Knowledge Base Revision in Description Logics

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Abstract. Ontology evolution is an important problem in the Semantic Web research. Recently, Alchourrón, Gärdenfors and Markinson's (AGM) theory on belief change has been applied to deal with this problem. However, most of current work only focuses on the feasibility of the application of AGM postulates on *contraction* to description logics (DLs), a family of ontology languages. So the explicit construction of a revision operator is ignored. In this paper, we first generalize the AGM postulates on revision to DLs. We then define two revision operators in DLs. One is the weakening-based revision operator which is defined by weakening of statements in a DL knowledge base and the other is its refinement. We show that both operators capture some notions of minimal change and satisfy the generalized AGM postulates for revision.

1 Introduction

Ontologies play a crucial role for the success of the Semantic Web [6]. One of the challenging problems for the development of ontology is ontology evolution, which is defined as the timely adaptation of an ontology to the arisen changes and the consistent management of these changes [10]. Ontology evolution is a very complex process, i.e. it consists of six phases [27]. In this paper, we consider an important phase called *semantics of change phase*, which prevents inconsistencies by computing additional changes that guarantee the transition of the ontology into a consistent state [27]. A center problem in this phase is inconsistency handling. There are various forms of inconsistencies, such as structural inconsistency, logical inconsistency and user-defined inconsistency. Among them, logical inconsistency in ontology evolution has attached lots of attention in recent years, where ontologies are represented by logical theories, such as description logics [21, 1, 8, 11, 10, 14, 19, 25].

AGM's theory of belief change [9] has been widely used to deal with logical inconsistency resulting from revising a knowledge base by newly received information. There are three types of belief change, i.e. *expansion*, *contraction* and *revision*. Expansion is simply to add a sentence to a knowledge base; contraction requires to consistently remove a sentence from a knowledge base and revision is the problem of accommodating a new sentence to a knowledge base consistently. Alchourrón, Gardenfors and Markinson proposed a set of postulates to characterize each belief change operator. The application of AGM' theory to description

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logics is not trivial because it is based on the assumptions that generally fail for DLs [7]. For example, a DL is not necessarily closed under the usual operators such as $\neg$ and $\wedge$ [8]. In [7, 8], the basic AGM postulates for contraction were generalized to DLs and the feasibility of applying the generalized AGM theory of contraction to DLs and OWL was studied. However, no explicit belief change operators were proposed in their papers. Furthermore, they did not consider the application of AGM postulates for revision in DLs.

In this paper, we first generalize the AGM postulates for revision to DLs. Instead of discussing the feasibility of applying the postulates, we propose two revision operators in DLs. One is the weakening-based revision operator which is defined by weakening of statements in a DL knowledge base. Since the weakening-based revision operator may result in counterintuitive results in some cases, we propose an operator to refine it. We show that both operators capture some notions of minimal change and satisfy the generalized AGM postulates on revision.

This paper is organized as follows. Section 2 gives a brief review of description logics. In Section 3, we generalize the Gärdenfors postulates on revision to DLs. We then propose two revision operators and discuss their logical properties in Section 4. In Section 5, we have a brief discussion on related work. Finally, we conclude the paper in Section 6 and give some further work.

2 Description Logics

In this section, we will introduce some basic notions of Description Logics (DLs), a family of well-known knowledge representation formalisms [3]. To make our approach applicable to a family of interesting DLs, we consider the well-known DL $\mathcal{ALC}$ [26], which is a simple yet relatively expressive DL. Let N_C and N_R be pairwise disjoint and countably infinite sets of *concept names* and *role names* respectively. We use the letters A and B for concept names, the letter R for role names, and the letters C and D for concepts. $\top$ and $\bot$ denote the universal concept and the bottom concept respectively. The set of $\mathcal{ALC}$ concepts is the smallest set such that: (1) every concept name is a concept; (2) if C and D are concepts, R is a role name, then the following expressions are also concepts: $\neg C$ (full negation), $C \sqcap D$ (concept conjunction), $C \sqcup D$ (concept disjunction), $\forall R.C$ (value restriction on role names) and $\exists R.C$ (existential restriction on role names).

An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of a set $\Delta^{\mathcal{I}}$, called the *domain* of $\mathcal{I}$, and a function $\cdot^{\mathcal{I}}$ which maps every concept C to a subset $C^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$ and every role R to a subset $R^{\mathcal{I}}$ of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ such that, for all concepts C, D, role R, the following properties are satisfied:

(1) $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$ and $\bot^{\mathcal{I}} = \emptyset$, $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$,
(2) $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$, $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$,
(3) $(\exists R.C)^{\mathcal{I}} = \{x | \exists\ y\ s.t. (x, y) \in R^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\}$,
(4) $(\forall R.C)^{\mathcal{I}} = \{x | \forall y (x, y) \in R^{\mathcal{I}} \text{ implies } y \in C^{\mathcal{I}}\}$.

A DL knowledge base consists of two components, the *terminological box (TBox)* and the *assertional box (ABox)*. A TBox is a finite set of terminological axioms of the form $C \sqsubseteq D$ (general concept inclusion or GCI for short) or $C \equiv D$ (equalities), where C and D are two (possibly complex) $\mathcal{ALC}$ concepts. An interpretation $\mathcal{I}$ satisfies a GCI $C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$, and it satisfies an equality $C \equiv D$ iff $C^{\mathcal{I}} = D^{\mathcal{I}}$. It is clear that $C \equiv D$ can be seen as an abbreviation for the two GCIs $C \sqsubseteq D$ and $D \sqsubseteq C$. Therefore, we take a TBox to contain only GCIs. We can also formulate statements about individuals. We denote individual names as $a,\ b,\ c$. A *concept* (*role*) assertion axiom has the form $C(a)$ $(R(a,b))$, where C is a concept description, R is a role name, and $a,\ b$ are *individual names*. To give a semantics to ABoxs, we need to extend interpretations to individual names. For each individual name a, $\cdot^{\mathcal{I}}$ maps it to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. The mapping $\cdot^{\mathcal{I}}$ should satisfy the *unique name assumption* (UNA), that is, if a and b are distinct names, then $a^{\mathcal{I}} \neq b^{\mathcal{I}}$. An interpretation $\mathcal{I}$ satisfies a concept axiom $C(a)$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$, it satisfies a role axiom $R(a,b)$ iff $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$. An *ABox* contains a finite set of concept and role axioms. A DL knowledge base K consists of a TBox and an ABox, i.e. it is a set of GCIs and assertion axioms. An interpretation $\mathcal{I}$ is a *model* of a DL (TBox or ABox) axiom iff it satisfies this axiom, and it is a model of a DL knowledge base K if it satisfies every axiom in K. In the following, we use $M(\phi)$ (or $M(K)$) to denote the set of models of an axiom ϕ (or DL knowledge base K). K is consistent iff $M(K) \neq \emptyset$. Two DL knowledge bases K_1 and K_2 are said to be element-equivalent iff there is a bijectin f from K_1 to K_2 such that for every ϕ in K_1, $M(f(\phi)) = M(\phi)$. Let K be an inconsistent DL knowledge base. A set $K' \subseteq K$ is a *conflict* of K if K' is inconsistent, and any sub-knowledge base $K'' \subset K'$ is consistent. Given a DL knowledge base K and a DL axiom ϕ, we say K *entails* ϕ, denoted as $K \models \phi$, iff $M(K) \subseteq M(\phi)$. We use $\mathcal{KB}$ to denote the set of all possible DL knowledge bases.

3 Generalizing the AGM Postulates for Revision to DLs

Let L be a propositional language constructed from a finite alphabet P of propositional symbols using the usual operators $\neg$ (not), $\vee$ (or) and $\wedge$ (and). An interpretation is a mapping from P to $\{true, false\}$. A *model* of a formula ϕ is an interpretation that makes ϕ true in the usual sense. $M(\phi)$ denotes the set of all the models of ϕ. A formula ϕ is satisfiable if $M(\phi) \neq \emptyset$. We denote the classical consequence relation by $\vdash$. Two formulas ϕ and ψ are equivalent, denoted as $\phi \equiv \psi$ iff $M(\phi) = M(\psi)$. In [17], AGM postulates for revision are rephrased as follows, where $\circ$ is a revision operator which is a function from a pair of formulas ψ and μ to a new formula denoted by $\psi \circ \mu$.

(R1) $\psi \circ \mu \vdash \mu$

(R2) If $\psi \wedge \mu$ is satisfiable then $\psi \circ \mu \equiv \psi \wedge \mu$

(R3) If μ is satisfiable then $\psi \circ \mu$ is also satisfiable

(R4) If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$ then $\psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2$

(R5) $(\psi \circ \mu) \wedge \phi$ implies $\psi \circ (\mu \wedge \phi)$

(R6) If $(\psi \circ \mu) \wedge \phi$ is satisfiable then $\psi \circ (\mu \wedge \phi)$ implies $(\psi \circ \mu) \wedge \phi$

We first define a revision operator in DLs. Before that, we need to introduce the notion of a *disjunctive DL knowledge base* (or DKB) in [19], which is defined as a set of DL knowledge bases. In the following, a DL knowledge base is viewed as a disjunctive DL knowledge base which contains a single DL knowledge base. In propositional logic, disjunction $\vee$ is a very important connective used to define revision operators. For example, the result of Dalal's revision operator is (syntactically) in disjunction form [5]. However, DL languages do not allow disjunctions of TBox statements with ABox statements. The semantics of DKBs is defined as follows [19]:

Definition 1. *A DKB $\mathcal{K}$ is satisfied by an interpretation $\mathcal{I}$ (or $\mathcal{I}$ is a model of $\mathcal{K}$) iff $\exists K \in \mathcal{K}$ such that $\mathcal{I} \models K$. $\mathcal{K}$ entails ϕ, denoted $\mathcal{K} \models \phi$, iff every model of $\mathcal{K}$ is a model of ϕ.*

Let $\mathcal{DKB}$ denote a set of (disjunctive) DL knowledge bases. A revision operator in DLs can be defined as follows.

Definition 2. *A knowledge base revision operator (or revision operator for short) in DLs is a function $\circ : \mathcal{DKB} \times \mathcal{KB} \to \mathcal{DKB}$ which satisfies the following condition: $\mathcal{K} \circ K' \models \phi$, for all $\phi \in K'$.*

That is, both the original knowledge base and the resulting knowledge base can be a DKB, Whist the newly received knowledge base must be an ordinary DL knowledge base (i.e. it is not a DKB).

We next generalize postulates (R1)-(R6) to DLs. The generalization is not as trivial as we have thought. The problem is that both the original knowledge base the result of revision may be a disjunctive DL knowledge base. To generalize (R1)-(R6), we need to define the conjunction of a disjunctive DL knowledge base and an ordinary DL knowledge base. A more simple way to generalize AGM postulates is to define them in a model-theoretic way as follows.

It is clear that (R1)-(R3) can be generalized in the following way. Let K be a (disjunctive) DL knowledge base and K' be a DL knowledge base, we have
(G1) $K \circ K' \models \phi$ for all $\phi \in K'$
(G2) If $M(K) \cap M(K') \neq \emptyset$, then $M(K \circ K') = M(K) \cap M(K')$
(G3) If K' is consistent, then $M(K \circ K') \neq \emptyset$

(G1) guarantees that the new information is inferred from the revised knowledge base. (G2) requires that when there is no conflict between K and K', the result of revision be equivalent to the "union" of K and K', i.e. the set of its models are $M(K) \cap M(K')$. (G3) is a condition preventing a revision from introducing unwarranted inconsistency.

The postulate (R4) is the principle of irrelevance of syntax. Its generalization has the following form:
(G4) If $M(K) = M(K_1)$ and $M(K') = M(K_2)$, then $M(K \circ K') = M(K_1 \circ K_2)$.

(G4) requires that the revised knowledge base be independent of the syntax of both original knowledge bases and new information. The rule (R4) (and its generalization (G4)) is (are) very strong condition(s) because many syntax-based revision operators in propositional logic do not satisfy it. It is interesting to consider a weakened version of (G4) as follows.
(G4)′ If K_1 and K_2 are element-equivalent and $M(K_1') = M(K_2')$, then $M(K_1 \circ K_1') = M(K_2 \circ K_2')$.

Finally, (R5) and (R6) are generalized as follows.

(G5) $M(K \circ K') \cap M(K'') \subseteq M(K \circ (K' \cup K''))$

(G6) If $M(K \circ K') \cap M(K'')$ is not empty, then $M(K \circ (K' \cup K''))$
$\subseteq M(K \circ K') \cap M(K'')$

We have the following definition.

Definition 3. *A revision operator $\circ$ is said to be AGM compliant if it satisfies (G1-G6). It is quasi-AGM compliant if it satisfies (G1)-(G3), (G4)$'$, (G5-G6).*

4 Revision Operators for DLs

4.1 Definition

In this subsection, we propose a revision operator for DLs and provide a semantic explanation of it.

In this paper, we only consider inconsistencies arising due to objects being explicitly introduced in the ABox. That is, suppose K and K' are the original knowledge base and the newly received knowledge base respectively, then for each conflict K_c of $K \cup K'$, K_c must contain an ABox statement. For example, we exclude the following case: $\top \sqsubseteq \exists R.C \in K$ and $\top \sqsubseteq \forall R.\neg C \in K'$. The handling of conflicting axioms in the TBox has been discussed recently in [25, 22]. In this paper, we discuss the resolution of conflicting information which contains assertional axioms in the context of knowledge revision.

In order to define our approach, we need to extend $\mathcal{ALC}$ with nominals $\mathcal{O}$ (also called *individual names* [24]). A nominal has the form $\{a\}$, where a is an individual name. It can be viewed as a powerful generalization of DL ABox individuals. The semantics of $\{a\}$ is defined by $\{a\}^{\mathcal{I}} = \{a^{\mathcal{I}}\}$ for an interpretation $\mathcal{I}$. Nominals are very important expressions and they are included in many important DLs, such as $\mathcal{SHOQ}$ [13].

We give a method to weaken a GCI first.

Definition 4. *Let $C \sqsubseteq D$ be a GCI. A weakened GCI $(C \sqsubseteq D)_{weak}$ of $C \sqsubseteq D$ has the form $(C \sqcap \neg\{a_1\} \sqcap ... \sqcap \neg\{a_n\}) \sqsubseteq D$, where n is the number of individuals to be removed from C. We use $d((C \sqsubseteq D)_{weak}) = n$ to denote the degree of $(C \sqsubseteq D)_{weak}$.*

It is clear that when $d((C \sqsubseteq D)_{weak}) = 0$, $(C \sqsubseteq D)_{weak} = C \sqsubseteq D$. The idea of weakening a GCI is similar to weaken an uncertain rule in [4]. That is, when a GCI is involved in conflict, instead of dropping it completely, we remove those individuals which cause the conflict.

The weakening of an assertion is simpler than that of a GCI. The weakened assertion ϕ_{weak} of an ABox assertion $\phi = C(a)$ is of the form $\phi_{weak} = \top(a)$ or $\phi_{weak} = \phi$. When $\phi_{weak} = \top(a)$, we have $\mathcal{I} \models \phi_{weak}$ for all $\mathcal{I}$. Therefore, when $\phi_{weak} = \top(a)$, we simply delete ϕ. Indeed, we denote ϕ_{weak} by $\top(a)$ when ϕ is to be deleted for convenience of theoretical analysis. The degree of ϕ_{weak}, denoted as $d(\phi_{weak})$, is defined as $d(\phi_{weak}) = 1$ if $\phi_{weak} = \top(a)$ and 0 otherwise.

Definition 5. *Let K and K' be two DL knowledge bases. Suppose K' is consistent and $K \cup K'$ is inconsistent. A DL knowledge base $K_{weak,K'}$ is a weakened knowledge base of K w.r.t K' if it satisfies:*

- *$K_{weak,K'} \cup K'$ is consistent, and*
- *There is a bijection f from K to $K_{weak,K'}$ such that for each $\phi \in K$, $f(\phi)$ is a weakening of ϕ.*

The set of all weakened base of K w.r.t K' is denoted by $Weak_{K'}(K)$.

Example 1. Let $K = \{bird(tweety), bird \sqsubseteq flies\}$ and $K' = \{\neg flies(tweety)\}$, where *bird* and *flies* are two concepts and *tweety* is an individual name. It is easy to check that $K \cup K'$ is inconsistent. Let $K_1 = \{\top(tweety), bird \sqsubseteq flies\}$, $K_2 = \{bird(tweety), bird \sqcap \neg\{tweety\} \sqsubseteq flies\}$, then both K_1 and K_2 are weakened bases of K *w.r.t* K'.

The degree of a weakened base is defined as follows.

Definition 6. *Let $K_{weak,K'}$ be a weakened base of a DL knowledge base K w.r.t K'. The degree of $K_{weak,K'}$ is defined as*

$$d(K_{weak,K'}) = \Sigma_{\phi \in K_{weak,K'}} d(\phi)$$

In Example 1, we have $d(K_1) = d(K_2) = 1$.

We now define a revision operator.

Definition 7. *Let $\mathcal{K}$ be a (disjunctive) DL knowledge base, and K' be a newly received DL knowledge base. The result of weakening-based revision of $\mathcal{K}$ w.r.t K', denoted as $\mathcal{K} \circ_w K'$, is defined as follows: If K' is inconsistent, then $\mathcal{K} \circ_w K' = \{K \cup K' : K \in \mathcal{K}\}$; Otherwise,*

$$\mathcal{K} \circ_w K' = \bigcup_{K \in \mathcal{K}} \{K' \cup K_{weak,K'} : \ K_{weak,K'} \in Weak_{K'}(K), \ and \\ \nexists K_i \in Weak_{K'}(K), d(K_i) < d(K_{weak,K'})\}.$$

If K' is inconsistent, the result of revision is an inconsistent disjunctive DL knowledge base. When K' is consistent, the result of revision of $\mathcal{K}$ by K' is a disjunctive DL knowledge base consisting of DL knowledge bases which are unions of K' and a weakened base of a DL knowledge base K in $\mathcal{K}$ with the minimal degree. In the following, we assume that the original knowledge bases are ordinary DL knowledge base. This assumption is used to simply our discussions.

We next consider the semantic aspect of our revision operator.

Definition 8. *Let $\mathcal{W}$ be a non-empty set of interpretations and $\mathcal{I} \in \mathcal{W}$, ϕ a DL axiom, and K a DL knowledge base. If ϕ is an assertion, the number of ϕ-exceptions $e^{\phi}(\mathcal{I})$ is 0 if $\mathcal{I}$ satisfies ϕ and 1 otherwise. If ϕ is a GCI of the form $C \sqsubseteq D$, the number of ϕ-exceptions for $\mathcal{I}$ is:*

$$e^{\phi}(\mathcal{I}) = \begin{cases} |C^{\mathcal{I}} \cap (\neg D^{\mathcal{I}})| & \text{if } C^{\mathcal{I}} \cap (\neg D^{\mathcal{I}}) \text{ is finite} \\ \infty & \text{otherwise.} \end{cases} \tag{1}$$

The number of K-exceptions for $\mathcal{I}$ is $e^K(\mathcal{I}) = \Sigma_{\phi \in K} e^{\phi}(\mathcal{I})$. The ordering $\preceq_K$ on $\mathcal{W}$ is: $\mathcal{I} \preceq_K \mathcal{I}'$ iff $e^K(\mathcal{I}) \leq e^K(\mathcal{I}')$, for $\mathcal{I}' \in \mathcal{W}$.

The definition of ϕ-exception originates from Definition 6 in [19]. However, in [19], it is used to define an ordering $\preceq^{\pi}_{K}$ on a set of interpretations with the same *pre-interpretation* $\pi = (\Delta^{\pi}, d^{\pi})$, where Δ^{π} is a domain and d^{π} is a *denotation function* which maps every individual name a to a different element in Δ^{π}.

We give a proposition to give a semantic explanation of our weakening-based revision operator.

Proposition 1. *Let K be a consistent DL knowledge base. K' is a newly received DL knowledge base. $\circ_w$ is the weakening-based revision operator. We then have*

$$M(K \circ_w K') = min(M(K'), \preceq_K).$$

Proposition 1 says that the models of the resulting knowledge base of our revision operator are models of K' which are minimal *w.r.t* the ordering $\preceq_K$ induced by K. So it captures some kind of minimal change. All proofs of this paper can be found in [23].

Example 2. Let $K = \{\forall hasChild.RichHuman(Bob), hasChild(Bob, Mary), RichHuman(Mary), hasChild(Bob, Tom)\}$. Suppose we now receive new information $K' = \{hasChild\ (Bob, John), \neg RichHuman(John)\}$. It is clear that $K \cup K'$ is inconsistent. Since $\forall hasChild.\ RichHuman(Bob)$ is the only assertion axiom involved in conflict with K', we only need to delete it to restore consistency, that is, $K \circ_w K' = \{\top(Bob), hasChild(Bob, Mary), RichHuman(Mary), hasChild(Bob, Tom), hasChild(Bob, John), \neg RichHuman\ (John)\}$.

We have the following proposition.

Proposition 2. *Given two DL knowledge bases K and K'. The weakening-based revision operator is not AGM-compliant but it is quasi-AGM compliant, that is, it satisfies postulates (G1), (G2), (G3), (G4′), (G5) and (G6).*

4.2 Refined Weakening-Based Revision

In the weakening-based revision, to weaken a conflicting assertion axiom, we simply delete it. The problem for this method of weakening is that it does not take the constructors of description languages, such as conjunction ($\sqcap$) and value restriction ($\forall R.C$), into account. This may result in counterintuitive conclusions. In Example 2, after revising K by K' using the weakening-based operator, we cannot infer that $RichHuman(Tom)$ because $\forall hasChild.RichHuman(Bob)$ is discarded, which is counterintuitive. From $hasChild(Bob, Tom)$ and $\forall hasChild.\ RichHuman(Bob)$ we should have known that $RichHuman(Tom)$ and this assertion is not in any conflict of $K \cup K'$. The solution for this problem is to treat *John* as an *exception* and that all children of *Bob* other than *John* are rich humans.

For an ABox assertion of the form $\forall R.C(a)$, it is weakened by dropping some individuals which are related to the individual a by the relation R, i.e. its weakening has the form $\forall R.(C \sqcup \{b_1, ..., b_n\})(a)$, where b_i $(i = 1, n)$ are individuals.

We give another example to illustrate the problem of the weakening method.

Example 3. Let $K = \{bird \sqcap flies(tweety), bird(chirpy)\}$ and $K' = \{\neg flies(tweety)\}$. Clearly, $bird \sqcap flies(tweety)$ is in conflict with $\neg flies(tweety)$ in K'. Let $\phi = bird \sqcap flies(tweety)$. The weakening of ϕ is $\phi_{weak} = \top(tweety)$.

In Example 3, to weaken ϕ, we simply delete it. However, $bird(tweety)$, which can be inferred from K, is not responsible for any conflict of $K \cup K'$. Therefore, it is counterintuitive to delete it. This intuition is based on the assumption of the independence of concept names. That is, we take concept names as the "basic unit of change".

Before defining the new weakening method, we need to define an atomic concept.

Definition 9. *A concept is an atomic concept iff it is either a concept name or is of one of the forms $\{a\}$, $\forall R.C$ or $\exists R.C$, where a is an individual name and C is a (complex) concept.*

We assume that each concept C occurring in the original DL knowledge base K is in conjunctive normal form, i.e., $C = C_1 \sqcap ... \sqcap C_n$ such that $C_i = C_{i1} \sqcup ... \sqcup C_{im}$, where C_{ij} is either an atomic concept or the negation of a concept name. Conjunctive normal forms can be generated by the following steps. First, we transform the concept C into its negation normal form by the following equalities: $\neg\neg C_i \equiv C_i$, $\neg(C_i \sqcap D_i) \equiv \neg C_i \sqcup \neg D_i$, $\neg(C_i \sqcup D_i) \equiv \neg C_i \sqcap \neg D_i$, $\neg(\exists R.C_i) \equiv \forall R.\neg C_i$, $\neg(\forall R.C_i) \equiv \exists R.\neg C_i$. Second, we move disjunction inward and conjunction outward according to De Morgan's law: $C_1 \sqcup (C_2 \sqcap C_3) \equiv (C_1 \sqcup C_2) \sqcap (C_1 \sqcup C_3)$. Suppose $C(a) \in K$, where C is a concept in conjunctive normal form, we assume that each concept assertion $C(a)$ is decomposed into ϕ_1, ..., ϕ_n such that $\phi_i = (C_{i1} \sqcup ... \sqcup C_{im})(a)$. Note that a cannot be moved inside the disjunction constructor because disjunction of ABox assertions is not allowed in DLs.

We now define a new weakening method. The idea is that we weaken a concept assertion by weakening its atomic concepts. That is, we have the following definition.

Definition 10. *Let $\phi = R(a,b)$ be a role assertion. A weakened relation assertion ϕ_{weak} of ϕ is defined as $\phi_{weak} = \top_R(a,b)$ or $\phi_{weak} = \phi$, where $\top_R$ is interpreted as $\top_R^{\mathcal{I}} = \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ for each interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$. Let $\phi = C(a)$ be a concept assertion. A weakened concept assertion ϕ_{weak} of ϕ is defined recursively as follows:*

1) if $C = A$ or $\neg A$ for a concept name A, then $\phi_{weak} = \top(a)$ or $\phi_{weak} = \phi$,
2) if $C = \exists R.D$, then $\phi_{weak} = \top(a)$ or $\phi_{weak} = \phi$,
3) if $C = \forall R.D$, then $\phi_{weak} = \forall R.(D \sqcup \{b_1, ..., b_n\})(a)$ or $\top(a)$,
4) if $C = \{b\}$, where b is an individual name, then $\phi_{weak} = \top(a)$ or $\phi_{weak} = \phi$,
5) if $C = C_{i1} \sqcup ... \sqcup C_{im}$, where C_{ij} is either an atomic concept or the negation of an atomic concept, then $\phi_{weak} = ((C_{i1})_{weak} \sqcup ... \sqcup (C_{im})_{weak})(a)$[1] if $(C_{ij})_{weak} \not\equiv \top$ for all j and $\phi_{weak} = \top(a)$ otherwise,

[1] According to 1), 2), 3), and 4), we have $(C_{ij})_{weak} = \top$ or C_{ij} if C_{ij} is either a concept name or the negation of a concept name or of the form $\exists R.D$ or $\{b\}$, and $(C_{ij})_{weak} = \forall R.(D \sqcup \{b_1, ..., b_n\})$ if C_{ij} is of the form $\forall R.D$.

Let us explain the part 5) of Definition 10. Since the concept of ϕ is in disjunctive form, if there exists a C_{ij} such that $(C_{ij})_{weak} \equiv \top$, then $C \equiv \top$. That is, the weakening of a disjunct concept of ϕ may influence the weakening of other disjuncts. When weakening a role assertion, we introduce the top role. However, in implementation, the top role does not exist in the resulting knowledge base because the role assertion is simply deleted if the role name is weakened into the top role. In this paper, we only consider the refinement of the weakening of ABox assertions. Similarly, we can also refine the weakening of TBox axioms.

We next define the degree of a weakened assertion.

Definition 11. *Let $\phi = R(a,b)$, then $d(\phi_{weak}) = 1$ if $\phi_{weak} = \top_R(a,b)$ and 0 otherwise. Let $\phi = C(a)$, then $d(\phi)$ is defined recursively as follows:*

1) if $C = A$ or $\neg A$ for a concept name A, then $d(\phi_{weak}) = 1$ if $\phi_{weak} = \top(a)$ and 0 otherwise,
2) if $C = \exists R.C$, then $d(\phi_{weak}) = 1$ if $\phi_{weak} = \top(a)$ and 0 otherwise,
3) if $C = \forall R.D$, then $d(\phi_{weak}) = n$ if $\phi_{weak} = \forall R.(D \sqcup \{b_1, ..., b_n\})(a)$ and $+\infty$ otherwise,
4) if $C = \{b\}$, where b is an individual name, then $d(\phi_{weak}) = 1$ if $\phi_{weak} = \top(a)$ and 0 otherwise,
5) if $C = C_{i1} \sqcup ... \sqcup C_{im}$, where C_{ij} is either an atomic concept or the negation of an atomic concept, then $d(\phi_{weak}) = max\{d(((C_{ij})_{weak})(a)) : j = 1, ..., m\}$,

In part 5) of Definition 11, we use *max* (instead of sum) to determine the degree of an assertion in "disjunction" form. This definition agrees with the semantic interpretations of disjunction in many logics such as fuzzy logic and possibilistic logic.

We call the weakened base obtained by applying weakening of GCIs in Definition 4 and weakening of assertions in Definition 10 as a refined weakened base. We then replace the weakened base by the refined weakened base in Definition 7 and get a new revision operator, which we call a refined weakening-based revision operator which is denote by $\circ_{rw}$. Let us go back to Example 2 again. According to our discussion before, $\forall hasChild.Rich\ Human(Bob)$ is the only assertion axiom involved in the conflict in K and John is the only *exception* which makes $\forall hasChild.RichHuman(Bob)$ in conflict with K', so $K \circ_{rw} K' = \{\forall hasChild.(RichHuman \sqcup \{John\})(Bob), hasChild(Bob, Mary), RichHuman(Mary), hasChild(Bob, Tom), hasChild(Bob, John), \neg RichHuman(John)\}$. We can then infer that RichHuman(Tom) from $K \circ_{rw} K'$.

We consider another example. Let $K = \{((\forall R.C) \sqcup D)(a), R(a,b)\}$ and $K' = \{\neg D(a), \neg C(b)\}$, where C and D are concept names. Clearly, $K \cup K'$ is inconsistent. We can either weaken $((\forall R.C) \sqcup D)(a)$ or $R(a,b)$ to restore consistency. To weaken $R(a,b)$, we can simply delete it, i.e. its weakening has the form $\top_R(a,b)$. We have $d(\top_R(a,b)) = 1$. For $\phi = ((\forall R.C) \sqcup D)(a)$, we should weaken $\forall R.C$ instead of D. This is because if we weaken D to $\top$ then $(\forall R.C) \sqcup D$ also needs to be weakened to $\top$. In this case, we have $d(\phi_{weak}) = +\infty$. In contrast, if we weaken $(\forall R.C) \sqcup D$ to $(\forall R.(C \sqcup \{b\})) \sqcup D$, then D does not need to be weakened. In this case, we have $d((\forall R.(C \sqcup \{b\})) \sqcup D)(a)) = 1$ and $d(\phi_{weak}) = 1$. Therefore, there are two weakened bases of K *w.r.t* K', i.e. $K_1 = \{((\forall R.(C \sqcup \{b\})) \sqcup D)(a), R(a,b)\}$ and $K_2 = \{((\forall R.C) \sqcup D)(a)\}$.

To give a semantic explanation of the refined weakening-based revision operator, we need to define a new ordering between interpretations.

Definition 12. *Let $\mathcal{W}$ be a non-empty set of interpretations and $\mathcal{I} \in \mathcal{W}$, ϕ a DL axiom, and K a DL knowledge base. If ϕ is a concept assertion, then the number of ϕ-exceptions for $\mathcal{I}$ is defined recursively as follows:*

1) if $\phi = A(a)$ or $\neg A(a)$ for a concept name A, then $e_r^\phi(\mathcal{I}) = 0$ if $\mathcal{I} \models \phi$ and 1 otherwise,
2) if $\phi = \exists R.C(a)$, then $e_r^\phi(\mathcal{I}) = 0$ if $\mathcal{I} \models \phi$ and 1 otherwise,
3) If ϕ is an assertion of the form $\forall R.C(a)$, the number of ϕ-exceptions for $\mathcal{I}$ is:

$$e_r^\phi(\mathcal{I}) = \begin{cases} |R^\mathcal{I}(a^\mathcal{I}) \cap (\neg C^\mathcal{I})| & \text{if } R^\mathcal{I}(a^\mathcal{I}) \cap (\neg C^\mathcal{I}) \text{ is finite} \\ \infty & \text{otherwise,} \end{cases} \tag{2}$$

where $R^\mathcal{I}(a^\mathcal{I}) = \{b \in \Delta^\mathcal{I} : (a^\mathcal{I}, b) \in R^\mathcal{I}\}$.
4) If $\phi = \{b\}(a)$, where b is an individual name, then $e_r^\phi(\mathcal{I}) = 0$ if $\mathcal{I} \models \phi$ and 1 otherwise,
5) $\phi = (C_{i1} \sqcup ... \sqcup C_{im})(a)$, where C_{ij} is either an atomic concept or the negation of an atomic concept, then $e_r^\phi(\mathcal{I}) = max\{e_r^{C_{ij}(a)}(\mathcal{I}) : j = 1, ..., m\}$.

If ϕ is a role assertion, then $e_r^\phi(\mathcal{I}) = 0$ if $\mathcal{I} \models \phi$ and 1 otherwise.

If ϕ is a GCI of the form $C \sqsubseteq D$, the number of ϕ-exceptions for $\mathcal{I}$ is:

$$e_r^\phi(\mathcal{I}) = \begin{cases} |C^\mathcal{I} \cap (\neg D^\mathcal{I})| & \text{if } C^\mathcal{I} \cap (\neg D^\mathcal{I}) \text{ is finite} \\ \infty & \text{otherwise.} \end{cases} \tag{3}$$

The number of K-exceptions for $\mathcal{I}$ is $e_r^K(\mathcal{I}) = \Sigma_{\phi \in K} e_r^\phi(\mathcal{I})$. The refined ordering $\preceq_{r,K}$ on $\mathcal{W}$ is: $\mathcal{I} \preceq_{r,K} \mathcal{I}'$ iff $e_r^K(\mathcal{I}) \leq e_r^K(\mathcal{I}')$, for $\mathcal{I}' \in \mathcal{W}$.

The following proposition gives the semantic interpretation of the refined weakening-based revision operator.

Proposition 3. *Let K be a consistent DL knowledge base. K' is a newly received DL knowledge base. $\circ_{rw}$ is the refined weakening-based revision operator. We then have*

$$M(K \circ_{rw} K') = min(M(K'), \preceq_{r,K}).$$

Proposition 3 says that the refined weakening-based operator can be accomplished with minimal change. The proof is similar to that of Proposition 1.

Proposition 4. *Let K be a consistent DL knowledge base. K' is a newly received DL knowledge base. We then have*

$$M(K \circ_{rw} K') \subseteq M(K \circ_w K').$$

By Example 3, $K \circ_{rw} K'$ and $K \circ_w K'$ are not equivalent. Thus, we have shown that the resulting knowledge base of the refined weakening-based revision contains more information than that of the weakening-based revision. However, the refined weakening-based revision need to convert every ABox assertion to its conjunctive normal form. In some cases this conversion can lead to an exponential explosion of the size of the ABox assertion. So the sizes of the revised DL knowledge bases of the refined weakening-based operator are exponentially larger than those of the weakening-based operator in the worst case.

The refined weakening-based revision operator is still not AGM compliant.

Proposition 5. *Given two DL knowledge bases K and K'. The refined weakening-based revision operator is not AGM-compliant but it is quasi-AGM compliant.*

5 Related Work

The importance of applying AGM theory on belief change to terminological systems has not been fully recognized until recent years. In his book [20], Nebel considered the revision problem in terminological logics in 1990. He proposed some revision operators based on several existing approaches on modification of a terminological knowledge base. When defining his revision operator, he presumed that the terminological knowledge is more relevant than the assertional knowledge. Recently, some work has been done to analyze the feasibility of applying AGM theory on belief change to DLs [16, 7, 8]. However, none of them considers the explicit construction of a revision operator. Furthermore, they did not consider the application of AGM postulates for revision in DLs where knowledge bases instead of knowledge sets are considered. The work in [16, 8] is based on the *coherence model*, i.e. both the original and the revised knowledge bases should be knowledge sets which are knowledge bases closed under logical consequence. In [7], Fuhrmann's postulates for knowledge base contraction is generalized to DLs. One may wonder if we can establish the relationship between revisions and contractions via the Levi and Harper identities. However, the problem is that Levi and Harper identities are not applicable in DLs [8]. In [19], some revision operators were proposed for revising a *stratified* DL knowledge base. The semantic aspects of these revision operators are also considered. To define their operators, an extra expression in DLs, called *cardinality restrictions* on concepts, is necessary. In contrast, our operators are based on nominals. Since cardinality restrictions can be encoded as nominals, our revision operators can be seen as a refinement of the revision operators in [19]. In [14], a general framework for reasoning with inconsistent ontologies was given based on *concept relevance*. A problem with their framework is that they do not consider the structure of DL language. For example, when a GCI is in conflict in a DL knowledge base, it is deleted to restore consistency. Our work is also related to the work in [1], where Reiter's default logic is embedded into terminological representation formalisms. In their paper, conflicting information is also treated as *exceptions*. To deal with conflicting default rules, they instantiated each rule using individuals appearing in the ABox and applied two existing default reasoning methods to compute all extensions. This instantiation step is not necessary for our revision operators. Furthermore, in [1], the resolution of conflicting ABox assertions was not considered. This work is also related to the work on updating DL ABoxes in [15]. They showed that in any standard DL in which nominals and the "@" constructor are not expressible, updated ABoxes cannot be expressed. They only consider a simple form of ABox update where the update information contains possibly negated ABox assertions that involve only atomic concepts and roles.

6 Conclusions and Further Work

In this paper, we have discussed the problem of applying AGM theory of belief revision to DLs. We first generalized the reformulated AGM postulates for revision to DLs. Then two revision operators were proposed by weakening assertion axioms and GCIs. We showed that both revision operators satisfy the generalized postulates and capture some notions of minimal change.

Several problems are left as further work. First, none of our revision operators is AGM compliant, that is, they do not satisfy (G4). We are looking for a revision operator satisfying all the AGM postulates. Second, to implement our revision operators, an important problem is to detect GCIs and and assertions which are responsible for the conflict. Some existing techniques on debugging of unsatisfiable classes (such as [25, 22]) can be adopted or generalized to deal with this problem. We will develop tableaux-based algorithms for implementing our revision operators. Based on the results in [25], it is expected that the computational complexity of our operators may not increase the complexity of consistency checking in the DL under consideration.

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