A Study on Optimal Configuration for the Mobile Manipulator: Using Weight Value and Mobility

Jin-Gu Kang[1] and Kwan-Houng Lee[2]

[1] Dept. Visual Broadcastion Media Keukdong College DanPyung-Ri 154-1,Gamgog Myun, Eumsung Gun Chungbuk, 467-900, Republic of Korea,
jgukang@kdc.ac.kr

[2] School of Electronics & Information Engineering, Cheongju University, Naedok-Dong Sangdang-Gu Cheongju-City, Chungbuk, 360-764, Republic of Korea,
khlee368@cju.ac.kr

Abstract. A Mobile Manipulator is redundant by itself. Using it's redundant freedom, a mobile manipulator can perform various task. In this paper, to improve task execution efficiency utilizing the redundancy, optimal configurations of the mobile manipulator are maintained while it is moving to a new task point. And using a cost function for optimality defined as a combination of the square errors of the desired and actual configurations of the mobile robot and of the task robot, the job which the mobile manipulator performs is optimized. Here, The proposed algorithm is experimentally verified and discussed with a mobile manipulator, PURL-II.

Keywords: Weight Value, Mobility, Gradient Method, Mobile robot.

1 Introduction

While a mobile robot can expand the size of the work space but does no work, a vertical multi-joints robot or manipulator can't move but it can do work. And at present, there has been a lot of research on the redundant robot which has more degrees of freedom than non-combination robots in the given work space, so it can have optimal position and optimized job performance[1][2]. While there has been a lot of work done on the control for both mobile robot navigation and the fixed manipulator motion, there are few reports on the cooperative control for a robot with movement and manipulation ability[3]. Different from the fixed redundant robot, the mobile manipulator has the characteristic that with respect to the given working environments, it has the merits of abnormal movement avoidance, collision avoidance, efficient application of the corresponding mechanical parts and improvement of adjustment. Because of these characteristics, it is desirable that one uses the mobile manipulator with the transportation ability and dexterous handling in difficult working environments[4]. This paper explains the mobile manipulator PURL-II which is a combination in series of a mobile robot that has 3 degrees of freedom for efficient job accomplishment and a task robot that has 5 degrees of

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freedom. We have analyzed the kinematics and inverse kinematics of each robot to define the 'Mobility' of the mobile robot as the most important feature of the mobile manipulator. We researched the optimal position and movement of robot so that the combined robot can perform the task with minimal joint displacement and adjust the weighting value using the this 'Mobility'. When the mobile robot performed the job with the cooperation of the task robot, we investigated the optimizing criteria of the task using the 'Gradient Method' to minimize the movement of the whole robots. The results that we acquired by implementing the proposed algorithm through computer simulation and the experiment using PURL-II are demonstrated.

2 Mobile Manipulator

2.1 Kinematics Analysis of the Mobile Manipulator

Each robot which is designed to accomplish each independent objective concurrently should perform its corresponding movement to complete the task. The trajectory is needed for kinematics analysis of the whole system, so that we can make the combination robot perform the task efficiently using the redundant degree of freedom generated by the combination of the two robots[5][6]. From Fig. 1, We can see the Cartesian coordinate of the implemented mobile/task robot system and the link coordinate system of each joint in space. This system is an independent mobile manipulator without wire. The vector $\dot{q}$ of the whole system joint variables can be defined $\dot{q}_t = [\dot{q}_{t1} \quad \dot{q}_{t2} \quad \dot{q}_{t3} \quad \dot{q}_{t4} \quad \dot{q}_{t5}]$ and $\dot{q}_m = [\dot{q}_{m6} \quad \dot{q}_{m7} \quad \dot{q}_{m8} \quad \dot{q}_{m9}]$ that represents the joint variable vector of the task robot. That is shown as

$$\dot{q} = \begin{bmatrix} \dot{q}_t \\ \dot{q}_m \end{bmatrix} = [\dot{q}_{t1} \quad \dot{q}_{t2} \quad \dot{q}_{t3} \quad \dot{q}_{t4} \quad \dot{q}_{t5} \quad \dot{q}_{m6} \quad \dot{q}_{m7} \quad \dot{q}_{m8} \quad \dot{q}_{m9}]^T \tag{1}$$

The linear velocity, and angular velocity of mobile robot in Cartesian space with respect to the fixed frame of the world frame can be expressed as (2).

$${}^0\dot{P}_m = \begin{bmatrix} {}^0V_m \\ {}^0\omega_m \end{bmatrix} = \begin{bmatrix} {}^0J_{m,v} \\ {}^0J_{m,\omega} \end{bmatrix} \dot{q}_m = {}^0J_m \dot{q}_m \tag{2}$$

In view of Fig. 1, let us represent the Jacobian of vector $\dot{q}_t$ (task robot joint variable) with respect to frame (1). These results are shown in (3) as follows[12] .

$${}^m\dot{P}_t = \begin{bmatrix} {}^mV_t \\ {}^m\omega_t \end{bmatrix} = \begin{bmatrix} {}^mJ_{t,v} \\ {}^mJ_{t,\omega} \end{bmatrix} \dot{q}_t = {}^mJ_t \dot{q}_t \tag{3}$$

Given the Jacobians, ${}^0J_m$ and ${}^mJ_t$, for each robot, if we express the Jacobian of the mobile manipulator as ${}^0J_t$; the linear velocity. Then angular velocity ${}^0\dot{P}_t = [{}^0V_t \quad {}^0\omega_t]^T$ from the end-effector to the world frame is represented as (4).

$$
\begin{aligned}
{}^{0}\dot{P}_{t} &= \begin{bmatrix} {}^{0}V_{t} \\ {}^{0}\omega_{t} \end{bmatrix} = \begin{bmatrix} {}^{0}V_{m} \\ {}^{0}\omega_{m} \end{bmatrix} + \begin{bmatrix} {}^{0}\omega_{m} + {}^{0}R_{1}\,{}^{1}V_{t} \\ {}^{0}R_{1}\,{}^{1}\omega_{t} \end{bmatrix} \\
&= {}^{0}J_{m}\dot{q}_{m} + {}^{0}J_{t}\dot{q}_{t} = \begin{bmatrix} {}^{0}J_{m} & {}^{0}J_{t} \end{bmatrix} \begin{bmatrix} \dot{q}_{m} \\ \dot{q}_{t} \end{bmatrix}
\end{aligned} \tag{4}
$$

Here ${}^{0}R_{1}$ is rotational transformation from world frame to the base frame of the task robot. Namely, in view of (2)-(4), the movements of mobile robot and task robot are involved with the movement of end-effector.

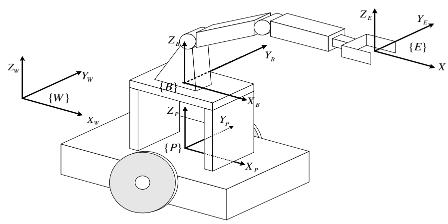


Fig. 1. Coordinate system of the mobile manipulator

3 Algorithm for System Application

3.1 Task Planning for Minimal Movement

Because the position of base frame of task robot varies according to the movement of mobile robot, through inverse kinematics, the task planning has many solutions with respect to the robot movement. and we must find the accurate solution to satisfy both the optimal accomplishment of the task and the efficient completion of the task.

In this paper, we have the objective of minimization of movement of the whole robot in performing the task, so we express the vector for mobile manipulator states as (5).

$$
q = \begin{bmatrix} q_{m} \\ q_{t} \end{bmatrix} \tag{5}
$$

where $q_m = [x_m \quad y_m \quad z_m \quad \theta_m]^T$ and $q_t = [\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4 \quad \theta_5]^T$ .Here, q is the vector for the mobile manipulator and consists of q_m representing the position and direction of mobile robot in Cartesian space and q_t, the joint variable to each n link of the task robot. Now to plan the task to minimize the whole movement of mobile manipulators, a cost function, L, is defined as

$$
\begin{aligned}
L &= \Delta q^{T} \Delta q = (q_{f} - q_{i})^{T}(q_{f} - q_{i}) \\
&= (q_{m,f} - q_{m,i})^{T}(q_{m,f} - q_{m,i}) + (q_{t,f} - q_{t,i})^{T}(q_{t,f} - q_{t,i})
\end{aligned} \tag{6}
$$

Here, $q_i = [q_{m,i} \quad q_{t,i}]^T$ represents the initial states of the mobile manipulator, and $q_f = [q_{m,f} \quad q_{t,f}]^T$ represents the final states after having accomplished the task. In the final states, the end-effector of the task robot must be placed at the desired position $X_{t,d}$. For that, equation (7) must be satisfied. In (7), we denote as $R(\theta_{m,f})$ and $f(q_{t,f})$, respectively, the rotational transformation to $X-Y$ plane and kinematics equation of task robot[7].

$$X_{t,d} = R(\theta_{m,f}) f(q_{t,f}) + X_{m,f} \tag{7}$$

where $X_{t,d}$ represents the desired position of task robot, and $X_{m,f}$ is the final position of mobile robot. We can express the final position of the mobile robot $X_{m,f}$ as the function of the desired coordinate $X_{t,d}$, joint variables $\theta_{m,f}$ and $q_{t,f}$, then the cost function that represents the robot movement is expressed as the $n \times 1$ space function of $\theta_{m,f}$ and $q_{t,f}$ as (8).

$$\begin{aligned} L = &\{X_{t,d} - R(\theta_{m,f}) f(q_{t,f}) - X_{m,i}\}^T \{X_{t,d} - R(\theta_{m,f}) f(q_{t,f}) - X_{m,i}\} \\ &+ \{q_{t,f} - q_{t,i}\}^T \{q_{t,f} - q_{t,i}\} \end{aligned} \tag{8}$$

In the equation (8), $\theta_{m,f}$ and $q_{t,f}$ which minimize the cost function L must satisfy the condition in (9).

$$\nabla L = \begin{bmatrix} \dfrac{\partial L}{\partial \theta_{m,f}} \\ \dfrac{\partial L}{\partial q_{t,f}} \end{bmatrix} = 0 \tag{9}$$

Because the cost function is nonlinear, it is difficult to find analytically the optimum solution that satisfies (9). So in this papers, we find the solution through the numeric analysis using the gradient method described by (10) .

$$\begin{bmatrix} \theta_{m,f(k+1)} \\ q_{t,f(k+1)} \end{bmatrix} = \begin{bmatrix} \theta_{m,f(k)} \\ q_{t,f(k)} \end{bmatrix} - \eta \, \nabla L \Big|_{\theta_{m,f(k)},\, q_{t,f(k)}} \tag{10}$$

This recursive process will stop, when $\|\nabla L\| < \varepsilon \approx 0$. That is, $\theta_{m,f(k)}$ and $q_{t,f(k)}$ are optimum solutions. Through the optimum solutions of $\theta_{m,f}$ and $q_{t,f}$ the final robot state q_f can be calculated as (11).

$$q_f = \begin{bmatrix} q_{m,f} \\ q_{t,f} \end{bmatrix} = \begin{bmatrix} X_{t,d} - R(\theta_{m,f}) f(q_{t,f}) \\ q_{t,f} \end{bmatrix} \tag{11}$$

There are several efficient searching algorithms. However, the simple gradient method is applied for this case.

3.2 Mobility of Mobile Robot

In this research, we define "mobility of the mobile robot" as the amount of movement of the mobile robot when the input magnitude of the wheel velocity is unity. That is, the mobility is defined as the corresponding quality of movement in any direction[8]. The mobile robot used in this research does move and rotate because each wheel is rotated independently under the control. The robot satisfies (12) with remarked kinematics by denoting left, right wheel velocities ($\dot{q}_{m,l}, \dot{q}_{m,r}$) and linear velocity and angular velocity (v_m, ω).

$$v_m = r\frac{\dot{q}_{m,l} + \dot{q}_{m,r}}{2} \tag{12a}$$

$$\omega = \frac{r}{l}\frac{\dot{q}_{m,l} - \dot{q}_{m,r}}{2} \tag{12b}$$

Rewriting (12a), (12b), we get (13a) and (13b).

$$\dot{q}_{m,r} = \frac{v_m + \omega\, l}{r} \tag{13a}$$

$$\dot{q}_{m,l} = \frac{v_m - \omega\, l}{r} \tag{13b}$$

Mobility is the output to input ratio with a unity vector, $\|v_m\| = 1$, or $\dot{q}^2_{m,l} + \dot{q}^2_{m,r} = 1$ and the mobility v_m in any angular velocity ω is calculated by (14).

$$v_m = r\sqrt{\frac{1}{2} - \omega^2\frac{l^2}{r^2}} \tag{14}$$

When the mobile robot has the velocity of unity norm, the mobility of mobile robot is represented as Fig. 2. It shows that the output, v and ω in workspace for all direction inputs that are variance of robot direction and movement. For any input, the direction of maximum movement is current robot direction when the velocities of two wheels are same[9]. At the situation, there does not occur any angular movement of the robot.

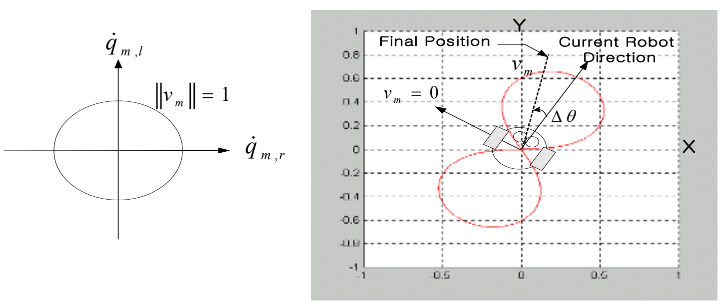


Fig. 2. Motion generation efficiency

3.3 Assigning of Weighting Value Using Mobility

From the mobility, we can know the mobility of robot in any direction, and the adaptability to a given task in the present posture of mobile robot. If the present posture of mobile robot is adaptable to the task, that is, the mobility is large to a certain direction. we impose the lower weighting value on the term in the cost function of (15) to assign large amount of movement to the mobile along the direction. If not, by imposing the higher weighting value on the term we can make the movement of mobile robot small. Equation (15) represents the cost function with weighting value

$$L = \{X_{t,d} - R(\theta_{m,f})f(q_{t,f}) - X_{m,i}\}^T W_m \{X_{t,d} - R(\theta_{m,f})f(q_{t,f}) - X_{m,i}\} + \{q_{t,f} - q_{t,i}\}^T W_t \{q_{t,f} - q_{t,i}\} \quad (15)$$

Here, W_m and W_t are weighting matrices imposed on the movement of the mobile robot and task robot, respectively. In the cost function, the mobility of mobile robot is expressed in the Cartesian coordinate space, so the weighting matrix W_m of the mobile robot must be applied. after decomposing each component to each axis in Cartesian coordinate system as shown in Fig. 3 and is represented as (16).

$$W_m = \begin{bmatrix} \omega_x & 0 & 0 & 0 \\ 0 & \omega_y & 0 & 0 \\ 0 & 0 & \omega_z & 0 \\ 0 & 0 & 0 & \omega_\theta \end{bmatrix} \quad (16)$$

Where, $\omega_x = \frac{1}{v \cdot \cos(\phi)\cos(\alpha) + e}$, $\omega_y = \frac{1}{v \cdot \sin(\phi)\sin(\alpha) + e}$, $\omega_z = \frac{k_1}{(z_d - f_z(q_t))^2}$, and $\omega_\theta = 1$.

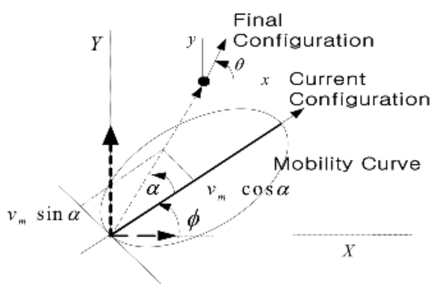


Fig. 3. Decomposing mobility

3.4 Mobile Robot Control

The mobile robot carries the task robot to the reachable boundary to the goal position, i.e., within the reachable workspace. We establish the coordinate system as shown in Fig. 4 so that the robot can take the desired posture and position movement from the initial position according to the assignment of the weighting value of the mobile robot to the desired position. After starting at the present position, (x_i, y_i), the robot reaches the desired position, (x_d, y_d). Here the current robot direction ϕ, the position error α from present position to the desired position, the distance error e to the desired position, the direction of mobile robot at the desired position θ are noted [9].

$$\dot{e} = -v\cos\alpha \tag{17a}$$

$$\dot{\alpha} = -\omega + \frac{v\sin\alpha}{e} \tag{17b}$$

$$\dot{\theta} = \frac{v\sin\alpha}{e} \tag{17c}$$

A Lyapunov candidate function is defined as in (18).

$$V = V_1 + V_2 = \tfrac{1}{2}\lambda e^2 + \tfrac{1}{2}(\alpha^2 + h\theta^2) \tag{18}$$

where V_1 means the error energy to the distance and V_2 means the error energy in the direction. After differentiating both sides in equation (18) in terms of time, we can acquire the result as in equation (19).

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = \lambda e\dot{e} + \left(\alpha\,\dot{\alpha} + h\theta\dot{\theta}\right) \tag{19}$$

Let us substitute equation (17) into the corresponding part in equation (19), it results in equation (20).

$$\dot{V} = -\lambda ev\cos\alpha + \alpha\left[-\omega + \frac{v\sin\alpha}{\alpha}\cdot\frac{(\alpha + h\theta)}{e}\right] \tag{20}$$

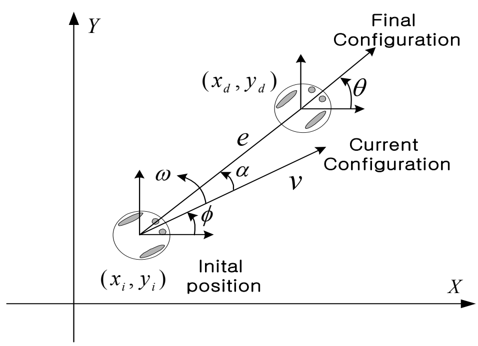


Fig. 4. Position movement of mobile robot by imposed weighting value

Note that $\dot{V} < 0$ is required for a given V to be a stable system. On this basis, we can design the nonlinear controller of the mobile robot as in (21).

$$v = \gamma\,(e\cos\alpha),\quad (\gamma > 0) \tag{21a}$$

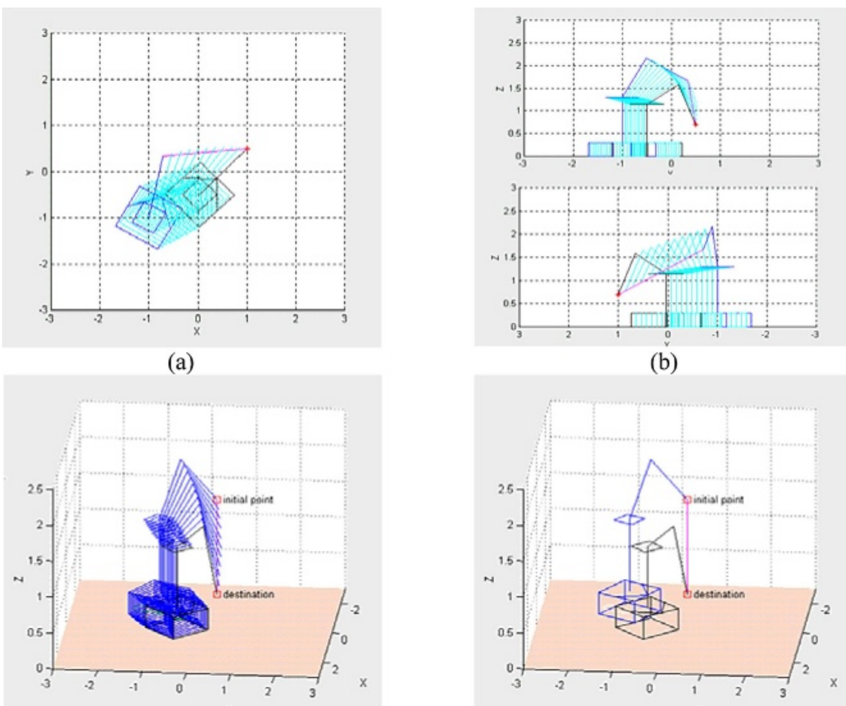
$$\omega = k\alpha + \gamma \frac{\cos\alpha \sin\alpha}{\alpha}(\alpha + h\theta), \; (k, \; h > 0) \tag{21b}$$

Therefore, using this controller for the mobile robot, $\dot{V}$ approaches to zero as $t \rightarrow \infty$; e and α also approach almost to zero as shown in (22).

$$\dot{V} = -\lambda(\gamma \cos^2\alpha)e^2 - k\alpha^2 \leq 0 \tag{22}$$

4 Simulation

For verifying the proposed algorithm, simulations were performed with PURL-II. Fig. 5 shows the simulation results with a 3 DOF task robot and a 3 DOF mobile robot. The goal is positioning the end-effect to (1, 0.5, 0.7), while initial configuration of mobile robot is (-1.0, -1.0, 1.3, 60°) and that of task robot is (18°, 60°, 90°). The optimally determined configuration of mobile robot is (0.0368, -0.497, 1.14, 44.1°) and that of task robot is (1.99°, 25.57°, 86.63°). Fig. 5 shows movements of the task robot in different view points.



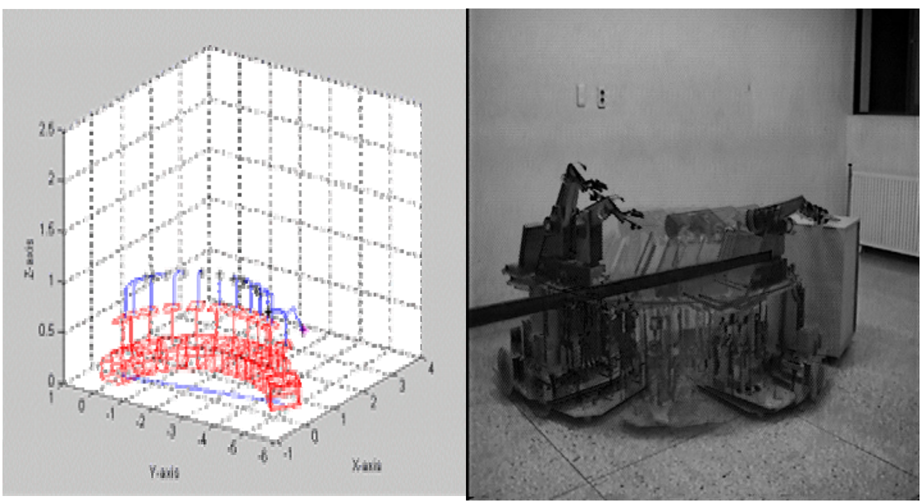
(a) (b)

(c) (d)

Fig. 5. The optimal position planning to move a point of action of a robot to (1, 0.5, 0.7)

5 Experiment

Before the real experiments, assumptions for moving manipulators operational condition are set as follows: 1. In the initial stage, the object is in the end-effect of the task robot. 2. The mobile robot satisfies the *pure rolling* and *non-slippage* conditions. 3. There is no obstacle in the mobile robot path. 4. There is no disturbance of the total system. And the task robot is configured as the joint angles of (18°, 60°, 90°), then the coordinate of the end-effect is set up for (0.02, 0.04, 1.3). From this location, the mobile manipulator must bring the object to (1, 1.5, 0.5). An optimal path which is calculated using the algorithm which is stated in the previous section has W_x = 10.0, W_y = 10.0, and W_z = 2.66. And at last the mobile robots angle is 76.52° from the X axis; the difference is coming from the moving of the right wheels 0.8m and the moving of the left wheels 1.4m. Next time, the mobile robot is different with the X axis by 11.64°with right wheels 0.4m moving and the left wheels 0.5m moving. Hence, the total moving distance of mobile robot is (1.2m, 1.9m), the total angle is 88.16°, and the each joint angle of task robot are (-6.45°, 9.87°, 34.92°). The experimental results are shown by the photography in Fig. 6. For the real experiment, the wheel *pure rolling* condition is not satisfied, also by the control the velocity through the robot kinematics, the distance error occurs from the cumulative velocity error. Using a timer in CPU for estimating velocity, timer error also causes velocity error. Hence consequently, the final position of end-effect is placed at (1.2, 1.5, 0.8) on object.



(a) Simulation (b) Experiment

Fig. 6. Response of robot posture

6 Conclusion

A new redundancy resolution scheme for a mobile manipulator is proposed in this paper. While the mobile robot is moving from one task (starting) point to the next task point, the task robot is controlled to have the posture proper to the next task, which can be pre-determined based upon TOMM[11]. Minimization of the cost function following the gradient method leads a mobile manipulator an optimal configuration at the new task point. These schemes can be also applied to the robot trajectory planning. The efficiency of this scheme is verified through the real experiments with PURL-II. The different of the result between simulation and experiment is caused by the error between the control input and the action of the mobile robot because of the roughness of the bottom, and is caused by the summation of position error through velocity control. In further study, it is necessary that a proper control algorithm should be developed to improve the control accuracy as well as efficiency in utilizing redundancy.

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