A Fault-Tolerant Default Logic*

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Abstract. Reiter's default logic can not handle inconsistencies and incoherences and thus is not satisfactory enough in commonsense reasoning. In the paper we propose a new variant of default logic named FDL in which the existence of extension is guaranteed and the trivial extension is avoided. Moreover, Reiter's default extensions are reserved and can be identified from the other extensions in FDL. Technically, we develop a paraconsistent and monotonic reasoning system based on resolution as the underlying logic of FDL. The definition of extension is also modified in the manner that conflicts between justifications of the used defaults and the conclusions of the extension, which we call justification conflicts, are permitted, so that justifications can not be denied by "subsequent" defaults and the existence of extension is guaranteed. Then we select the desired extensions as preferred ones according to the criteria that justification conflicts should be minimal.

1 Introduction

Reiter's default logic[1] is a most advocated nonmonotonic reasoning system. It augments classical logic by *defaults* that differ from standard inference rules in sanctioning inferences that rely on given as well as absent information. Knowledge is represented in default logic by a *default theory* $\langle D, W \rangle$ consisting of a set of defaults D and a set of formulas W. Formulas in W are the *axioms* of the default theory. A default $\frac{\alpha:\beta_1,\cdots,\beta_n}{\gamma}$ has two types of antecedents: α is called the *prerequisite* and is established if α is derivable, and for $1 \leq i \leq n$, β_i is called a *justification* and is established if $\neg\beta_i$ can not be derived. If both conditions hold, the default is *applicable* and the *consequent* γ is concluded. An *extension* of a default theory which is a fixpoint of the belief revision operator w.r.t. the default theory can be viewed as a belief set described by the default theory. For clarity, we use the term *default logic* to refer to Reiter's default logic and call extension in Reiter's default logic *default extension*.

Despite its simple syntax and powerful expressivity, Reiter's default logic is not satisfactory enough in the following two aspects. On the one hand, a default theory has only a trivial default extension that includes every formula once the axioms in the default theory have contradictions (see Proposition 3). That is,

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contradictions in the axioms damage the meaningful information of a default theory. Such contradictions are called *inconsistencies* and are represented by the curve labeled with 1 in Figure 1. A default theory is *inconsistent* if it has inconsistencies, otherwise it is *consistent*.

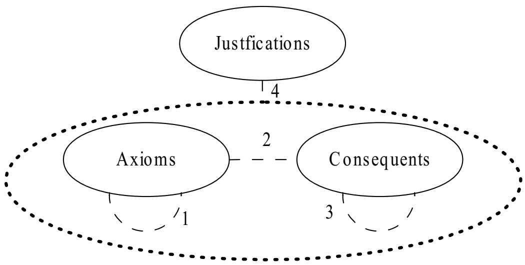

Fig. 1. Inconsistencies and Incoherences

On the other hand, some contradictions in a default theory cause the nonexistence of extension. We call such contradictions *incoherences*. A default theory is *incoherent* if it has no default extension, otherwise it is *coherent*. Incoherences may be categorized into three sorts shown in the following three examples respectively.

Example 1. $T = \langle D, W \rangle$, where $D = \{\frac{:B}{\neg A}\}$ and $W = \{A\}$. In T incoherences occur between W and the consequents of applicable defaults, which are represented by the line labeled with 2 in Figure 1.

Example 2. $T = \langle D, W \rangle$, where $D = \{\frac{:B}{C}, \frac{:D}{\neg C}\}$ and $W = \emptyset$. In T incoherences occur in the consequents of applicable defaults, which are represented by the curve labeled with 3 in Figure 1.

Example 3. $T = \langle D, W \rangle$, where $D = \{\frac{:\neg B}{A}\}$ and $W = \{A \rightarrow B\}$. In T incoherences occur between the justifications of used defaults and the consequences of (W and the consequents of used defaults), which are represented by the line labeled with 4 in Figure 1.

Some researchers hold the view that triviality and the nonexistence of extension of a given default theory are not shortcomings of default logic, and sometimes they are useful when default logic is used as a problem solver. For instance, a planning problem may be represented by a default theory whose default extensions correspond to the solutions of the problem and the incoherences of this default theory tell us that the problem has no solution. With this viewpoint, they have made efforts to find characterizations of default theories that have default extensions[1, 2, 3, 4].

The above viewpoint is reasonable in some aspects but does not seem to be sound when it turns to commonsense reasoning, since commonsense is always inconsistent and incoherent and we expect that some meaningful conclusions can still be reached even the knowledge is inconsistent or incoherent.

According to the above analysis, some researchers believe that Reiter's default logic is not fault-tolerant enough. Some of them refer to paraconsistent logic[5, 6, 7, 8], multi-valued logic[9, 10, 11, 12] in particular, to overcome inconsistencies, such as Bi-default logic[13] and four-valued default logic[14]. To deal with incoherences, some researchers modify Reiter's definition of extension to guarantee the existence of extension, among which are justified default logic[15], constrained default logic[16] and cumulative default logic[17].

The above five default reasoning systems are good attempts to extract meaningful information from a default theory with inconsistencies or incoherences, but they fail to handle inconsistencies and incoherences simultaneously. Moreover, justified default logic and constrained default logic can not identify default extensions from the other extensions and therefore they can not solve some problems that Reiter's default logic can, e.g. they are not suitable as problem solvers.

In the paper we propose a default reasoning system called *FDL*(shorted for Fault-tolerant Default Logic) in which every default theory has at least one extension and the trivial extension is avoided. Thus FDL can extract meaningful information from a default theory with inconsistencies and incoherences, which indicates that it can perform better than other default reasoning systems in commonsense reasoning. Besides its fault-tolerance, we also show that Reiter's default extensions are reserved and can be identified from the other extensions, which makes FDL able to solve the problems that Reiter's default logic can.

To overcome inconsistencies, it is natural that the underlying logic of FDL should be paraconsistent. But a paraconsistent logic is not adequate, since if it is nonmonotonic, the existence of extension can not be ensured—using an applicable default may make the prerequisite of a used default not derivable and thus inapplicable. Besides, to reserve Reiter's default extensions, the underlying logic should coincide with classical logic as to consistent sets of formulas. Since most existing paraconsistent logics which coincide with classical logic as to consistent sets of formulas are nonmonotonic, we need to develop a paraconsistent and monotonic one. To guarantee the existence of extension, we have to go further—a paraconsistent and monotonic underlying logic can only resolve contradictions represented by 1, 2 and 3 in Figure 1. To resolve contradictions represented by 4 in Figure 1, the definition of extension needs to be modified. In FDL, the modification is in the manner that justifications of the used defaults should be most consistent with the conclusions of the default theory, i.e. justification conflicts which are minimized are permitted.

To sum up, our work may be divided into the following steps:

1. Develop a paraconsistent and monotonic reasoning system as the underlying logic to handle inconsistencies.
2. Modify the definition of default extension so that FDL can tolerate incoherences and the existence of extension is guaranteed.
3. Select desired extensions.

Step 1 and 2 make FDL able to tolerate inconsistencies and incoherences, but at the cost that some counter-intuitive extensions may appear. With Step 3, these counter-intuitive extensions are excluded and desired ones are reserved. The trick—first guarantee the existence, then select desired ones—is also involved in preferred models[18, 19, 11] of multi-valued logic.

The rest of the paper is structured as follows. In Section 2 we briefly review Reiter's default logic. In Section 3 we introduce the paraconsistent and monotonic underlying reasoning system of FDL. Our system is represented in Section 4. Some properties of FDL are also studied in this section. We compare our work with others in Section 5 and conclude the paper in Section 6.

2 Default Logic

We start by completing our initial introduction to Reiter's default logic.

Throughout this paper, let $\mathcal{L}$ be a propositional language. We use Greek and uppercase letters to represent the formulas and the atoms in $\mathcal{L}$ respectively. Each atom A and its negation $\neg A$ are called *literals* which are represented by lowercase letters. The connectives in $\mathcal{L}$ are defined in the canonical manner. We write $\vdash$ for the provability relation in classical logic. The set of consequences of S is defined as $Cn(S) = \{\alpha \in \mathcal{L} \,|\, S \vdash \alpha\}$, where S is a set of formulas in $\mathcal{L}$.

A default is *normal* if it is of the form $\frac{\alpha:\beta}{\beta}$. Let D be a set of defaults. By $Pre(D)$, $Just(D)$ and $Con(D)$, we denote the sets of prerequisites, justifications and consequents of the defaults in D respectively. A set of defaults D and a set of formulas W form a *default theory* $\langle D, W\rangle$, which is *normal* if all defaults in D are normal. For simplicity, we assume that W and D are both finite. A default theory may induce one, multiple or even no default extensions in the following way.

Definition 1 ([1]). *Let $T = \langle D, W\rangle$ be a default theory. For any set of formulas S, $\Gamma(S)$ is the smallest set of formulas such that*

1. $\Gamma(S) = Cn(\Gamma(S))$.
2. $W \subseteq \Gamma(S)$.
3. *If* $\frac{\alpha:\beta_1,\cdots,\beta_n}{\gamma} \in D$, $\alpha \in \Gamma(s)$, *and* $\neg\beta_1 \notin S$, $\cdots$, $\neg\beta_n \notin S$, *then* $\gamma \in \Gamma(S)$.

A set of formulas E is a default extension *of $\langle D, W\rangle$ if $\Gamma(E) = E$.*

The *set of generating defaults* for E w.r.t. default theory T is defined as

$$GD(E,T) = \left\{ \frac{\alpha : \beta_1, \cdots, \beta_n}{\gamma} \in D \,|\alpha \in E, \neg\beta_1 \notin E, \cdots, \neg\beta_n \notin E \right\}$$

Proposition 1 ([1]). *If E is a default extension of default theory $T = \langle D, W\rangle$, then $E = Cn(W \cup Con(GD(E,T)))$.*

Proposition 2 ([1]). *Let $T = \langle D, W\rangle$ be a default theory. For any set of formulas E, define $E_0 = W$ and for each $i \geq 0$*

$$E_{i+1}=Cn(E_i)\cup\left\{\gamma\left|\frac{\alpha:\beta_1,\cdots,\beta_n}{\gamma}\in D,\ \textit{where}\ \alpha\in E_i,\neg\beta_1\notin E,\cdots,\neg\beta_n\notin E\right.\right\}$$

Then E is a default extension of T iff $E=\bigcup_{i=0}^{\infty}E_i$.

Proposition 3 ([1]). *Default theory $T=\langle D,W\rangle$ has only a trivial default extension iff W is inconsistent.*

Proposition 4 ([1]). *Each normal default theory has at least one default extension.*

3 The Underlying Logic

Since we base the underlying logic on resolution, we have to convert formulas into clauses. If a formula contains inconsistencies, there would be more than one set of clauses corresponding to it and the result of resolution is different. Hence a "normal" form of clauses is necessary.

Definition 2. *Let α be a formula which is not a tautology in $\mathcal{L}$ and $P_1,\cdots,P_n$ be all atoms occurring in α. We say formula $\beta=(l_{11}\vee\cdots\vee l_{1n})\wedge\cdots\wedge(l_{m1}\vee\cdots\vee l_{mn})$ is a* principal conjunctive normal form *of α if*

1. *α is equivalent to β.*
2. *l_{ij} is P_j or $\neg P_j$ for $1\le i\le m$ and $1\le j\le n$.*

If α is a tautology, define the principal conjunctive normal form of α to be t.

Definition 3. *Let α be a formula in $\mathcal{L}$. The* set of principal clauses *of α is defined as $C(\alpha)=\{\{l_1,\cdots,l_n\}|l_1\vee\cdots\vee l_n$ is a conjunct of a principal conjunctive normal form of $\alpha\}$. Let S be a set of formulas in $\mathcal{L}$. The set of principal clauses of S is $C(S)=\bigcup_{\alpha\in S}C(\alpha)$.*

Lemma 1. *Each set of formulas has exactly one set of principal clauses.*

Definition 4. *A set of clauses S is* resolution closed *if $\{A\}\cup C_1\in S$ and $\{\neg A\}\cup C_2\in S$ imply $C_1\cup C_2\in S$, provided that $C_1\cup C_2$ is not empty and A is an atom. The smallest set that includes S and is resolution closed is called the* resolution closure *of S written as $RC(S)$.*

Definition 5. *A set of clauses S is* appending closed *if*

1. *$\{t\}\in S$.*
2. *If $C_1\in S$ and $C_1\subseteq C_2$, then $C_2\in S$.*

The appending closure *of S written as $AC(S)$ is the smallest set that includes S and is appending closed.*

Definition 6. *Let α be a formula and S be a set of formulas in $\mathcal{L}$. If $C(\alpha)\subseteq AC(RC(C(S)))$, we say that α is* monotonically derivable *from S written as $S\vdash_{mc}\alpha$. The* monotonic closure *of S is defined as $MC(S)=\{\alpha\in\mathcal{L}|S\vdash_{mc}\alpha\}$.*

Example 4. Assume S=$\{A, A \rightarrow B\}$. Then $C(S)$=$\{\{A\}, \{\neg A, B\}\}$, $RC(C(S))$= $\{\{A\}, \{\neg A, B\}, \{B\}\}$, $AC(RC(C(S)))$ =$\{\{t\}, \{A\}, \{\neg A, B\}, \{B\}, \cdots\}$. Since $C(A \wedge B)$=$\{\{A, B\}, \{A, \neg B\}, \{\neg A, B\}\} \subseteq AC(RC(C(S)))$, $S \vdash_{mc} A \wedge B$. It can be verified that $MC(S) = Cn(S)$. Now let $S' = S \cup \{\neg A\}$. Then $C(S')$=$\{\{\neg A\}, \{A\}, \{\neg A, B\}\}$, $RC(C(S'))$=$\{\{\neg A\}, \{A\}, \{\neg A, B\}, \{B\}\}$, $AC(RC(C(S')))$= $\{\{t\}, \{\neg A\}, \{A\}, \{\neg A, B\}, \{B\}, \cdots\}$. Therefore $MC(S')$=$\{\neg A, A, A \rightarrow B, B, A \vee B, \neg A \vee B, A \wedge \neg A, \cdots\}$. It is readily to verify that $MC(S) \subseteq MC(S')$. Also $S' \nvdash_{mc} C$ for any atom C not occurring in S', which indicates that MC is paraconsistent.

From the above example, we notice that inconsistencies can be conquered with MC. Moreover, MC is monotonic, as stated by the following proposition.

Proposition 5 (Monotonicity of MC). *If $S \subseteq S'$, then $MC(S) \subseteq MC(S')$.*

Proposition 6. *If S is classically consistent, then $MC(S) \equiv Cn(S)$.*

Proposition 6 implies that, although MC is strictly weaker than Cn(since MC is paraconsistent), they are identical as to consistent sets of formulas.

In [20], Lehmann suggests that the reasoning relation $\vdash_p$ in a plausibility logic should satisfy the following conditions:

1. Inclusion: $\Gamma \vdash_p \alpha$ if $\alpha \in \Gamma$.
2. Right Monotonicity: If $\Gamma \vdash_p \alpha$, then $\Gamma \vdash_p \alpha \vee \beta$ for any formula β.
3. Cautious Left Monotonicity: If $\Gamma \vdash_p \alpha$ and $\Gamma \vdash_p \beta$, then $\Gamma \cup \{\alpha\} \vdash_p \beta$.
4. Cautious cut: If $\Gamma \cup \{\alpha\} \vdash_p \beta$ and $\Gamma \vdash_p \alpha$, then $\Gamma \vdash_p \beta$.

It can be verified that $\vdash_{mc}$ satisfies all of the above conditions but cautious cut. It means that a derived formula can not be used as a lemma to infer new formulas, i.e. cumulativity does not hold as to $\vdash_{mc}$. Therefore $\vdash_{mc}$ is not a plausibility logic in the above sense.

4 Fault-Tolerant Default Logic

4.1 Alternative Extension

To this point, we modify the definition of default extension.

Definition 7. *Let $T = \langle D, W \rangle$ be a default theory. For the sets of formulas E, J and E', we say that default $\delta = \frac{\alpha:\beta_1,\cdots,\beta_n}{\gamma}$ is* applicable *to E' w.r.t. E and J if*

1. *$\alpha \in E'$.*
2. *$\neg\beta_i \notin E$ or there is a formula equivalent to $\neg\beta_i$ in $E \cap J$ for each $1 \leq i \leq n$.*

For sets of formulas E and J, we say pair $\langle E, J \rangle$ is *smaller* than $\langle E', J' \rangle$ written as $\langle E, J \rangle \leq \langle E', J' \rangle$ if $E \subseteq E'$ and $J \subseteq J'$. $\langle E, J \rangle$ is *consistent* if E is consistent.

Definition 8. *Let* $T = \langle D, W\rangle$ *be a default theory.* $\Gamma(\langle E, J\rangle)$ *is the smallest pair* $\langle E', J'\rangle$ *such that*

1. $W \subseteq E'$
2. $E' = MC(E')$
3. *If* $\frac{\alpha:\beta_1,\cdots,\beta_n}{\gamma} \in D$ *is applicable to* E' *w.r.t.* E *and* J*, then* $\gamma \in E'$, $\neg\beta_1 \in J'$, $\cdots$, $\neg\beta_n \in J'$.

A pair $\langle E, J\rangle$ *is an* alternative extension *of* $T = \langle W, D\rangle$ *if* $\langle E, J\rangle = \Gamma(\langle E, J\rangle)$.

With the similar approach in [21], we can verify that Γ and alternative extension are well-defined.

Theorem 1. *Let* $T = \langle D, W\rangle$ *be a default theory. For the sets of formulas* E *and* J*, define* $E_0 = MC(W)$, $J_0 = \emptyset$*, and for* $i \geq 0$

$$E_{i+1} = MC(E_i) \cup \left\{\gamma \,\middle|\, \frac{\alpha : \beta_1, \cdots, \beta_n}{\gamma} \in D \text{ is applicable to } E_i \text{ w.r.t. } E \text{ and } J\right\}$$

$$J_{i+1}=J_i\cup\left\{\neg\beta_1, \cdots, \neg\beta_n \,\middle|\, \frac{\alpha : \beta_1, \cdots, \beta_n}{\gamma} \in D \text{ is applicable to } E_i \text{ w.r.t. } E \text{ and } J\right\}$$

Then $\langle E, J\rangle$ *is an alternative extension of* T *iff* $E = \bigcup_{i=0}^{\infty} E_i$ *and* $J = \bigcup_{i=0}^{\infty} J_i$.

Definition 9. *The* set of generating defaults *for pair* $\langle E, J\rangle$ *w.r.t. default theory* T *written as* $GD(E, J, T)$ *is*

$$GD(E, J, T) = \left\{\delta \,\middle|\, \delta = \frac{\alpha : \beta_1, \cdots, \beta_n}{\gamma} \in D \text{ is applicable to } E \text{ w.r.t. } E \text{ and } J\right\}$$

Theorem 2. *For the sets of formulas* E *and* J*, if* $\langle E, J\rangle$ *is an alternative extension of default theory* $T = \langle D, W\rangle$*, then* $E = MC(W \cup Con(GD(E, J, T)))$ *and* $J = \{\neg\beta | \beta \in Just(GD(E, J, T))\}$.

The following two examples indicate that alternative extension could tolerate all contradictions shown in Figure 1.

Example 5. Consider default theory $T = \langle D, W\rangle$, where $D = \{\frac{B:C}{D}\}$ and $W = \{A, \neg A, A \to B\}$. According to Proposition 3, T has only a trivial default extension. Contrarily, since the underlying logic of FDL is paraconsistent, it has a nontrivial alternative extension $\langle MC(\{A, \neg A, A \to B, D\}), \{\neg C\}\rangle$.

Example 6. Consider the default theories in Example 1, 2 and 3. They have $\langle MC(\{A, \neg A\}), \{\neg B\}\rangle$, $\langle MC(\{C, \neg C\}), \{\neg B, \neg D\}\rangle$ and $\langle MC(\{A, A \to B, B\}), \{\neg\neg B\}\rangle$ as their alternative extensions respectively.

4.2 Operational Considerations

In [22], an operational semantics is assigned to Reiter's default logic. We can do the similar thing to alternative extension. Let $T = \langle D, W\rangle$ be a default theory and $\Pi = \langle d_1, d_2, \cdots\rangle$ be a sequence of defaults from D. We denote the initial segment of Π of length k by Π_k, provided that the length of Π is at least k, and define $In(\Pi_i) = MC(W \cup Con(\Pi_i))$, $Out(\Pi_i) = \{\neg\beta | \beta \in Just(\Pi_i)\}$.

Definition 10. *A default* $d = \alpha : \beta_1, \cdots, \beta_n/\gamma$ *is* applicable *to a sequence* Π *if*

1. $\alpha \in In(\Pi)$.
2. $\neg\beta_i \notin In(\Pi)$ *or there is a formula equivalent to* $\neg\beta_i$ *in* $In(\Pi) \cap Out(\Pi)$ *for each* $1 \leq i \leq n$.

Definition 11. *A sequence* $\Pi = \langle d_1, d_2, \cdots \rangle$ *in* D *is a* process *if* d_k *is applicable to* Π_{k-1} *for every* $k \geq 1$*. Process* Π *is* closed *if every* $d \in D$ *that is applicable to* Π *already occurs in* Π*.*

Lemma 2. *Each default theory has at least one closed process.*

Since justifications of used defaults in a process can not be invalidated by subsequent defaults, we have Lemma 3 and Theorem 3.

Lemma 3. *If* Π *is a closed process of default theory* T*, then* $\langle In(\Pi), Out(\Pi)\rangle$ *is an alternative extension of* T*.*

Theorem 3 (Semimonotonicity). *If* $T = \langle D, W\rangle$ *and* $T' = \langle D', W\rangle$ *are two default theories, where* $D \subseteq D'$*, and* $\langle E, J\rangle$ *is an alternative extension of* T*, then* T' *must have an alternative extension* $\langle E', J'\rangle$ *such that* $\langle E, J\rangle \leq \langle E', J'\rangle$*.*

From Lemma 2 and 3, we immediately have

Theorem 4 (Existence of Alternative Extension). *Each default theory has at least one alternative extension.*

For an alternative extension $\langle E, J\rangle$, if there is a closed process Π such that $In(\Pi) = E$ and $Out(\Pi) = J$, we say $\langle E, J\rangle$ has Π *corresponding* to it. Although Definition 11 and Lemma 3 imply that each closed process corresponds to an alternative extension, there are some alternative extensions that have no process corresponding to them, i.e. some alternative extensions are not constructive.

Example 7. Consider default theory $T = \langle D, W\rangle$, where $D = \{\frac{:A}{B}, \frac{:\neg B}{\neg A}\}$ and $W = \emptyset$. T has just two closed processes: $\langle\frac{:A}{B}\rangle$ and $\langle\frac{:\neg A}{\neg B}\rangle$. There is no closed process corresponding to alternative extension $\langle MC(\{\neg A, B\}), \{\neg A, \neg\neg B\}\rangle$.

4.3 The Largest and the Minimal Alternative Extensions

Minimality does not hold as to alternative extension.

Example 8. Let $D = \{\frac{:A}{B}, \frac{:\neg B}{C}\}$ and $W = \emptyset$. It can be verified that default theory $T = \langle D,\ W\rangle$ has two alternative extensions: $\langle E_1,\ J_1\rangle$=$\langle MC(\{B\}),\ \{\neg A\}\rangle$ and $\langle E_2,\ J_2\rangle$=$\langle MC(\{B,\ C\}),\ \{\neg A,\ \neg\neg B\}\rangle$. Since $E_1 \subset E_2$, $J_1 \subset J_2$, $\langle E_1,\ J_1\rangle < \langle E_2,\ J_2\rangle$.

This is not occasional. As a matter of fact, we have

Theorem 5. *If* $\langle E_k, J_k\rangle$ *are alternative extensions of default theory* T *for each* $k = 1, 2, \cdots$*, then there must exist the smallest alternative extension* $\langle E, J\rangle$ *of* T

that is bigger than $\langle E_k, J_k\rangle$ *for each* k *such that* $E = \bigcup_{i=0}^{\infty} F_i$ *and* $J = \bigcup_{i=0}^{\infty} K_i$*, where* $F_0 = \bigcup_k E_k$*,* $K_0 = \bigcup_k J_k$*, and for* $i \geq 0$

$$F_{i+1} = MC(F_i) \cup \left\{\gamma \,\middle|\, \frac{\alpha : \beta_1, \cdots, \beta_n}{\gamma} \text{ is applicable to } F_i \text{ w.r.t. } F_i \text{ and } K_i\right\}$$

$$K_{i+1} = K_i \cup \left\{\neg\beta_1, \cdots, \neg\beta_n \,\middle|\, \frac{\alpha : \beta_1, \cdots, \beta_n}{\gamma} \text{ is applicable to } F_i \text{ w.r.t. } F_i \text{ and } K_i\right\}$$

The above theorem indicates that all the alternative extensions of a default theory form a *complete upper semilattice* w.r.t. $\leq$.

Corollary 1 (Existence of the Largest Alternative Extension). *Each default theory* T *has an alternative extension* $\langle E, J\rangle$ *such that for each alternative extension* $\langle E', J'\rangle$ *of* T*,* $\langle E', J'\rangle \leq \langle E, J\rangle$*.*

Definition 12. *For an alternative extension* $\langle E, J\rangle$ *of default theory* T*, if there is no alternative extension of* T *that is smaller than* $\langle E, J\rangle$*, then* $\langle E, J\rangle$ *is a* minimal alternative extension *of* T*.*

Theorem 6. *If* $\langle E, J\rangle$ *is a minimal alternative extension of default theory* T*, then there must be a closed process* Π *such that* $E = In(\Pi)$ *and* $J = Out(\Pi)$*.*

The above theorem does not hold vice versa. See the following example.

Example 9. Consider default theory $T = \langle D, W\rangle$, where $D = \{\frac{:A}{B}, \frac{:C}{\neg A}\}$ and $W = \emptyset$. Sequence $\langle \frac{:A}{B}, \frac{:C}{\neg A}\rangle$ is a closed process of T. Therefore $\langle E, J\rangle = \langle MC(\{\neg A, B\}), \{\neg A, \neg C\}\rangle$ is an alternative extension. However, it is not a minimal alternative extension(alternative extension $\langle MC(\{\neg A\}), \{\neg C\}\rangle$ is smaller than $\langle E, J\rangle$).

4.4 Preferred Extension

In Example 8, we note that some alternative extensions are counter-intuitive. In this subsection, we exclude those counter-intuitive ones according to the criteria that the justification conflicts should be minimal.

Definition 13. *Let* T *be a default theory. An alternative extension* $\langle E', J'\rangle$ *of* T *is called a* preferred extension *if* $E' \cap J'$ *is minimal in* $\{E \cap J | \langle E, J\rangle$ *is an alternative extension of* $T\}$*.*

Example 10. Let $T = \langle D, W\rangle$ be a default theory, where $D = \{\frac{:A}{B}, \frac{:\neg B}{\neg A}, \frac{B:C}{C}, \frac{\neg A:B}{B}\}$ and $W = \emptyset$. T has three alternative extensions: $\langle E_1, J_1\rangle = \langle MC(\{B, C\}), \{\neg A, \neg C\}\rangle$, $\langle E_2, J_2\rangle = \langle MC(\{\neg A, B, C\}), \{\neg B, \neg\neg B, \neg C\}\rangle$ and $\langle E_3, J_3\rangle = \langle MC(\{\neg A, B, C\}), \{\neg A, \neg\neg B, \neg C\}\rangle$. Since $E_1 \cap J_1 = \emptyset$, $E_2 \cap J_2 = \{\neg\neg B\}$ and $E_3 \cap J_3 = \{\neg A, \neg\neg B\}$, $\langle E_1, J_1\rangle$ is a preferred extension of T, while the other two are not.

Example 11. Let $T = \langle D, W\rangle$ be a default theory, where $D = \{\frac{:A}{\neg A}\}$ and $W = \emptyset$. T has only one alternative extension $\langle MC(\{\neg A\}), \{\neg A\}\rangle$ which is also the only preferred extension of T.

Theorem 7 (Existence of Preferred Extension). *Each default theory has at least one preferred extension.*

Theorem 8. *Each preferred extension is a minimal alternative extension.*

The above theorem does not hold vice versa, as shown by the following example.

Example 12. Let $D = \{\frac{:A}{B}, \frac{B:C}{\neg A}, \frac{:\neg B}{\neg A \wedge \neg C}\}$ and $W = \emptyset$. Default theory $T = \langle D, W\rangle$ has $\langle E_1, J_1\rangle = \langle MC(\{\neg A, B\}), \{\neg A, \neg C\}\rangle$, $\langle E_2, J_2\rangle = \langle MC(\{\neg A, \neg C\}), \{\neg\neg B\}\rangle$ and $\langle E_3, J_3\rangle = \langle MC(\{\neg A, B, \neg C\}), \{\neg A, \neg\neg B, \neg C\}\rangle$ as its alternative extensions, where $\langle E_1, J_1\rangle$ and $\langle E_2, J_2\rangle$ are minimal alternative extensions, but $\langle E_1, J_1\rangle$ is not a preferred extension, whereas $\langle E_2, J_2\rangle$ is.

Corollary 2 (Minimality of Preferred Extension). *If $\langle E, J\rangle$ and $\langle E', J'\rangle$ are both preferred extensions of default theory T and $\langle E, J\rangle \leq \langle E', J'\rangle$, then $E = E'$ and $J = J'$.*

Theorem 9. *E is a consistent default extension of default theory T iff T has a consistent preferred extension $\langle E, J\rangle$ such that $E \cap J = \emptyset$.*

Semimonotonicity does not hold as to preferred extension(see the following example). Therefore semimonotonicity implies the existence of extension, but not vice versa.

Example 13. Let $W = \emptyset$ and $D = \{\frac{:A}{B}, \frac{:\neg B}{\neg A}\}$, $D' = \{\frac{:A}{B}, \frac{:\neg B}{\neg A}, \frac{B:\neg A}{\neg A}\}$. Default theory $\langle D, W\rangle$ has two preferred extensions: $\langle E_1, J_1\rangle = \langle MC(\{B\}), \{\neg A\}\rangle$ and $\langle E_2, J_2\rangle = \langle MC(\{\neg A\}), \{\neg\neg B\}\rangle$, while $\langle D', W\rangle$ has only one preferred extension: $\langle E, J\rangle = \langle MC(\{\neg A\}), \{\neg\neg B\}\rangle$. Although $D \subseteq D'$, $\langle D', W\rangle$ has no preferred extension bigger than $\langle E_1, J_1\rangle$.

Definition 14. *If $\langle E, J\rangle$ and $\langle E', J'\rangle$ are preferred extensions of default theory T such that $E \cap J = E' \cap J'$ and $\{\alpha | \alpha \wedge \neg\alpha \in E\} \subset \{\alpha | \alpha \wedge \neg\alpha \in E'\}$, we say $\langle E, J\rangle$ is* more consistent *than $\langle E', J'\rangle$. If T has no preferred extension more consistent than $\langle E, J\rangle$, $\langle E, J\rangle$ is a* most consistent preferred extension *of T.*

Theorem 10 (Existence of Most Consistent Preferred Extension). *Each default theory has at least one most consistent preferred extension.*

From Theorem 9, we have

Corollary 3. *E is a default extension of consistent and coherent default theory T iff there is a set of formulas J such that $\langle E, J\rangle$ is a most consistent preferred extension of T and $E \cap J = \emptyset$.*

To this point, we have discussed a set of extensions, the inclusion relations among which are shown in Figure 2. The figure also indicates that to compute all the preferred extensions and most consistent preferred extensions, it suffices to consider only closed processes which are constructive.

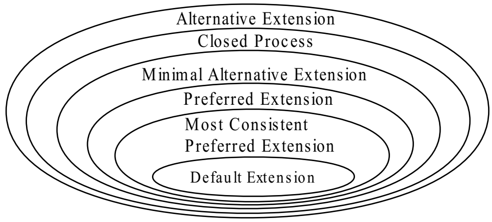

Fig. 2. Inclusion Relations

Theorem 11. *If $T = \langle D, W \rangle$ is a normal default theory where W is consistent, then E is a default extension of T iff T has a closed process Π such that $In(\Pi) = E$ and $In(\Pi) \cap Out(\Pi) = \emptyset$.*

The above theorem implies that as to a consistent normal default theory, closed processes, minimal alternative extensions, preferred extensions, most consistent preferred extensions and default extensions are identical. Theorem 9 and Corollary 3 imply that as to a consistent and coherent default theory, default extensions are identical with consistent preferred extensions as well as most consistent preferred extensions.

5 Related Work

The main idea of the paper is inspired by Reiter's seminal paper[1], some variants of it[15, 17, 16] and Linke *et al*'s work on default logic[4]. The formal definition of default extension is more delicate than could have been expected. This is due to the context-sensitive nature of justifications. In fact, a default's justifications can be refuted only when all default consequents contributing to a default extension are known. This is why the non-refutability of a justification is verified w.r.t. the final default extension. In such an approach, default extensions are not constructive and one is obliged to inspect the entire set of defaults to decide whether a default can be applied.

Reiter[1], Lukaszewicz[15] and Linke[4] have tried to reduce this kind of global checks to local ones to make extensions constructive. In normal default logic, defaults are restricted to be normal so that local checks are adequate. Linke *et al* replace such global checks by the strictly necessary ones. In justified default logic, a default is applicable only if its prerequisite is derivable and its justifications and consequent are consistent with used defaults. Therefore global checks are unnecessary in justified default logic.

By avoiding this kind of global checks, normal default logic and justified default logic are seminormal and the existence of extension is guaranteed. But since the underlying logic of the two are not paraconsistent, the trivial extension can not be avoided.

Taking no account of the existence of extension, bi-default logic[13] adopts the approach of signed system[8]. It translates a default theory into two related default theories which are both consistent. The two related default theories comprise a bi-default theory. By dividing the inconsistencies into two parts, bi-default logic overcomes triviality. Compared with bi-default logic, the approach taken by Yue *et al*[14] seems to be more natural. They define four-valued models for an arbitrary default theory. Since four-valued logic is paraconsistent, all four-valued models are nontrivial. Similar to bi-default logic, a transformation is introduced to translate the original default theory to a consistent signed one. What is interesting is, it is proved that four-valued models of the original default theory can be gained by computing the default extensions of the translated default theory. Unfortunately, the relationship between four-valued models and extension is not clear.

The paper is an attempt to resolving inconsistencies and incoherences simultaneously. Thus it needs to go further than the above default logics. In FDL, we also try to avoid global checks: the condition of justification establishing is weaker than in justified default logic and the original default logic, which makes "subsequent" defaults would never invalidate used defaults. Moreover, since the syntax is not modified and default extensions are reserved, FDL retains the simplicity and powerful expressivity of Reiter's default logic.

6 Conclusion

Our main contribution in the paper is, based on a paraconsistent and monotonic reasoning system, we generalize Reiter's default logic, i.e., each default theory has at least one extension in FDL and Reiter's default logic coincides with FDL(when most consistent preferred extension is used) as to a consistent and coherent default theory.

Although FDL has some nice properties, the computation of alternative extensions and preferred extensions is under discussion. Besides, the relationship between FDL and other default reasoning systems is not so clear, and more research will be devoted to it.

Not only do inconsistencies and incoherences occur in default logic, but also they exist in other reasoning systems, in which logic programming is a case in point. In the future work, we will apply the idea of FDL to other reasoning systems.

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