# **Semantic Querying of Mathematical Web Service Descriptions**

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**Abstract.** This paper describes a semantic extension to the Mathematical Services Query Language (MSQL). MSQL is a language for querying registry-published mathematical Web service descriptions expressed in the Mathematical Services Description Language (MSDL). The semantic extension allows queries in MSQL to be based on the underlying semantics of service descriptions; the MSQL engine processes these queries with the help of an automated reasoner.

## **1 Introduction**

Semantic-based discovery of Web services is one of the crucial issues that are currently receiving considerable attention in the field of the Semantic Web. In the case of mathematical Web services, this issue is more subtle due to the fact that they operate within semantically rich domains on objects that need proper encoding and specification.

A *mathematical Web service* is a Web service that offers the solution to a mathematical problem (based on e.g. a computer algebra system or on an automated theorem prover). In the MathBroker project [\[12\]](#page-14-0), we have developed a framework for mathematical services based on standards such as XML, SOAP, WSDL, and OpenMath. We have developed the XML-based Mathematical Services Description Language (MSDL) [\[8\]](#page-13-0) to adequately describe mathematical services and their constituent entities. The description of a mathematical service in MSDL may contain information related to the type of the problem, the algorithm(s) used to solve the problem, related problems, machines executing the problem, etc. A skeleton of a service description in MSDL is shown in Figure [1.](#page-1-0)

To facilitate the process of publishing and discovering mathematical services, we have developed an ebXML-based mathematical registry [\[3\]](#page-13-1) where MSDL descriptions of services are published such that clients can discover them by browsing or querying the registry (and consequently receive corresponding WSDL descriptions). Since the querying facilities of the registry do not support contentbased querying, we have developed the content-based Mathematical Services

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```
<monet:definitions>
  <mathb:machine_hardware name="perseus">
    ...
 </mathb:machine_hardware>
  <monet:problem name="integration">
    ...
  </monet:problem>
  <monet:algorithm name="RischAlg">
    ...
  </monet:algorithm>
  <monet:implementation name="RImpl">
    ...
  <monet:hardware href=".../perseus"/>
  <monet:algorithm href=".../RischAlg"/>
  </monet:implementation>
  <monet:service name="RRISC">
      ...
   <monet:problem href=".../integration"/>
   <monet:implementation href=".../RImpl"/>
  </monet:service>
</monet:definitions>
```
<span id="page-1-0"></span>**Fig. 1.** A Skeleton of a Service Description

Query Language  $(MSQL)$  [\[1,](#page-13-3)[4\]](#page-13-4) which is able to perform queries at the syntactical structure of a MSDL service description. However, mathematical objects respectively their MSDL descriptions are semantically rich and MSQL does not capture these semantic structures and their relations. This limits the effectiveness of service discovery since it is not based on the semantic information contained in MSDL descriptions. In this paper we present an extension to MSQL that addresses the semantic information contained in service descriptions. This extension adds a number of constructs to the language in order to express predicate logic formulas and adds a semantic evaluator to the MSQL engine to process these formulas with the help of an automated reasoner. The rest of this paper briefly describes the syntactic structure of MSQL (Section [2\)](#page-1-1), the semantic extension to MSQL (Section [3\)](#page-3-0), the MSQL engine architecture and implementation (Section [4\)](#page-7-0), and finally reviews related work (Section [5\)](#page-11-0).

#### <span id="page-1-1"></span>**2 The MSQL Syntactic Structure**

The Mathematical Services Query Language is a language designed and implemented to query registry-published services based on the contents of their MSDL descriptions. It provides the functionality to interface to a registry and retrieve service descriptions on which queries are performed. Its implementation is based on a formally defined semantics [\[1\]](#page-13-3).

A query in MSQL conforms to the following syntax:

```
SELECT EVERY|SOME <entity>
FROM <classificationConcept>
WHERE <expression>
ORDERBY <expression> ASCENDING|DESCENDING
```
The query has four main clauses:

- **–** The *SELECT* clause selects EVERY or SOME description of the type specified by *entity* from a given classification scheme in the registry. The *entity* types defined by MSDL are *problem*, *algorithm*, *implementation*, *realization* (*including a WSDL service description*), and *machine*.
- **–** The *FROM* clause determines the classification scheme from which the specified description is to be selected. Every service respectively its description in the registry is classified according to predefined classification schemes in the registry. The *FROM* clause limits the range of descriptions to be retrieved for querying to those classified under *ClassificationConcept*.
- **–** The *WHERE* clause applies its *expression* parts to each candidate document retrieved from the registry. The expression of the *WHERE* clause is a logical condition: if it is evaluated to *true*, the document is considered as (part of) the result of the query.
- **–** The *ORDERBY* clause sorts the resulting documents in *ASCENDING* or *DESCENDING* order based on the comparison criteria resulting from the evaluation of its *expression* on each document.

MSQL is designed such that it has a minimal set of expressions that are sufficient to construct logical statements on the contents of the target MSDL descriptions and that it is able to address the structure of such descriptions. MSQL expressions include: path expressions that can access every part of an MSDL document; expressions involving logical, arithmetic, and comparative operators; conditional expressions; quantified expressions; functions; and variable bindings. The following is a sample MSQL query that illustrates the usage of some of these expressions.

**Example 1.** *Find every service in "/GAMS/Symbolic Computation" such that, if it has an implementation, it runs on a machine called "perseus", otherwise its interface is on this machine.*

```
SELECT EVERY service
FROM /GAMS/Symbolic Computation
WHERE
 if not (/service[empty(//implementation)])
then
   let $d := doc(//implementation/@href) in
    $d/hardware[contains(@name, "perseus")]
else
 //service-interface-description[contains(@href, "perseus")]
ORDERBY /service/@name descending
```
This query asks for every service description classified under "/GAMS/Symbolic Computation" that satisfies the WHERE expression. The resulting documents are to be sorted in descending order according to their names. The conditional expression (if .. then .. else) is used to decide if the current service document node has an implementation. If this is the case, it takes from the service document the URI of such implementation document, retrieves it from the registry (let \$d := doc(//implementation/@href)), and checks if this implementation is related to the machine perseus. If this is not the case, it checks in the else branch, if the service has its interface on the said machine. The let clause is used to assign a document to the variable d which is then used as part of the path expression. The doc function returns the root node of the document whose name appears as its argument. Its argument is a URI that is used as the address of the required document in the registry. The contains function returns *true* if its first argument value contains as part of it its second argument value.

Although MSQL provides the functionality to express and perform queries on the syntactic structure of MSDL descriptions, it does not provide the functionality to express and perform queries on their semantic content. In the next section, we present an extension to MSQL that addresses this limitation.

#### <span id="page-3-0"></span>**3 A Semantic Extension to MSQL**

The Mathematical Services Description Language (MSDL) is capable of representing not only syntactic structures, but also semantic information. This information is expressed in OpenMath [\[6\]](#page-13-5), an XML-based standard format for representing mathematical objects in a semantics-preserving way. To illustrate this approach, we first present a sample description to show the underlying semantics of MSDL and then show how a query that operates on this semantics can be constructed .

Consider a description of the mathematical problem of indefinite integration (Figure [2\)](#page-4-0). It consists of the following pieces of semantic information:

- **–** Input: *<sup>f</sup>* : <sup>R</sup> <sup>→</sup> <sup>R</sup> (lines 3 to 13) which expresses the type <sup>R</sup> <sup>→</sup> <sup>R</sup> of the input and gives it the local name *f*.
- **–** Output: *<sup>i</sup>* : <sup>R</sup> <sup>→</sup> <sup>R</sup> which expresses the type <sup>R</sup> <sup>→</sup> <sup>R</sup> of the output and gives it the local name *i*.
- **–** Post-condition: *i* = *indefint*(*f*) (lines 17 to 28) which states that the output *i* equals the indefinite integral of the input *f*.

The semantic information expressed in this problem description can be used as a basis for discovering suitable services published in the mathematical registry. Suppose a client wants to solve a problem with the following specification:

- $-$  Input:  $a : \mathbb{R} \to \mathbb{R}$
- $-$  Output:  $\mathbf{b} : \mathbb{R} \to \mathbb{R}$
- **–** Post-condition: *diff* (*b*) = *a* (which states that the differentiated output equals the input).

```
1 <problem name="indefinite-integration">
2 \left<body\right>3 <input name="f">
4 <signature>
5 <OMOBJ>
6 <OMA>
7 <OMS cd="sts" name="mapsto"/>
8 <OMS cd="setname1" name="R"/>
9 <OMS cd="setname1" name="R"/>
10 </OMA>
11 </OMOBJ>
12 </signature>
13 </input>
14 <output name="i">
15 ...
16 </output>
17 <post-condition>
18 <OMOBJ>
19 <OMA>
20 <OMS cd="relation1" name="eq"/>
21 <OMV name="i"/>
22 <OMA>
23 <OMS cd="calculus1" name="indefint"/>
24 <OMV name="f"/>
25 </OMA>
26 </OMA>
27 </OMOBJ>
28 </post-condition>
29 </body>
30 </problem>
```
<span id="page-4-0"></span>**Fig. 2.** An MSDL Problem Description

The client would thus like to find some service which solves a problem *p* such that

$$
type(input_p) = \mathbb{R} \to \mathbb{R} \quad \wedge \tag{1}
$$

$$
type(output_p) = \mathbb{R} \to \mathbb{R} \quad \wedge \tag{2}
$$

<span id="page-4-1"></span>
$$
\forall a \in \mathbb{R} \to \mathbb{R}, b \in \mathbb{R} \to \mathbb{R} \ (post_p(a, b) \Rightarrow \text{diff}(b) = a)
$$
 (3)

where formulas [\(1\)](#page-4-1) and [\(2\)](#page-4-1) state that the types of the input and output shall be  $\mathbb{R} \to \mathbb{R}$  and the universally quantified subformula [\(3\)](#page-4-1) states that the postcondition  $post_p$  of the problem  $p$  implies that the differentiation of the output *b* equals the input *a*. The truth of this statement depends on knowledge available about the operation *diff*, e.g. a knowledge base may contain the formula  $diff(indefint(a)) = a$  which semantically relates the operators  $diff$  and  $indefint$ .

To express such a formula in MSQL, we extended the grammar of MSQL as shown in Figure [3](#page-5-0) by adding two clauses:

```
<msqlQuery> ::= 'SELECT' ( 'EVERY' | 'SOME' ) <entity>
                       ( 'FROM' <classification> )?
                       ( 'WHERE' <msqlExpr> )?
                       ( 'ORDERBY' <msqlExpr )?;
                     ...
        <msqlExpr> ::= ... | <typematch> | <semanticExpr>;
       <typematch> ::= 'typematch' (omObjExpr, omObjExpr);
   <semanticExpr> ::= 'satisfy' ( <omObjExpr> );
       <omObjExpr> ::= <omApplication> | <omAttribution> | <omBinding>
                          | <omInt> | <omVar> | <omString> | <omSymbol>
                          | <var>;
  <omApplication> ::= 'oma' '(' <omObjExpr> (, <omObjExpr> )* (
                          <varReplacement> )? ')';
  <omAttribution> ::= 'omattr' '(' <omObjExpr>, ( <omObjExpr>
                          <omObjExpr> )(, ( <omObjExpr> <omObjExpr> ))*
                          ( <varReplacement> )? ')' ;
       <omBinding> ::= 'ombind' '(' <omObjExpr> '[' <omBoundVariable>
                            (, omBoundVariable )* ']' <omObjExpr>
                            ( <varReplacement> )? ')';
<omBoundVariable> ::= 'omvar' ':' ( <var> | <omVar> ) '@' '('<omObjExpr>,
                          <omObjExpr> ( <varReplacement> )? ')';
 \langle \text{varReplacent} \rangle ::= '[' \langle \text{omObjExpr} \rangle'/' \langle \text{var} \rangle (, \langle \text{omObjExpr} \rangle'/'
                          \langle var \rangle )* ']';
            <omInt> ::= 'omi' ':' <number>;
            \langle \text{omVar} \rangle ::= 'omv' ':' ( \langle letter> | \langle \text{var} \rangle );
        <omString> ::= 'omstr' ':' <letter> ;
        <omSymbol> ::= 'oms' ':' <letter> ':' <letter>;
              \langle var \rangle ::= '\$\rangle' \langle letter \rangle;
                     ...
```
**Fig. 3.** The MSQL Semantic Extension Grammar

- <span id="page-5-0"></span>**–** The clause 'typematch(a,b)' states that type *a* matches (i.e. equals or is a special version of) type *b*.
- **–** The clause 'satisfy e' states that the semantic interpretation of the predicate logic formula *e* (encoded as an OpenMath expression) yields *true*.

The <semanticExpr> rule and its subrules define the grammar of predicate logic formulas based on the classification of OpenMath objects into *basic* objects and *compound* objects [\[6\]](#page-13-5). *Basic* objects include *Integers*, *Strings*, *Variables*, and *Symbols*. *Compound* objects include *Application*, *Attribution*, and *Binding*. The syntax is defined such that expressions are written in a prefix notation which is internally transformed to OpenMath syntax. For instance the <omBinding> subrule (see also Example 2) expresses an OpenMath *Binding* object which is constructed from an OpenMath object (the binder), and from zero or more variables (the bound variables) followed by another OpenMath object (the body). The MSQL expression

```
oma(oms:relation1:eq, oma(oms:calculus1:diff, omv:b), omv:a)
```
is thus transformed to the OpenMath XML object

```
<OMA<OMS name="eq" cd="relation1"/>
  <OMA<OMS name="diff" cd="calculus1"/>
    <OMV name="b"/>
  </OMA>
 <OMV name="a"/>
</OMA>
```
**Example 2.** Our request to *find some service with problem* p *such that the type checks [\(1\)](#page-4-1) and [\(2\)](#page-4-1) and the subformula [\(3\)](#page-4-1) are satisfied* can be expressed by the following MSQL query:

```
SELECT SOME service
FROM /GAMS/Symbolic Computation
WHERE let $p:= doc(//problem/@href) in
          a:=$p//input/@name,
          $b:= $p//output/@name,
         $ta:= $p//input/signature/OMOBJ,
         $tb:= $p//output/signature/OMOBJ,
       $post:= $p//post-condition/OMOBJ in
 (typematch(oma(oms:sts:mapsto(oms:setname1:R,
                       oms:setname1:R)), $ta)) and
 (typematch($tb, oma(oms:sts:mapsto(oms:setname1:R,
                            oms:setname1:R)))) and
 (satisfy(ombind(oms:quant1:forall
  [omvar:$a@(oms:sts:type, $ta),
   omvar:$b@(oms:sts:type, $tb)]
  oma(oms:logic1:implies, $post,
      oma(oms:relation1:eq,
          oma(oms:calculus1:diff, omv:$b), omv:$a)))))
```
Variable \$p represents the problem description of the service retrieved from the registry by the *doc* function according to the problem *href* provided as part of the service description. Variables \$a and \$b represent the names of the input and the output of the problem. Variables \$ta and \$tb represent the types of the input and the output of the problem. Variable \$post represents the postcondition of the problem.

The two typematch expressions correspond to formulas [\(1\)](#page-4-1) and [\(2\)](#page-4-1). They check if type  $\mathbb{R} \to \mathbb{R}$  matches the type \$ta of the input and if the type \$tb of the output matches type  $\mathbb{R} \to \mathbb{R}$ .

The satisfy expression corresponds to the universally quantified subformula [\(3\)](#page-4-1).

In the next section, we explain how the query is handled by the MSQL engine.

## <span id="page-7-0"></span>**4 The MSQL Architecture and Implementation**

MSQL including its semantic extension has been implemented as MSQL engine [\[1,](#page-13-3)[4\]](#page-13-4) and has been incorporated into the MathBroker framework [\[12\]](#page-14-0) for service publication and discovery.

## **4.1 Architecture**

Figure [4](#page-7-1) illustrates the architecture of the MSQL engine which consists of the following components:



<span id="page-7-1"></span>**Fig. 4.** The MSQL Engine Architecture

- **–** The **MSQL Engine** which has the MSQL query functionality. It consists of the following components:
	- The **Query Processor** which receives the query from the client, decomposes it into processable parts, and hands each part to the corresponding component.
	- The **Parser** receives the query from the processor and parses it according to the MSQL syntax. If the query does not comply with the syntax, an error message is returned to the processor which forwards the message to the client.
	- The **Registry Handler** receives from the processor the *entity* and *classificationConcept* parts of the query. It composes a registry query to retrieve *EVERY* /*SOME* description document of the given *entity* type classified under the given *classificationConcept*.
- The **Syntactic Expression Evaluator** receives from the Query Processor the syntactic expression part of the query and evaluates it against each description document retrieved from the registry. It returns to Query Processor those documents for which the expression evaluates to *true*.
- The **Semantic Expression Evaluator** receives from the Query Processor the semantic expression part of the query and evaluates it against each description document retrieved from the registry. It returns to the Query Processor those documents for which the expression evaluates to *true*. Unlike the Syntactic Expression Evaluator, the Semantic Expression Evaluator does not perform the whole evaluation by itself. It rather takes the semantic expression, converts it into OpenMath format (see Figure [5\)](#page-10-0), retrieves from the Knowledge Base the axiom(s) and type declaration(s) needed to reason about the semantic expression and sends all of them to the Reasoner Interface. As required by the Reasoner Interface, the axioms are represented in OpenMath format and the declarations are represented in OMDoc [\[16\]](#page-14-1) format.
- The **Result Quantifier and Sorter** receives from the Query Processor SOME/EVERY document filtered by the two evaluators, orders (if needed) the documents according to the ORDERBY expression, and returns them as the query result to the Client.
- **–** The **Registry** which stores a collection of published MSDL documents of different *entity* types and classifies them according to some registry-predefined classification schemes. Query requests to the registry are handled by the Query Manager of the registry.
- **–** The **Reasoner Interface** receives from the Semantic Expression Evaluator the semantic expression part of the query in OpenMath, the  $axiom(s)$  in OpenMath, and the declaration(s) in OMDoc and converts each one to the format required by the Automated Reasoner and hands them to the reasoner. It gets the answer from the reasoner and sends it to the Semantic Expression Evaluator. The Reasoner Interface used is a component of the RISC ProofNavigator [\[19\]](#page-14-2).
- **–** The **Automated Reasoner** reasons about semantic expressions based on the  $axiom(s)$  and declaration(s) given and returns the answer to the Reasoner Interface. The Automated Reasoner currently used is the Cooperating Validity Checker Lite (CVCL) [\[5\]](#page-13-6).
- **–** The **Knowledge Base** holds declarations of OpenMath symbols that may be used in semantic queries together with axioms that describe the semantics of that symbols.

#### **4.2 Performing the Semantic Query**

Based on this architecture, we summarize the actions taken to perform the query in Example 2:

**–** The MSQL Engine receives the query from the Client and hands it to the Query Processor

- **–** The Query Processor asks the Parser to parse the query according to the MSQL syntax. If the query does not comply with the MSQL syntax, an error message is returned to the user.
- **–** The Query Processor decomposes the query to processable parts. It hands the registry-related part (the *entity service* and the *classificationConcept "/GAMS/Symbolic Computation"*) to the registry handler.
- **–** The Registry Handler forms a registry query based on the *entity* and the *classificationConcept*, connects to the Registry and hands the registry query to the Query Manager of the Registry which performs the query and returns a set of candidate *service* documents to the Registry Handler.
- **–** The Query Processor asks the Syntactic Expression Evaluator to evaluate the syntactic expression part on the current *service* document. The Syntactic Expression part consists of a *let* expression which has six assignment subexpressions. The evaluations of these subexpressions assign values to variables \$a, \$b, \$ta, \$tb, and \$post representing input, output, input type, output type, and post-condition respectively. These variables are used in the semantic expression of the query.
- **–** The Query Processor asks the Semantic Expression Evaluator to evaluate the semantics expression against the (same) current *service* document. The Semantic Evaluator performs the following steps:
	- It performs the type checking required by the two typematch expressions. If the result of the check is *true* it proceeds to the next step. Otherwise it returns *false* and the Query Processor proceeds to perform the query on the next candidate document.
	- It converts the satisfy expression to OpenMath format. The Open-Math representation of the satisfy expression is shown in Figure [5.](#page-10-0) The conversion also takes care of variable substitution (e.g., variable \$a is substituted by the input name *f* ).
	- It retrieves from the Knowledge Base the declarations of the symbols *diff* and *indefint* represented in OMDoc. The two symbols occur in the OpenMath representation of the satisfy formula after variable substitution. The declaration of the *diff* symbol is shown in Figure [6.](#page-11-1) The *indefint* symbol has a similar declaration.
	- It retrieves from the Knowledge Base the axiom  $diff(indefint(a)) = a$ . This axiom is represented by the following quantified formula in Open-Math format (similar to the OpenMath format of the *satisfy* expression)

$$
\forall f \in \mathbb{R} \to \mathbb{R} \ (indefint(df(f)) = f)
$$

• It hands the satisfy expression (in OpenMath), the declarations (in OM-Doc), and the axiom (in OpenMath) to the Reasoner Interface which converts each of them to the syntax required by the reasoner. The reasoner decides about the truth value of the expression based on the given axiom and declarations and returns the answer to the RISC ProofNavigator which in turn returns the answer to the Semantic Expression Evaluator.

```
1 <OMOBJ>
 2 <OMBIND>
3 <OMS name="forall" cd="quant1"/>
 4 < OMBVAR>
     5 <OMATTR>
 6 <OMATP>
 7 <OMS name="type" cd="sts"/>
          < OMA>9 < COMS name="mapsto" cd="sts"/><br>10 < COMS name="R" cd="setname1"/>
           10 <OMS name="R" cd="setname1"/>
11 <OMS name="R" cd="setname1"/>
12 </OMA>
13 </OMATP>
14 <0MV name="f"/><br>15 </0MATTR>
       15 </OMATTR>
16 <OMATTR>
17
18 <OMV name="i"/>
19 </OMATTR>
20 </OMBVAR><br>21 <0MA>
    < OMA>22 <OMS name="implies" cd="logic1"/>
      < OMA>24 <CMS name="eq" cd="relation1"/><br>25 <CMW name="i"/>
25 \langle OMV name="i"/><br>26 \langle OMA>
      < \capMA>27 < OMV name="indefint" cd="calculus1"/><br>28 < OMV name="f"/>
        <OMV name="f"/>
29 </OMA>
30 </0MA><br>31 <0MA
     \langlenma>
32 <OMS name="eq" cd="relation1"/>
33 < OMA > <br>34 < OMV
        34 <OMV name="diff" cd="calculus1"/>
35 <OMV name="i"/>
       </MA>37 <OMV name="f"/>
38 </0MA><br>39 </0MA>
    </OMA40 </OMBIND>
41 </OMOBJ>
                                                  Lines 5 to 15 represent the conversion of
                                                  the binder expression
                                                    omvar:$a@(oms:sts:type, $ta)
                                                  with the variables $a and $b substituted
                                                 by their values. It represents the declaration
                                                   f : \mathbb{R} \to \mathbb{R}Lines 21 to 39 represent the conversion of
                                                  the satisfy subexpression
                                                    oma(oms:logic1:implies, $post,
                                                          oma(oms:relation1:eq,
                                                                 oma(oms:calculus1:diff,
                                                                    omv:$b), omv:$a))
                                                  with the variables $post, $a, and $b ap-
                                                  propriately substituted by their values. It
                                                  represents the implication
                                                   i = indefint(f) \Rightarrow diff(i) = f.
```
**Fig. 5.** OpenMath Representation of the satisfy Expression in Example 2

<span id="page-10-0"></span>**–** If the evaluation of the semantic expression yields *true*, the Query Processor returns the current *service* document to the Result Quantifier and Sorter which returns it to the Client as the ultimate result (because of the SOME clause) of the query. If the evaluation is *false* the Query Processor proceeds to process the query on the next candidate *service* document. If the evaluation is *false* for all candidate documents, then no document is returned as a result of the query.

#### **4.3 A Prototype Implementation**

A prototype of the architecture has been implemented in Java making use of the registry [\[3\]](#page-13-1) for publishing service descriptions, a component of the RISC ProofNavigator [\[19\]](#page-14-2) as the Reasoner Interface, and the Cooperating Validity Checker Lite (CVCL) [\[5\]](#page-13-6) as the Automated Reasoner.

The implementation of the MSQL engine is based on a formal definition [\[1\]](#page-13-3) using denotational semantics [\[18\]](#page-14-3). The implementation consists of a set of

```
<omdoc:omgroup>
<omdoc:symbol kind="object" name="calculus1_diff">
 <omdoc:type system="simply_typed"
  xml:id="calculus1_diff_type">
  <om: OMA><om:OMS cd="sts" name="mapsto"/>
     <om: OMA><om:OMS cd="sts" name="mapsto"/>
       <om:OMS cd="setname1" name="R"/>
       <om:OMS cd="setname1" name="R"/>
     </om:OMA>
     <om:OMA>
       <om:OMS cd="sts" name="mapsto"/>
       <om:OMS cd="setname1" name="R"/>
       <om:OMS cd="setname1" name="R"/>
     </om:OMA>
    </om:OMA</omdoc:type>
</omdoc:symbol>
</omdoc:omgroup>
```
**Fig. 6.** The declaration of variable diff in OMDoc Format

<span id="page-11-1"></span>evaluation classes each of which corresponds to one component of the MSQL engine with a set of methods each of which corresponds to one equation in the denotational semantics. The signature of a method corresponds to the signature of the semantic function. For example, the equation

$$
\mathbf{E}[V] \ d \ n \ r = lookup(d, [V])
$$

with the semantic function

```
\mathbf{E}: Expression \times Declaration \times Node \times Register \times Value
```
is implemented by the Java method with the signature

evaluateVariableExpr(ChildAST expression, Declaration declaration, Node node, Registry registry)

The prototype implementation of the MSQL engine including its API can be found in [\[2\]](#page-13-7).

#### <span id="page-11-0"></span>**5 Related Work**

The semantic-based discovery of Web services has recently received growing interest. The METEOR-S Web Service Discovery Infrastructure (MWSDI) [\[20\]](#page-14-4) aims to provide efficient publication and discovery mechanisms in a federation of registries. It uses an ontology-based approach to organize registries, enabling semantic classification of services based on domains. Registries support semantic publication of services which is used during the discovery process.

The Ontology Web Language for Services (OWL-S) [\[17\]](#page-14-5) allows Semantic Web tools to process Web services in order to enable software agents to automatically discover, invoke, compose, and monitor Web services. In [\[11\]](#page-14-6), OWL-S is used in conjunction with WSDL to add semantic descriptions to a Web service. When a registry is used for the publication and discovery of a service, OWL-S is used in this respect to add capability matching to the registry.

The Web Service Modeling Ontology (WSMO) [\[21\]](#page-14-7) is a formal language for semantically describing Web services to facilitate the automation of discovering, composing, and invoking such services. WSMO uses a formal logic for describing its own elements such as ontologies, descriptions, goals, and mediators.

Few approaches have focused on the semantic description and discovery of mathematical Web services. In the MONET project [\[14\]](#page-14-8), ontologies [\[7\]](#page-13-8) are used to model service descriptions as well as queries on these descriptions. These ontologies are ontological conversions of MSDL descriptions written in OWL [\[13\]](#page-14-9) and are used by a component within the MONET architecture called *Instance Store* [\[10\]](#page-14-10) which uses the Description Logic reasoner RACER [\[9\]](#page-14-11) for matching queries to appropriate services. The reasoning process in the case of MONET is based on a restricted form of first order logic which is more tractable for automated reasoning but strictly less expressive. In our semantic queries, we use full predicate logic which is a highly expressive language.

A matching-based discovery approach [\[15\]](#page-14-12) to registry-published mathematical services performs matchmaking between representations of tasks (client requests) and capabilities (service descriptions). The approach applies a normalization process on a task. It then compares the normalized task with a registered capability calculating a similarity value that is used in the matchmaking process. Task normalization amounts to carrying out a sequence of transformations on the task description rewriting all logical parts in disjunctive normal form, flattening arguments of n-associative operations, and consistent variable renaming.

The similarity value is calculated based on the matching of the capability precondition (or the task postcondition) and the capability postcondition (or the task precondition). Matchmaking is performed by: registering capabilities in the database, taking a description of a task normalizes it, and returns an ordered list of the capabilities from the registry database based on their calculated similarity.

The matching process used in the discovery is ultimately based on the syntactic similarity traced between tasks and capabilities. In our case, the decision is based on logical implications between statements extracted from descriptions, which is strictly more general.

#### **6 Conclusion and Future Work**

The semantic extension of MSQL supports semantic-based discovery of registry published mathematical services. Semantic queries formed in predicate logic capture the underlying semantical structures of mathematical service specifications. The MSQL engine performs semantic-based queries with the help of an automated reasoner which takes predicate logic formulas, decides their validity, and returns the answer to the engine.

The syntax of MSQL and the presented query examples reveal an apparent difficulty in forming queries on target documents. Such a difficulty is alleviated partly by the fact that MSQL has a relatively small number of constructs for forming queries and by the fact that its queries operate on documents that possess common structures imposed by a schema. Thus, a user-friendly tool for forming queries on target MSDL documents should be developed.

A future extension to the presented framework may involve service compositions: when a client submits a service request, a broker agent determines suitable service compositions satisfying the client request and returns the description of a composition rather than that of a single service. To find the suitable candidate services, the agent might form MSQL queries based on information contained in the client request, send them to the MSQL engine, and make composition decisions based on the results returned by the MSQL engine.

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