Dual Adaptive ANN Controllers Based on Wiener Models for Controlling Stable Nonlinear Systems

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Abstract. This paper presents two nonlinear adaptive predictive algorithms based on Artificial Neural Network (ANN) and a Wiener structure for controlling asymptotically stable nonlinear plants. The first algorithm is based on the minimization of a cost function taking into account the future tracking error and the Certainty Equivalence (CE) principle, under which the estimated parameters are used as if they were the true parameters. In order to improve the performance of the adaptive algorithm, we propose to use a cost function, considering not only the future tracking error, but also the effect of the control signal over the estimated parameters. A simulated chemical reactor example illustrates the performance and feasibility of both approaches.

1 Introduction

Adaptive control of discrete nonlinear systems, using flexible nonlinear parameterization like Artificial Neural Networks, have received a great deal of attention [1]. Most of these works have relied on the use of inverse model approach assuming that the system has a stable inverse and is affine in the control signal. These assumptions have limited their range of applications. The adaptive algorithms can be classified, in general, as *indirect* or *direct* ones. The former adapts the parameter of the controller with respect to some performance index, while the latter calculates the parameter of the controller based on an identified model of the plant. Adaptive controllers that are based on the CE principle completely ignore the uncertainty associated to the parameters. This may lead to inadequate transient and poor parameter convergence. Some authors have addressed these problems, in the context of inverse control, by modelling the parameters as random variables and taking into account their uncertainty in the control law [2] [3]. An algorithm that takes into account not only the control objective, but also the effect of the control signal on the convergence of the estimation algorithm is called adaptive *dual* control system [4].

In order to overcome the limitation of inverse control approaches, nonlinear predictive strategies have been proposed. As any control design tool, it requires

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dynamical models of the nonlinear systems to be controlled; if these models are not available, then some empirical ones, such as: Neural Networks, NARMAX, Volterra [5], and Wiener [6], can be considered. The latter are particularly useful in representing open loop stable nonlinear processes; without introducing the stability problems associated to general recursive nonlinear models.

Adaptive Predictive controllers based on Wiener type of models and the Certainty Equivalence principle, have been proposed by several authors [6][7]. In addition, Wiener structure can be combined with ANN to provide a powerful modelling framework [8].

In general, if the parameters are modelled as random variables, then the problem posed by the predictive controllers cannot even be solved numerically, because it requires the prediction of the posterior densities. These densities can not be evaluated, since the estimated mean depends on the future output. It has been suggested, in [9], to approximate these densities by only propagating the covariance matrix. This approximation has given good results in the linear context. Hence, in this work, we explore the use of a Wiener structure combined with an ANN model, to design a dual adaptive non-linear predictive controller for stable unknown linear system, with a stochastic additive disturbance acting at the output. The dual characteristic means that the controller is able to *cautiously* track the desired reference signal; and at the same time, it *excites* the system to improve its identification, so that the performance of the overall controller can be improved in future time intervals.

The paper is organized as follows: section 2 describes the characteristic of the nonlinear model to be considered in the design of the predictive controller. Section 3 introduces the adaptive model based on ANNs, and section 4 the design of the adaptive controllers based on the CE principle and the prediction of the posterior densities. Section 5 illustrates the performance of both algorithms. Finally, in section 6 some conclusions are drawn.

2 The Nonlinear Model

The general model considered in this work can be represented by the following state space representation

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k) \tag{1}$$

$$y(k) = h(\mathbf{x}(k)) \tag{2}$$

where $y(k)$ is the measured variable, $u(k)$ is the input variable, $\mathbf{x}(k)$ is a vector of dimension N and $h(\mathbf{x}(k))$ a continuous function. The matrix $\mathbf{A}$ and vector $\mathbf{b}$ define the dynamic of the models. It has been demonstrated that this type of structure can approximate any stable nonlinear system with any degree of accuracy [10][6]. There are two popular choices for the matrix $\mathbf{A}$ and vector $\mathbf{b}$: orthonormal filters, which are suitable for process modelling [10], and gamma filters more suitable for signal processing [11]. This approach leads naturally to a Wiener model, as it was originally proposed by Wiener [12]. The general

structure of the Wiener model, using Laguerre filters, can be described in state space equations, as eq. (1) with :

$$\mathbf{A}_c = -\mu \begin{bmatrix} 1 & 0 & \dots & 0 \\ 2 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 2 & \dots & 2 & 1 \end{bmatrix}, \mathbf{b}_c = \sqrt{(}2\mu) \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1] \end{bmatrix} \tag{3}$$

where μ is the scale factor. The discrete parameters are:

$$\mathbf{A} = e^{\mathbf{A}_c T_m}, \mathbf{b} = (\mathbf{A} - \mathbf{I})\mathbf{A}_c^{-1}\mathbf{b}_c, \tag{4}$$

where T_m is the sampling time.

3 An ANN Model

Let's consider that the unknown nonlinear function $h(\mathbf{x}(k))$ can be represented as a parameterized nonlinear function:

$$h(\mathbf{x}(k)) = n(\mathbf{x}(k), \theta) \tag{5}$$

where θ represents a vector of unknown parameters. The nonlinear function is approximated by a two layer neural network as follows :

$$y(k) = \mathbf{W}_1 \sigma(\mathbf{V}_1 \mathbf{x}(k) + \mathbf{v}_0) + w_0. \tag{6}$$

where σ is a sigmoidal function. The parameters are collected in a vector of parameters θ.

Thus, the system (1) can be described by:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k) \tag{7}$$

$$y(k) = n(\mathbf{x}(k), \theta) + \eta(k) \tag{8}$$

where θ is the unknown vector of parameter, and $\eta(k)$ is an independent, identically distributed Gaussian random variable, with a distribution given by $N(0, \sigma^2)$. We will assume that the parameters are modelled as a random variable with a normal prior distribution given by $N(\theta(0), \mathbf{P}(0))$, where $\theta(0)$ and $\mathbf{P}(0)$ define the initial mean and covariance matrix respectively. The on-line computation of the conditional mean and covariance of θ can be carried out by the following Extended-Kalman filter:

$$\theta(k+1) = \theta(k) + \frac{\mathbf{m}(k)'\mathbf{P}(k)e(k)}{1 + \mathbf{m}(k)'\mathbf{P}(k)\mathbf{m}(k)} \tag{9}$$

$$\mathbf{P}(k+1) = \mathbf{P}(k) - \frac{\mathbf{P}\mathbf{m}(k)\mathbf{m}(k)'\mathbf{P}(k)}{1 + \mathbf{m}(k)'\mathbf{P}(k)\mathbf{m}(k)} \tag{10}$$

where $\mathbf{m}(k) = \nabla_\theta n(\mathbf{x}(k), \theta)$. Note that the covariance matrix depends only on the input signal, $u(k)$, through the variables $\mathbf{m}(k)$.

4 The Predictive Control Strategies

Predictive control can be seen as a dynamic programming problem [13]. Thus, the general predictive control in the stochastic dynamic programming setting, finds a set of future control signals $\mathbf{u}(k) = [u(k) \dots u(k+T)]'$, so that, Bellman's equations are satisfied:

$$J^{k,k+T}(\Im_k) = min_{\mathbf{u}(k)} E\left[q_0(w(k+1) - n(\mathbf{x}(k),\theta))^2 + r_0 u(k)^2 + J^{k,k+T}(\Im_{k+1})|\Im_k\right] \tag{11}$$

where $\Im_k = \{y(0), \dots, y(k), n(\mathbf{x}(0)), \dots, n(\mathbf{x}(k))\}$ defines the information vector at time k, $J_k^{k+T}(\Im_k)$ is the optimal cost to go at step k, with $J_{k+T}^{k+T+1}(\Im_T) = 0$. Once the solution is obtained, only the first value is applied to the process, and the minimization is carried out each sampling time. The problem posed by (11) can not be solved, because it requires the knowledge about the future values of the posterior densities, which depend on the future control signals and future output of the process. The solution of (11) gives as a result a control signal that not only takes into account the control objective, but also the effect of the input over the parameter estimation algorithm. Several approximations to (11) can be formulated in order to obtain some practical solutions.

4.1 Certainty Equivalence Controller

The predictive controller based on the CE assumption calculates a set of future control signals without considering the uncertainty in the parameters; so that:

$$min_{\mathbf{u}(k)} J_{CE}^{k,k+T}(\Im_k) = E\left[\sum_{i=0}^{T} q_i(w(k+i+1) - n(\mathbf{x}(k+i+1),\theta))^2 + r_i u(k+i)^2\right], \tag{12}$$

where q_i and r_i are weighting factors, and $w(k)$ is the reference signal. Under the receding horizon principle, only the first control is applied to the system. As all signals are deterministic, the expectation is just:

$$min_{\mathbf{u}(k)} J_{CE}^{k,k+T}(\Im_k) = \sum_{i=0}^{T} q_i(w(k+i+1) - n(\mathbf{x}(k+i+1),\theta))^2 + r_i u(k+i)^2 \tag{13}$$

subject to the system equations. To reduce the dimension of the optimization problem several approaches can be applied. For instance, if the future control signal is assumed constant [14]; i.e. $u(k) = u(k+1) \dots = u(k+T)$, then the problem is reduced to one dimensional optimization problem by considering the predictions as follows:

$$n(k+j) = n(\mathbf{A}^j \mathbf{x}(k) + \sum_{i=0}^{j-1} \mathbf{A}^{j-i} \mathbf{b} u(k), \theta(k)) \tag{14}$$

.

In addition, if we consider $r_i = q_i = 0$, $i = 0, .., T-1$ and $q_T = 1$, $r_T = 1$, the final index will be:

$$min_{u(k)} J_{CE}^{k,k+T}(\Im_k) = (w(k+1+T) - n(\mathbf{A}^j \mathbf{x}(k) + \sum_{i=0}^{j-1} \mathbf{A}^{j-i} \mathbf{b} u(k), \theta(k)))^2 \quad (15)$$

In general, a simple line search algorithm can be used to obtain the solution to this optimization problem.

4.2 An Approximate Dual Controller

The dual strategy has associated the cost function (11), which in order to be optimized in terms of $\mathbf{u}(k)$ requires the knowledge of the posterior densities $N(\theta(k+i), \mathbf{P}(k+i))$. Unfortunately, these densities can not be evaluated since the estimated mean depends on the future output. In [9] has been suggested to approximate these densities by $N(\theta(k), \mathbf{P}(k+i))$. Thus, taking the expectation, the following approximation can be found:

$$\begin{aligned} min_{\mathbf{u}(k)} J_{CE}^{k,k+T}(\Im_k) \quad &= \sum_{i=0}^{T} q_i (w(k+i+1) - n(\mathbf{x}(k+i+1), \theta))^2 + ... \\ &r_i u(k+i)^2 + q_i \mathbf{m}(k+1+i)' \mathbf{P}(k+1+i) \mathbf{m}(k+1+i) \quad (16) \end{aligned}$$

The above cost function subject to the system model equations (8) and the covariance equation (10) can be minimized with respect to the future control signal $\mathbf{u}(k)$. An active strategy is obtained, since the future values of the covariance matrix, are included in the index. In this way, the control signal will also try to bring the uncertainty of the parameters to some low level.

5 Simulation Results

In this section, the algorithms are applied to control two reactions in series ($A \rightarrow B \rightarrow C$) in a Continuous Stirred Tank Reactor [10]. The desired product is the intermediate product B. The differential equations describing the system are given by :

$$\dot{x}_1 = 1 - x_1 - E_3 e^{-E_1/x_3} x_1 + E_4 e^{-E_2/x_3} x_2 \quad (17)$$

$$\dot{x}_2 = -x_2 + E_3 e^{-E_1/x_3} x_1 - E_4 e^{-E_2/x_3} x_2 \quad (18)$$

$$\dot{x}_3 = u - x_3 + .005(E_3 e^{-E_1/x_3} x_1 - E_4 e^{-E_2/x_3} x_2) \quad (19)$$

with $E_1 = 50$, $E_2 = 70$, $E_3 = 300000$, and $E_4 = 60 \cdot 10^6$; where x_1 and x_2 are dimensionless concentrations of A and B, x_3 is the dimensionless temperature of the jacket surrounding the reactor. In order to model the relationship between the concentration of the desired product; i.e. x_2, and the control signal, we have considered a scale factor $\mu = .6$, sampling time $T_m = 1$, and six Laguerre filters. A neural network with 6 inputs and 4 hidden units, was trained on line. The measured concentration,$y(kT)$, considered a zero mean noise signal, $\eta(kT)$,

$$y(kT) = x_2(kT) + \eta(kT). \quad (20)$$

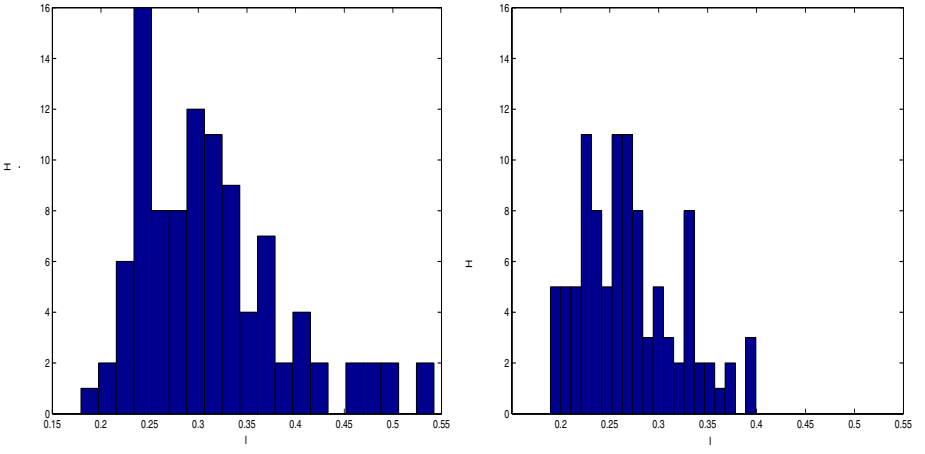

Fig. 1. Distribution of the cost function: CE controller and Dual controller

As a measure of the control performance the following index is estimated for different realizations of the noise signal:

$$I = \sqrt{\frac{1}{N}\sum_{k=0}^{N}(y_d(kT_m) - y(kT_m))^2}, \tag{21}$$

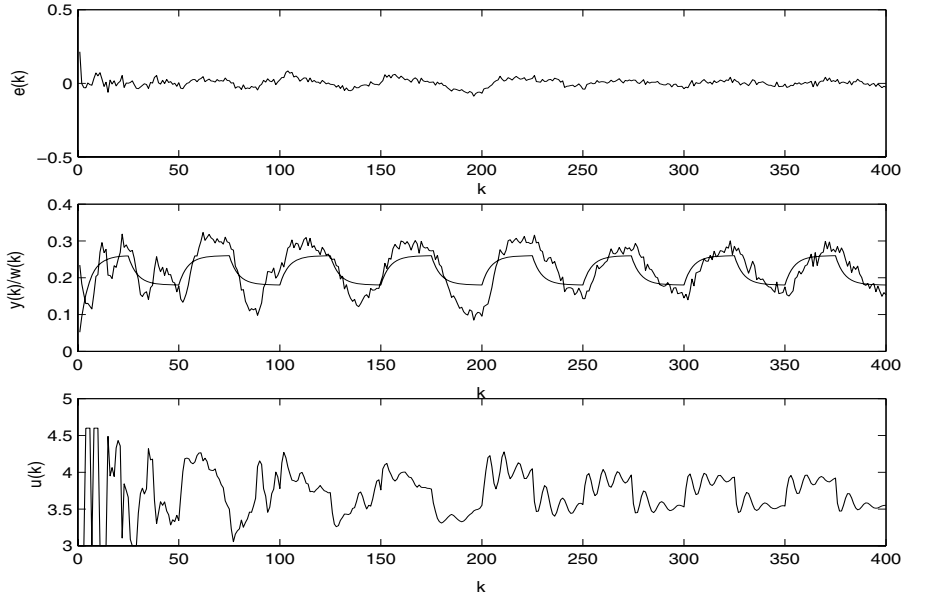

Fig. 2. A CE predictive controller

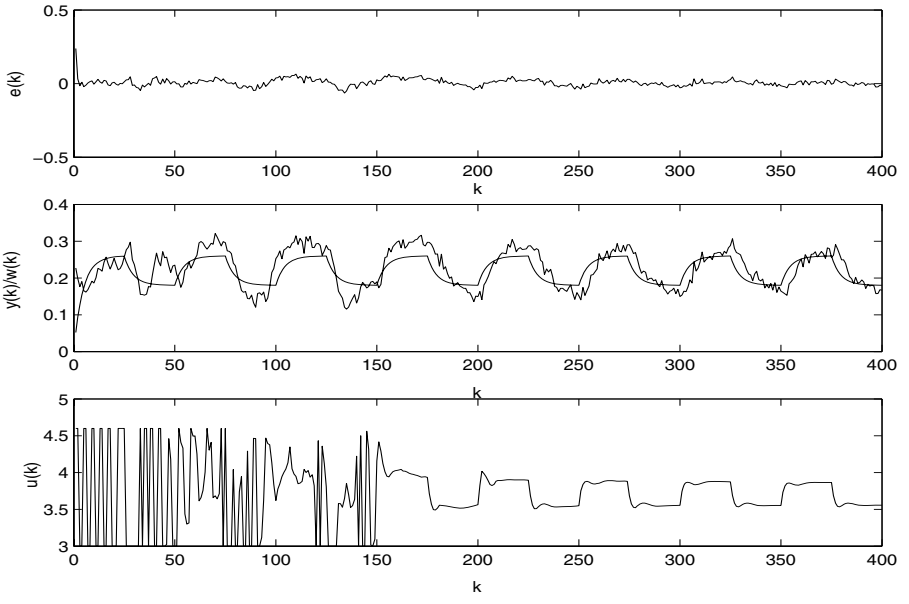

Fig. 3. A Dual predictive controller

where N is the number of samples considered in the cost function, $y_d(kT)$ and $y(kT)$ are the desired set-point and measured concentration respectively. The controller prediction horizon was set to $T = 5$ and the same initial conditions were used for all the simulations. Performing Monte Carlo simulations, the index I was evaluated for 100 realizations of the measurement noise. Figure 1 shows the distribution of the cost function for both controllers, clearly the dual controller shifts the distribution toward smaller values of the cost function. Figure 2 shows the behavior of the controller based on the CE principle, and figure 3 the one of the dual controller, both figures were obtained for the same noise realization. By comparing both figures, we can see that the latter provides larger excitation signals, but without compromising tracking performance. This extra excitation, at initial stages of the adaptive process, means smaller identification errors and better tracking performance in future time instants.

6 Final Remarks

We have presented a methodology to design an adaptive predictive controller based on an Artificial Neural Network model, considering the minimization of a cost function taking into account the future tracking errors and the effect of the control signal on the parameter estimation algorithm. The parameters of the ANN are considered as random variables and the network training algorithm is based on an Extended Kalman-filter. The obtained result shows that this controller provides more excitation to the system at initial stages than a

controller based on the CE principle. This key feature gives as a result a better control performance in future time intervals. Future works consider the real-time implementation of this type of controllers.

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