a hybrid model that integrates BNs, CA and Geographic Information Systems (GIS) to model land use change. The transition rules of the CA model are generated from a graphical formalism where the key land use drivers are represented by nodes and the dependencies between them are expressed by conditional probabilities extracted from historical spatial datasets. The results indicate that the proposed model is able to realistically simulate and forecast spatio-temporal process of land use change. Further, it forms the basis for new synergies in CA model design that can lead to improved model outcomes.

1 Introduction

Geographic Information Systems (GIS) are well established as tools for the storage, handling, analyzing, and visualizing of spatial data [1, 2]. Despite these advantages, GIS is not well developed for handling the temporal component of data [3]. The current GIS have a limited capacity to handle short time step iterations needed for modeling dynamic spatial phenomena. There is, therefore, an urgent need to understand the spatio-temporal process itself, how to formalize the process and the problem as a geographic model, and the method to solve the problem within GIS frameworks.

The development of GIS based procedures that can handle the dynamics of geographic phenomena through the integration of complex systems theory is an important area of GIScience research. Complexity theory has been used extensively to model land use change processes together with cities and their evolution. Land use change and urban growth are characterized by a large number of interacting components and have several key signatures such as fractal dimensionality, self-similarity, self-organization and emergence that make them suitable to be modeled as complex systems [4-6]. For more than two decades, complex systems theory and cellular automata (CA) have been used to handle the dynamics, complexity, and self-organizing properties of land use change processes. The spatial complexity and dynamics of land use

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change is represented by selecting various configurations of the basic elements of the CA model design - cell space, cell size, neighborhood size and type, transition rules and temporal increments [7-9]. Transition rules are the most important element of the CA as they control the behavior of the cells and their evolution into the future states. Recently, there has been a concerted effort to improve the transition rules [10, 11] or combine other approaches such as multi-criteria evaluation, principal component analysis and neural networks among others [12-14] to advance GIS-based CA.

Bayesian Networks (BNs) provides an alternative approach based on artificial intelligence that can overcome some of the challenges of CA models. In the late 1970s and 1980s, rule-based approaches were common in artificial intelligence (AI). Although neural networks were popularized later, they have failed when sufficient data are not available for data learning. In the late 1980s, BNs were seen as an efficient way to deal with data uncertainty [15, 16]. The early applications of BN were in medical diagnosis and genetics, but recently their use have expanded to areas such as environmental studies [17] and geographic information systems [18].

The Bayesian Networks are considered as probabilistic network-graphical models that use probability and graph theory in their implementation [19]. The advantage of probabilistic networks is that they provide explicit representations of dependencies or independencies between variables without scientific numeric or functional details [20]. As a probabilistic network, the BN representation was originally designed to model the uncertain knowledge of an expert to deal with complex systems and data uncertainty. In BN models, simple parts (such as land use drivers) of the complex system (such as land use change process) are constructed using graph theory. The parts are then combined to each other using probability theory [21]. BNs are used for probabilistic inference and based on Bayes' Theorem, which is a mathematical formula to calculate probabilities among several variables that are causally related [22].

The objective of this study is to improve existing CA models by proposing a hybrid GIS-based Bayesian network cellular automata model. The theoretical framework for this study is the integration of cellular automata with Bayesian networks since the formulation of cellular automata transition rules should depend on how driving factors of land use change are perceived. Thus, land use change is seen as the result of the interplay between land use drivers that mimics the complexity of a spatial process. Sensitivity analysis was used to test the model for changes occurring in the model outcomes when the number of nodes and land use classes are changed.

2 The BN-CA Model

Cellular automata are discrete spatial models [9]. They consist of an array of cells, each of which has cell states. The state of a cell at consecutive time $t+1$ is a function of its state, its neighbourhood, and set of transition rules at an initial time t. Using this function, transition rules are applied to each cell to determine what state it should change to during a time transition. This step is repeated over the whole cell array. The cell state can be mathematically represented as:

$$S_i(t+1) = f(N_i(t), S_i(t), T) \tag{1}$$

where $S_i(t)$ and $S_i(t+1)$ are the states of cell i at the initial and consecutive time t and $t+1$ respectively; $N_i(t)$ is the neighborhood state of cell i at time t; T are the transition rules. In general, the transition rules are in the form of <IF, THEN, ELSE> statements. For example, IF an event occurs in the neighborhood of a cell, THEN some-other-event occurs to the cell [23].

In land use change modeling, these rules represent how change occurs in the real world. Since there is no standard procedure or method for defining transition rules, they are reformulated in the spatial modeling literature using probabilistic expressions, accessibility algorithms, logistic regression, linguistic variables, multi-criteria evaluation methods, and neural network structure among others. However, when processes and changes in the urban area are difficult to describe (e.g. large number of factors affecting land use change), then most of these methods fail to adequately capture the land use change process. Neural Networks (NN) [14] have been used to improve the capability of CA models to deal with multiple land uses. With the NN-CA models, the parameters required for the simulation are determined by a training procedure and no transition rules are required. However, they are black-box models when incorporated in the CA structure. Explicit knowledge about the modeled land use change process is not provided. In addition, the optimal structure for the numbers of network layers and neurons is still unclear for a specific application [24]. Hence, there is a need to make the design and implementation of existing CA models more explicit.

The alternative method of Bayesian Networks is proposed to simulate land use change in cellular automata spatial models. A Bayesian network is the pair *(G, P)* where G is a Directed Acyclic Graph (DAG) where nodes represent variables, arcs between nodes represent probabilistic dependencies, and *P* is a multivariate probability distribution defined on variables that correspond to the nodes of *G*. A graph is called *directed* if the graph links have directions. A directed graph is *acyclic* if the graph contains no directed cycles.

The elements of BN are [19]:

1. Variables: A set of *variables* and a set of *directed edges* between variables,
2. States: Each set contains a finite set of mutually exclusive states,
3. Structure: The variables coupled with the directed edges form a DAG,
4. CPT: Each variable *A* with parents *B*1, *B*2… *Bn* has a Conditional Probability Table (CPT) which includes *P* (*A* | *B*1, *B*2… *Bn*).

If directed edges (arcs) in the DAG are assumed to represent causality, then BNs are sometimes called causal networks. However, when building BN models, users and experts do not need to see the links as causal relationships. Rather they should ensure that links correspond to qualitative relationships. In order to explain the basic BN ideas, consider the example in Figure 1. The circles are nodes and they are the variables. They represent the most important factors about the particular phenomenon. They are linked so that a change in one will result in a chain reaction of impacts on all the linked variables in the direction of the links, assuming that the links represent causality. Each variable is probabilistically independent of its non-parents given its parents. Therefore, the absence of a direct link between A and G means that the influence of A on G results from other variables (e.g. C, D, E). The design of the network such as deciding which factors link to each other is based on how the phenomenon being modeled is perceived. For simplicity, assume that all the nodes are binary variables that take a value of either true (T) or false (F). While the states of the variables can be discrete, they can also be real valued, integer valued, or multivariate [25]. For each node, there is a conditional probability function that relates this node to its parents. For instance, the probabilistic relationship between D and its parent C is the *conditional probability distribution of D given C.* This is expressed in the conditional probability table shown in Figure 1. In the table, P_{00} means probability of D being false given C is false, that is $P(D = false \mid C = false)$.

The important characteristic of Bayesian Networks is their explicit representation of the conditional dependence and independence between variables [26]. The probability of every possible event as defined by the values of all the variables is called the joint probability distribution. It has been shown [19] that the joint probability distribution induced by the DAG can be factorized into the conditional distribution of each variable with respect to its parents (Equation 3):

$$P(X_1 \ldots X_n) = \prod_{1}^{n} P(X_i \mid pa_i) \tag{2}$$

where pa_i is the set of direct parents for variable X_i. Therefore, the probability distribution represented by the example network from Figure 1 is:

$$P(A,B,\ldots,G) = P(A)P(B)P(C \mid A,B)P(D \mid C)P(E \mid C)P(F \mid D,E)P(G \mid C,D,E) \tag{3}$$

2.1 Model Framework

In the proposed model, the transition rules *T* in the equation (1) are equal to *(G, P)* and the cell states are defined as:

$$S_i(t+1) = f(S_i(t), T(G,P)), \tag{4}$$

which implied that the cell state at time *t+1* depends on the current cell state and transition rules defined by *G* and *P*. The proposed model eliminates the use of neighborhood type and size in the transition rules since it is proved that CA model outcomes are sensitive to the changing neighborhood size and type [27]. In the next section, the details of the model building process will be elaborated.

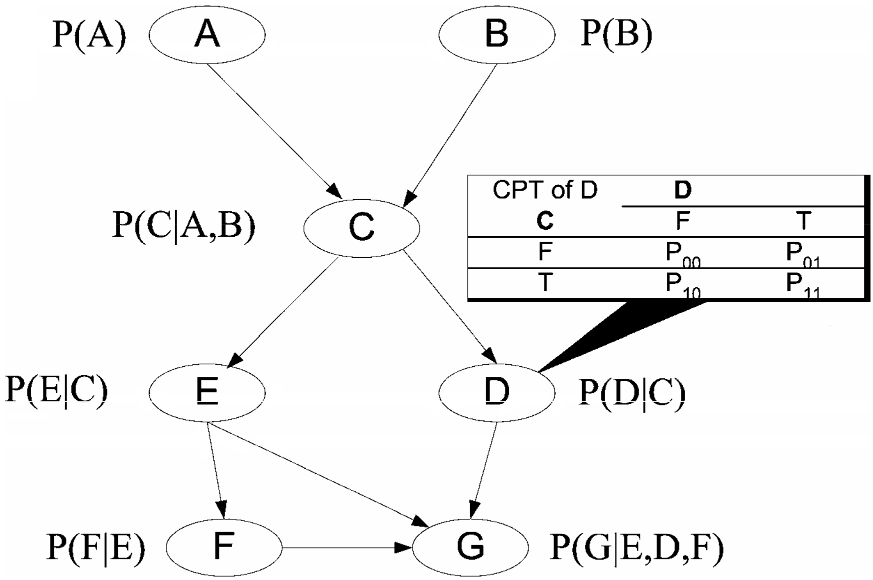

Fig. 1. A simple Bayesian network structure containing seven variables

2.2 Building the BN-CA Model

Defining Input Raster GIS Layers and Bayesian Network Nodes. Multiple GIS layers provide the data input to this model and they represent the key land use drivers that affect the land use change. The GIS operations such as reclassification, distance calculations, suitability evaluation, buffering, and overlay are used to derive suitable areas and constraints as input layers. These input GIS layers are defined as nodes in the BN structure. In addition to land use driver nodes, future land use state is defined as a node in the BN structure. Consequently, probabilities of each future land use state conditional on land use drivers are calculated. The values of the nodes are represented as discrete or continuous or both.

Structuring the Bayesian Network. Once the variables and their values are identified, the next step is to identify the structure of the BN and how the nodes are connected together. The structure should capture the relationships between variables. Nodes should be linked such that if one affects or causes the other, there should be an arc between them. The direction of the arc represents the causation.

The experts can construct the structure and quantify probabilities in the network by using their knowledge. Nevertheless, some researchers [28] have argued that instead of trusting experts, the BN should be constructed from the observed data in a learning process. This provides a better way to interpret the existing (observed) data in an accurate manner and to better understand the phenomenon that is being modeled. In land use change modeling, this gives an indication of what variables are the main factors in the land use change process.

Learning in BN depends on whether the network structure is known and whether the variables are all observable [29, 30]. In this study, the BN structure is extracted using the data at two temporal snapshots of the study site to find the factors underlying the land use change process. Figure 2 depicts how the BN sub-model is conceptualized in terms of observed data and predicted data. The data of the site are observed but the BN structure is not known. In this case, given the set of variables, the links must be found and then parameters as the values in the Conditional Probability Table (CPT) must be estimated using the observed data (data at time t and $t+\Delta t$ of the site).

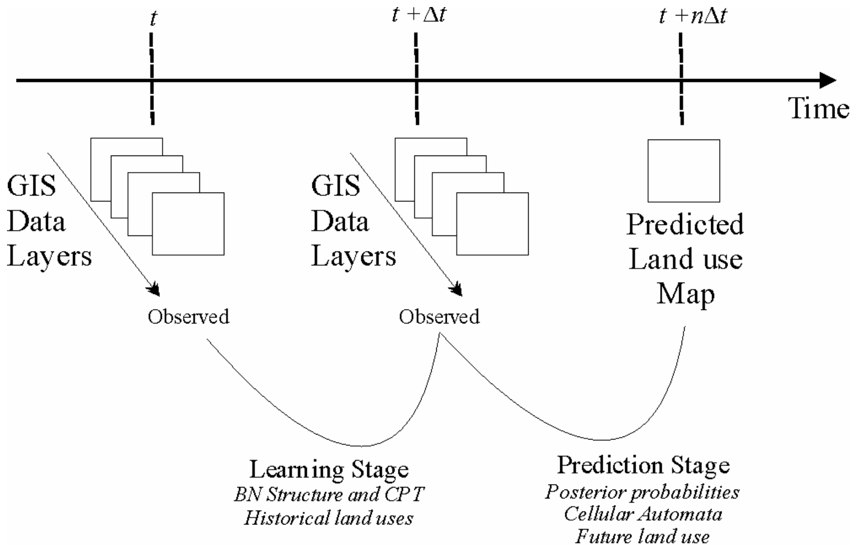

Fig. 2. GIS based BN-CA model: conceptual framework

The K2 algorithm [31] which uses Bayesian scoring method was used in this study to learn the BN structure from two observed land use datasets. The K2 algorithm provides an efficient way to structure learning from complete observable data [32]. The algorithm starts with a network with no links; then for each node, it incrementally adds parents whose addition increases the score of the resulting structure until the addition of parents does not increase the score.

Estimating Conditional Probability Table (CPT) in the BN Structure. In the process of BN structure learning, there are several possible ways to obtain estimates for the conditional probabilities in the CPT. It is possible to use subjective probabilities, usually encoded from expert knowledge when the data available for a particular variable are limited or non-existent. The alternative to subjective probabilities is learning, which is achieved by calculating the conditional probability table values using estimation techniques such as Maximum Likelihood Estimation (MLE) and Bayesian estimation.

In this study, since the network structure was learned from observed data in the previous step, the structure is known and complete data are available to derive the probabilities. This is the most studied case of learning BN in the research literature. MLE estimation was applied to compute the probabilities.

Inference - Predicting Future Land Use. The last step in building the BN sub-model of the BN-CA hybrid model is the prediction done by inference. The computation of a probability of interest given a model is called probabilistic inference [33]. In this step, observed variable states are entered as evidence in the BN to calculate the revised probabilities of interest given the evidence. In this study, the Junction Tree Algorithm was used to calculate the inference in the BN [34]. As shown in Figure 2, the observed data at time $t+\Delta t$ are entered into the BN as evidences and the probabilities of each land use state at time $t+n\Delta t$ are obtained.

For each cell, the BN inference is used to obtain the probabilities of each predicted land use state. Then, the transition rule of the CA changes each cell's land use state to one with the highest potential. Each cell is subject to inference and transition rule in each iteration.

3 Model Implementation and Simulation Results

3.1 Study Site

A hypothetical study site was created at 25m spatial resolution with 900x1363 cells. The two hypothetical snapshots of this site with the temporal interval of 10 years (Δt=10years, n=2) were used in the simulations of the proposed model. Figure 3 depicts the new residential development that was created along the undeveloped land. In these land use maps, there are ten common land use classes, namely residential, industrial, commercial, office, schools, recreation, green areas, agriculture, undeveloped land and others (such as water bodies, roads).

3.2 Data Preparation

In the hybrid model, the first step is to identify the key variables-land use drivers that affect the land use change process since changes in the types of land use are induced by these drivers. A total of ten spatial variables were identified as key factors that affect future land use change. These variables are: distance to education facilities, distance to existing transportation network, distance to commercial centers, distance to employment centers, distance to recreational facilities, distance to green areas, constraints (water areas, steep sloped, flood areas), land use policies, land ownership and current land use (Table 1). These layers as variables were obtained by using raster GIS analyses. The Euclidean distances in the ArcGIS software were used to calculate the distance variables and to classify them into three main classes: 'good', 'medium' and 'low' accessibility to the land use driver being considered.

Fig. 3. Hypothetical site with land use classes at time *t* and *t+10* years

Table 1. Input variables of the proposed model

Land use drivers/variables	Raster GIS calculations
Distance to education	Euclidean distance from every cell to the nearest schools, universities, colleges.
Distance to existing transportation network	Euclidean distance from every cell to the nearest street network
Distance to commercial centers	Euclidean distance from every cell to the nearest commercial areas, such as shopping malls
Distance to employment centers	Euclidean distance from every cell to the nearest office spaces and business areas
Distance to recreational facilities	Euclidean distance from every cell to the nearest recreational attractions, such as tourist attraction sites, stadiums, etc
Distance to green areas	Euclidean distance from every cell to the nearest green areas, such as residential parks, hillsides, etc
Constraints	Physical and policy constraints to the development such as conservation areas, flood plains, steep slopes, etc
Land use policies	Government policy and plans on future land use
Land ownership	Public land ownership (land owned by schools, states, forest service, etc) and private land ownership
Current land use	Current land use

3.3 Simulations

The proposed BN-CA model was applied to the hypothetical study site with a spatial resolution of 25m and temporal resolution of 10 years. The study site data at time t and $t+10$ years were input to the model and the land use map at $t+20$ years was predicted (Δt=10 years, n=2 iterations that were generated). The algorithm of the proposed model was coded in the Matlab software using some of the functions of the Bayes Net Toolbox [35]. The proposed BN-CA model uses a loose coupling architecture with the ArcGIS and Matlab software.

All the variables, namely land use drivers were incorporated into the BN as nodes. In addition to those, as a last node, predicted land use was defined in the BN structure to obtain the probabilities of each future land use state conditional on the others. The distance variables have three states (good, medium, and low accessibility), constraints and ownership have two states (public and private ownerships), and policy and land use variables have ten states representing the land use classes.

Bayesian Network employs two important operations: *explanation* and *prediction.* The explanation part was accomplished by a BN learning procedure in which the structure of the network was constructed and values of the CPT were estimated from the observed data at initial time t and consecutive time $t+10$ years. Although all cells could be used for structure and parameter learning, 1000 cells from the raster GIS layers at time t were chosen randomly for the learning algorithms. When the number of cells required for learning increases, the sample complexity increases and creates computational complexity. Also, the performance of the learning improves with increasing the number of observed cells. The result of the structure learning procedure is shown in Figure 4, which explains the underlying processes and interactions in the

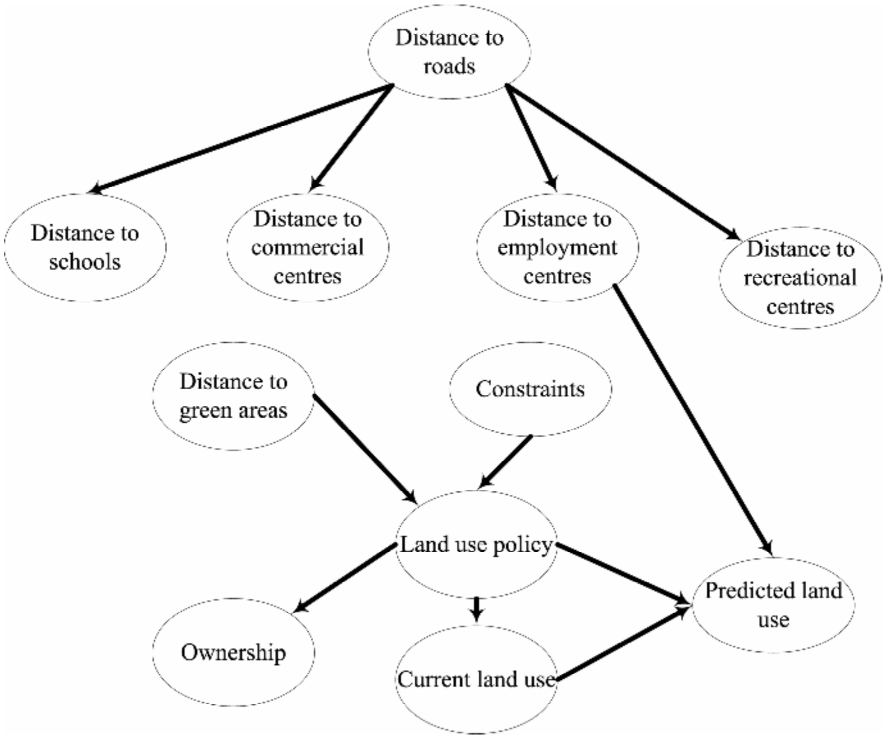

Fig. 4. Learned BN structure

study site. Distance to existing transportation network affects other distance variables, except green areas. In addition, distance to employment centers, land use policies and current land use directly affects the future land use change.

Prediction was employed by the probabilistic inference and cellular automata transition rules. With the inference algorithm, the probabilities of each future land use state were calculated. Then, transition rule decides the change from one land use state to another depending on the highest probability. Figure 5 illustrates the land use states predicted for 20 years, at *t+20* generated by the proposed model. The results imply that the predicted land use generated by the model show similar growth to the current land use change trend where new residential areas appeared on the undeveloped land.

4 Model Sensitivity

Sensitivity Analysis is useful in spatial modeling to identify what parts of the model are critical and which ones are less likely to be important to the results [27, 36]. The proposed model's sensitivity was tested by running the model with different configurations. There are two sensitivity areas of the model: *node sensitivity* and *classification sensitivity*.

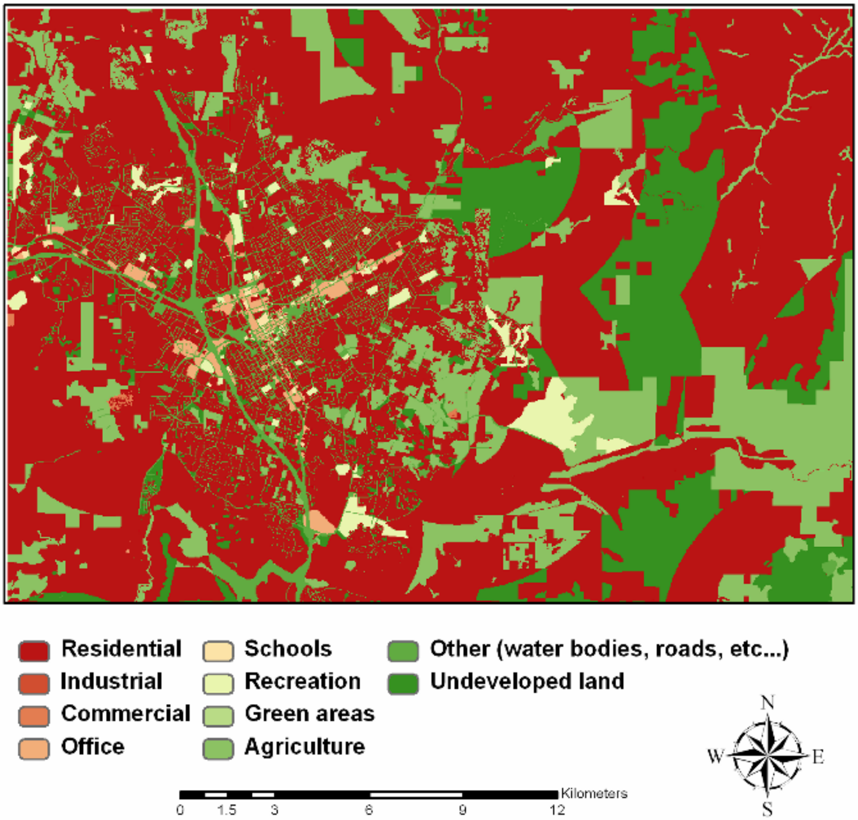

Fig. 5. Predicted land use generated by the proposed hybrid model at time *t+20* years with using eleven nodes

In testing the node sensitivity of the model, different number of nodes was incorporated into the BN sub-model. As a result, different simulation results were obtained. For example, Figure 6 shows the predicted land use map when seven nodes (only six distance nodes and future land use node) were employed. It can be seen from the figure that the resultant map has only seven land use classes and the model did not generate residential growth. In addition, large school areas were generated by the model. This shows that model is very sensitive to the number of nodes. During the modeling process, nodes should be chosen meaningfully. The key is to determine the most important nodes. It should be noted that the number of the nodes ought to be pruned due to computational complexity that can arise from large number of nodes. This is because as the number of nodes increases, the joint probability distribution grows exponentially and creates computational complexity in the calculation of probabilities.

Apart from the node sensitivity, there is also classification sensitivity on how land use categorization and classification affect the land use change models [37]. In this

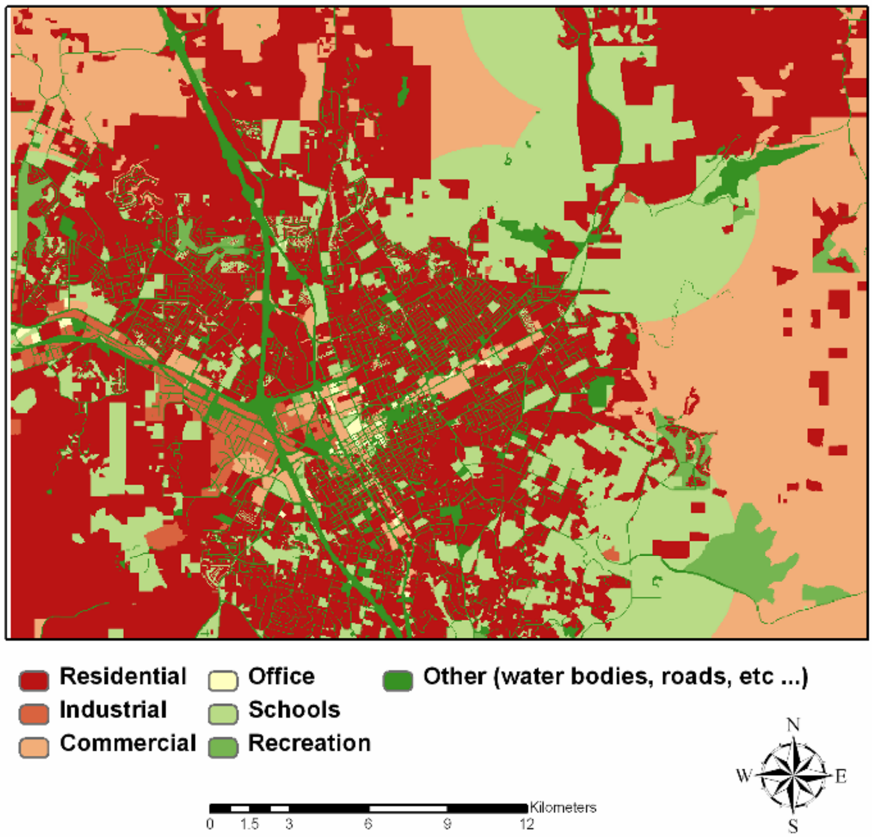

Fig. 6. Predicted land use at time *t+20* years when seven nodes are used in the model

study, simulations were performed with a variety of land use classes; from three to fourteen, and the outcomes of the model with different land use classes were analyzed. Figure 7 depicts two of the outcomes, using three land use classes (Figure 7a) and using fourteen classes (Figure 7b). Visual comparison of these maps and the simulation obtained for ten classes (Figure 5) illustrates that land use classification affects the residential growth since the generated directions and shapes are different.

In addition to the land use classification sensitivity examination, the model's sensitivity to the number of distance classes was investigated. Initial simulation was employed with three and five class distance layers (Figure 5 and 8 respectively). The simulation outcomes were compared and it can be seen that the generated pattern is different. Figure 8 depicts that large residential areas are created at the expense of green areas in the inner core of the urban area.

The results of sensitivity analysis emphasize the importance of the node selection and the land use classification in the modeling process. The GIS based BN-CA model is sensitive to the changes in the number of nodes, the number of distance classes, and the number of land use classes.

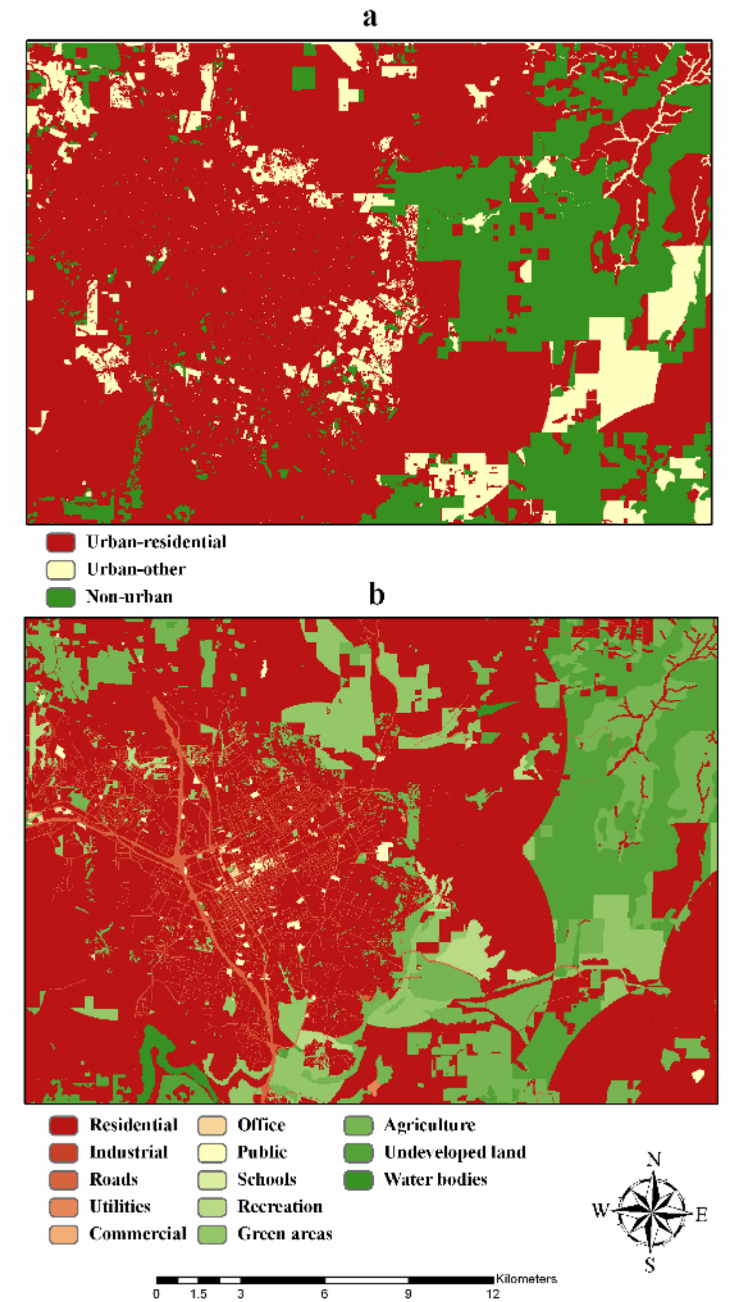

Fig. 7. Predicted land use at time *t+20* years with: a) three and b) fourteen land use classes

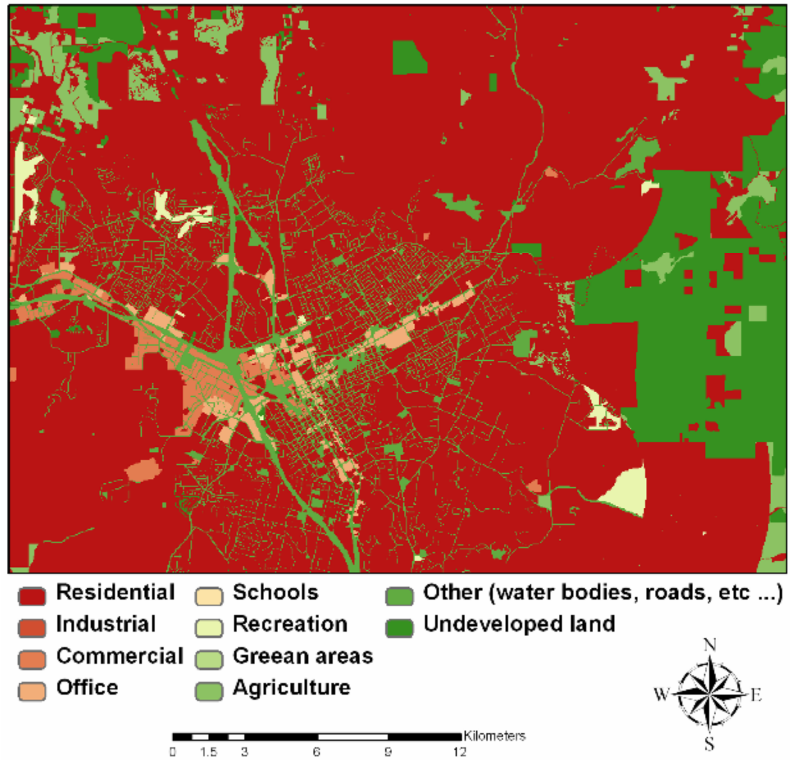

Fig. 8. Predicted land use at time *t+20* years when five class distance layers were used

5 Conclusion

This study developed a novel hybrid model that incorporated Bayesian Networks, Cellular Automata and GIS to predict dynamic land use changes in an urban environment. The developed model addressed some drawbacks of existing GIS-based CA models. First, the developed model is a dynamic GIS model that uses spatial complexity theory. Second, it makes explicit the information about the process of land use change. Third, the transition rule definition is relatively simple. Fourth, the model is explanatory and predictive which makes it useful for land use change modeling. Fifth, the model handles datasets with a large number of variables and relationships. In most probabilistic models, each variable in the model directly affects the outcome of the model. However, in the real world it is possible that one variable can affect another directly or indirectly through several variables such as in a Bayesian Networks. This characteristic is important in land use change modeling as it is difficult to define model parameters and transition rules when many variables affect the change. Finally, the model allows ease in calibration due to the learning procedure. Causal relationships are learned from available data by using Bayesian Networks.

In the developed model, the CA transition rules are represented by a specific Bayesian Network. Land use drivers are represented by nodes and dependencies between them are represented by conditional probabilities. After the probabilities of the future land use state given the other variables states are calculated, the decision whether the current cell state will be converted or not to the other state is made. The developed model was applied to a hypothetical study site. The results showed that Bayesian networks are suitable for deriving the complex relationships between land use drivers that cause land use change. Moreover, the developed model is capable of producing various scenarios of land use change. With different scenarios through the inclusion of planning policies in the BN structure, it can be used to make predictions in response to policy changes. Further, when some of the variables change (e.g. new roads, new commercial centers), the model can be updated by recalculating the probabilities instead of re-running the model. This emphasizes the ability of the GIS-based BN-CA model to represent and respond to changing configurations.

Sensitivity analysis allowed model testing and the evaluation of the changes occurring in the model outcomes when the number of nodes and land use classes were changed. The results of the sensitivity analysis indicate that the model is sensitive to the changes in the number of nodes and land use classes. Due to the use of hypothetical datasets, validation of the proposed model is not accomplished. This implies that detailed calibration and validation procedures have to be established in GIS-based BN-CA model applications, and this forms the basis of ongoing research in the development of hybrid GIS BN-CA models.

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