

Evolving Spatiotemporal Coordination in a Modular Robotic System

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Abstract. In this paper we present a novel information-theoretic measure of spatiotemporal coordination in a modular robotic system, and use it as a fitness function in evolving the system. This approach exemplifies a new methodology formalizing co-evolution in multi-agent adaptive systems: information-driven evolutionary design. The methodology attempts to link together different aspects of information transfer involved in adaptive systems, and suggests to approximate direct task-specific fitness functions with intrinsic selection pressures. In particular, the information-theoretic measure of coordination employed in this work estimates the generalized correlation entropy K_2 and the generalized excess entropy E_2 computed over a multivariate time series of actuators' states. The simulated modular robotic system evolved according to the new measure exhibits regular locomotion and performs well in challenging terrains.

1 Introduction

Innovations in distributed sensor and actuator technologies, as well as advances in multi-agent control theory and studies of self-organization, support rapid growth in applications of complex adaptive multi-agent systems (MAS), such as modular robotics, multi-robot teams, self-assembly, etc. In particular, modular robots built of several similar building blocks (modules) become more and more attractive due to high versatility in their shapes, locomotion modes, tasks, and manipulation abilities [3,26,22,21,7]. This multi-faceted versatility increases robustness, adaptability, and scalability required in practical systems, ranging from search and rescue to space exploration. These requirements are achieved through a distribution of sensing, actuation and computational capabilities throughout the MAS such as a modular robotic system. This distribution forms a complex multi-agent network, enabling the desired responses to self-organize within the system, without central control. However, the main challenge with developing a self-organizing MAS is a design methodology for systematically inter-connecting a set of global system-level tasks, functions, etc. with localized sensors, behaviors, and actuators.

In this paper we further develop such a methodology originally sketched in [14], aiming at formalizing “taskless adaptation” of co-evolving multiple agents (robotic modules, network nodes, swarm elements, etc.). The co-evolution can be achieved in two

ways: via task-specific objectives or via generic intrinsic selection criteria. The generic information-theoretic criteria may vary in their emphasis: for example, we may focus on maximization of information transfer in perception-action loops [11,12]; minimization of heterogeneity in agent states, measured with the variance of the rule-space's entropy [25,17] or Boltzmann entropy in swarm-bots' states [1]; stability of multi-agent hierarchies [17]; efficiency of computation (computational complexity); efficiency of communication topologies [15,16]; efficiency of locomotion and distributed actuation [14,6,22,21], etc. The solutions obtained by information-driven evolution can be judged by their degree of approximation of direct evolutionary computation, where the latter uses task-specific objectives and depends on hand-crafting fitness functions by human designers. A good approximation will indicate that the chosen criteria capture the information-theoretic core of selection pressures. The main theme, however, is that different selection criteria incorporate information transfer within specific channels, and selecting some of these channels and not the others would guide information-driven evolutionary design.

Following [14] we apply here an information-theoretic measure of spatiotemporal coordination in a modular robotic system to an evolution of a sufficiently simple system: a modular limbless, wheelless snake-like robot (Snakebot) [22,21] without sensors. The only design goal of Snakebot's evolution, reported by Tanev and his colleagues, is fastest locomotion. Our immediate goal is information-theoretic approximation of this direct evolution. Specifically, we construct measures of spatiotemporal coordination of distributed actuators used by a Snakebot in locomotion. The measures are based on the generalized correlation entropy K_2 (a lower bound of Kolmogorov-Sinai entropy) and its excess entropy E_2 computed over a multivariate time series of actuators' states. The experiments reported by [14] confirmed that maximal coordination is achieved synchronously with fastest locomotion. In this paper we replace the direct measure with the information-theoretic measure of spatiotemporal coordination, and use the latter exclusively in evolving the Snakebot.

The following Section places this methodology in the context of previous studies, describes the proposed measures, and presents results, followed by conclusions.

2 Information Transfer as an Intrinsic Selection Pressure

An example of an intrinsic selection pressure is the acquisition of information from the environment: there is evidence that pushing the information flow to the information-theoretic limit (i.e., maximization of information transfer) can give rise to intricate behavior, induce a necessary structure in the system, and ultimately adaptively reshape the system [11,12]. The central hypothesis of Klyubin *et al.* is that there exists "a local and universal utility function which may help individuals survive and hence speed up evolution by making the fitness landscape smoother", while adapting to morphology and ecological niche. The proposed general utility function, *empowerment*, couples the agent's sensors and actuators via the environment. Empowerment is the perceived amount of influence or control the agent has over world, and can be seen as the agents potential to change the world. It can be measured via the amount of Shannon information that the agent can "inject into" its sensor through the environment, affecting future

actions and future perceptions. Such a perception-action loop defines the agent’s actuation channel, and, technically, empowerment is defined as the capacity of this actuation channel: the maximum mutual information for the channel over all possible distributions of the transmitted signal. “The more of the information can be made to appear in the sensor, the more control or influence the agent has over its sensor” — this is the main motivation for this local and universal utility function [12].

Heterogeneity in agent states is another generic pressure related to intrinsic coordination and self-organization. For example, it was measured with the variance of the rule-space’s entropy [25] and applied to evolve the *spatiotemporal stability* of multi-cellular patterns in a sensor/communication network embedded within a self-monitoring impact sensing test-bed of an aerospace vehicle [9,17,24]. The study of spatiotemporal stability in evolving impact boundaries — continuously connected multi-cellular circuits, self-organizing in presence of cell failures and connectivity disruptions around damaged areas — employs both task-dependent graph-theoretic and generic information-theoretic measures in separating chaotic regimes from ordered dynamics. The task-dependent measure captured the impact boundary’s connectivity in terms of the size of the average connected boundary fragment — an analogue of a largest connected sub-graph and its standard deviation over time. The intrinsic information-theoretic measure captured the diversity of transition rules invoked by the network cells during an impact boundary formation, using the Shannon entropy of the rules’ frequency distribution:

$$H(X^t) = - \sum_{i=1}^m \frac{X_i^t}{n} \log \frac{X_i^t}{n},$$

where n is the system size (the total number of cells), and X_i^t is the number of times the transition i was used at time t across the system. Both measures concurred in identifying complex dynamics, pointing to the same phase transition between chaos and order, for particular regions in a parameter-space. The entropy $H(X^t)$ can also be interpreted as the joint state transition entropy $H(S^t, S^{t+1})$, where S^t is the state of the cell at time t [17]. This opens a way to consider information transfer

$$I(S^t; S^{t+1}) = H(S^t) - H(S^t, S^{t+1}),$$

within the channel between a cell and itself at the next time-step.

An investigation of Baldassarre *et al.* [1], characterized coordinated motion in a swarm collective as a self-organized activity, and measured the increasing organization of the group on the basis of Boltzmann entropy. In particular, the emergent *common direction* of motion, with the chassis orientations of the robots spatially aligned, was observed to allow the group to achieve high coordination. Baldassarre *et al.* proposed a method to capture the spatial alignment via Boltzmann entropy by dividing the state space of the elements of the system into cells (e.g., cells of 45° each, corresponding to chassis orientations), measuring the number of elements in each cell for a given macrostate m , computing the number w_m of microstates that compose m , and calculating Boltzmann entropy of the macrostate as $E_m = k \ln[w_m]$, where k is a scaling constant. This constant is set to the inverse of the maximum entropy which is equal to the entropy of the macrostate where all the elements are equally distributed over the cells. The results indicate that “independently of the size of the group, the disorganization of the group initially decreases with an increasing rate, then tends to decrease with

a decreasing rate, and finally reaches a null value when all the robots have the same orientation” [1].

In this work, we advance from a purely spatial characterization (such as Boltzmann entropy of a macrostate distributing chassis orientations over the cells) to a spatiotemporal measure. The entropy measure proposed in our work is intended not only to capture spatial alignment of different modules, but also to account for temporal dependencies among them, such as travelling or standing waves in multi-segment chains observed by Ijspeert *et al.*. Importantly, we plan to focus on channels where information transfer contributes to a selection pressure.

We refer here to one more example of a selection pressure — efficiency of communication topologies — which can be interpreted as in terms of information transfer. One feasible average measure of a complex network’s heterogeneity is given by the entropy of a network defined through the link distribution. The latter can be defined via the simple degree distribution — the probability P_k of having a node with k links. Similarly, one can capture the average uncertainty of the network as a whole, using the joint entropy based on the joint probability of connected pairs $P_{k,k'}$. Ultimately, the amount of correlation between nodes in the graph can be calculated via the mutual information measure, the information transfer [19], as

$$I(P; P') = H(P) - H(P|P') = \sum_{k=1}^m \sum_{k'=1}^m P_{k,k'} \log \frac{P_{k,k'}}{P_k P_{k'}}.$$

The reviewed examples highlight the possible role of information transfer in guiding selection of efficient perception-action loops, spatiotemporally stable multi-cellular patterns, and well-connected network topologies. We intend to demonstrate that spatiotemporal coordination in a modular robotic system can also be captured as information transfer, and apply such a measure to the system’s evolution.

Before presenting our approach, we briefly review some studies of the relation between locomotion and rhythmic inter-modular coordination. Dorigo [7] describes an experiment in swarm robotics (SWARM-BOT) which also complements standard self-reconfigurability with task-dependent cooperation. Small autonomous mobile robots (s-bots) aggregate into specific shapes enabling the collective structure (a swarm-bot) to perform functions beyond capabilities of a single module. The swarm-bot forms as a result of self-organization “rather than via a global template and is expected to move as a whole and reconfigure along the way when needed” [7]. One basic ability of a swarm-bot, immediately relevant to our research, is *coordinated motion* emerging when the constituent independently-controlled modules coordinate their actions in choosing a common direction of motion. Our focus is on how much locomotion can be “patterned” in an aggregated structure. Regardless of an environment (aquatic, terrestrial or aerial), locomotion is achieved by applying forces generated by the rhythmic contraction of muscles attached to limbs, wings, fins, etc. Typically, a locomotory gait is efficient when all the involved muscles contract and extend with the same frequency in different phases. For example, Yim *et al.* [26] investigated a snake-like (serpentine) sinusoid gait, where forward motion is essentially achieved by propagating a waveform travelling down the length of the chain. Tanev and his colleagues [22,21] demonstrated emergence of side-winding locomotion with superior speed characteristics for the given morphology as well as adaptability to challenging terrain and partial damage.

3 Spatiotemporal Coordination of Actuators

Snakebot is simulated as a set of identical spherical morphological segments, linked together via universal joints. All joints feature identical angle limits, and each joint has two attached actuators. In the initial standstill position of Snakebot, the rotation axes of the actuators are oriented vertically (vertical actuator) and horizontally (horizontal actuator). These actuators perform rotation of the joint in the horizontal and vertical planes respectively. No anisotropic friction between the morphological segments and the surface is considered. Open Dynamics Engine (ODE) was chosen to provide a realistic simulation of the mechanics of Snakebot. Given this representation, the task of designing the fastest Snakebot locomotion can be rephrased as developing temporal patterns of desired turning angles of horizontal and vertical actuators for each joint, maximizing the overall speed. Previous experiments of evolvable locomotion gaits with fitness measured as either velocity in any direction or velocity in forward direction [22] indicated that side-winding locomotion — locomotion predominantly perpendicular to the long axis of Snakebot (Figures 1 and 2) — provides superior speed characteristics for the considered morphology. The actuators states (horizontal and vertical turning angles) are constrained by the interactions between segments and the terrain. The *actual turning angles* provide an underlying time series for our information-theoretic analysis: horizontal turning angles $\{x_t^i\}$ and vertical turning angles $\{y_t^i\}$ at time t , where i is the actuator index, S is the number of joints, $1 \leq i \leq S$, and T is the considered time interval, $1 \leq t \leq T$. Since we deal with actual rather than ideal turning angles, the underlying dynamics in the phase-space may include both periodic and chaotic orbits.

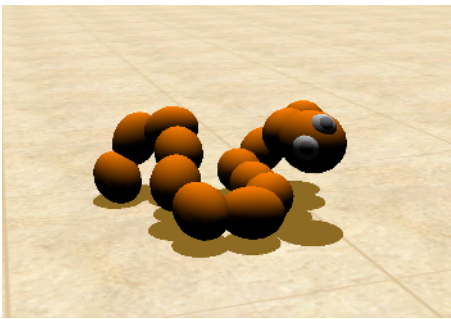


Fig. 1. Side view of the Snakebot

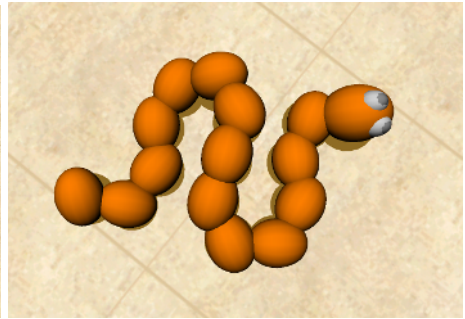


Fig. 2. Top view of the Snakebot

We intend to estimate “irregularity” for each of the multivariate time series $\{x_t^i\}$ and $\{y_t^i\}$. Each of these time series, henceforth denoted for generality $\{v_t^i\}$, contains both spatial and temporal patterns, and minimizing the irregularity over both space and time dimensions should ideally uncover the extent of spatiotemporal coordination among actuator states.

For any given actuator i , a simple characterisation of the “regularity” of the time series $\{v_t^i\}$ is provided by the auto-correlation function. However, the auto-correlation is limited to measuring only linear dependencies. We consider instead a more general

approach. One classical measure is the Kolmogorov-Sinai (KS) entropy, also known as metric entropy [13]: it is a measure for the rate at which information about the state of the system is lost in the course of time. In other words, it is an entropy per unit time, an entropy rate or entropy density. Suppose that the d -dimensional phase space is partitioned into boxes of size r^d . Let $P_{i_0 \dots i_{d-1}}$ be the joint probability that a trajectory is in box i_0 at time 0, in box i_1 at time Δt , ..., and in box i_{d-1} at time $(d-1)\Delta t$, where Δt is the time interval between measurements on the state of the system (in our case, we may assume $\Delta t = 1$, and omit the limit $\Delta t \rightarrow 0$ in the following definitions). The KS entropy is defined by

$$K = - \lim_{r \rightarrow 0} \lim_{d \rightarrow \infty} \frac{1}{d\Delta t} \sum_{i_0 \dots i_{d-1}} P_{i_0 \dots i_{d-1}} \ln P_{i_0 \dots i_{d-1}}, \tag{1}$$

and more precisely, as a supremum of K on all possible partitions. This definition has been generalized to the order- q Rényi entropies K_q [18]:

$$K_q = - \lim_{\Delta t \rightarrow 0} \lim_{r \rightarrow 0} \lim_{d \rightarrow \infty} \frac{1}{d\Delta t(q-1)} \ln \sum_{i_0 \dots i_{d-1}} P_{i_0 \dots i_{d-1}}^q. \tag{2}$$

It is well-known that $K = 0$ in an ordered system, K is infinite in a random system, and K is a positive constant in a deterministic chaotic system. Grassberger and Procaccia [10] considered the correlation entropy K_2 in particular, and capitalized on the fact $K \geq K_2$ in establishing a sufficient condition for chaos $K_2 > 0$. Their algorithm estimates the entropy rate K_2 for a univariate time series. For our analysis we need to introduce a spatial dimension across multiple Snakebot’s actuators. An estimate of the spatiotemporal entropy density can be obtained as

$$K = - \lim_{d_s \rightarrow \infty} \lim_{d_t \rightarrow \infty} \frac{1}{d_s} \frac{1}{d_t} \sum_{V(d_s, d_t)} p(V(d_s, d_t)) \ln p(V(d_s, d_t)), \tag{3}$$

where $V(d_s, d_t)$ are “patterns” of spatial size d_s and time length d_t [2]. Our objective, an estimate of spatiotemporal generalized correlation entropy, can be obtained as

$$K_2 = - \lim_{d_s \rightarrow \infty} \lim_{d_t \rightarrow \infty} \frac{1}{d_s} \frac{1}{d_t} \ln \sum_{V(d_s, d_t)} p^2(V(d_s, d_t)). \tag{4}$$

In achieving this objective, we follow Grassberger-Procaccia method [10] of computing correlation integrals, but use the multivariate time series with S actuators (joints) and T time steps in the following approximation:

$$K_2^{d_s d_t}(S, T, r) = \ln \frac{C_{d_s d_t}(S, T, r)}{C_{d_s(d_t+1)}(S, T, r)} + \ln \frac{C_{d_s d_t}(S, T, r)}{C_{(d_s+1)d_t}(S, T, r)}, \tag{5}$$

where correlation integrals are generalized as

$$C_{d_s d_t}(S, T, r) = \frac{1}{(T-1)T(S-1)S} \sum_{l=1}^T \sum_{j=1}^T \sum_{g=1}^S \sum_{h=1}^S \Theta(r - \|\mathbf{V}_l^g - \mathbf{V}_j^h\|). \tag{6}$$

Here Θ is the Heaviside function (equal to 0 for negative argument and 1 otherwise), and the vectors \mathbf{V}_l^g and \mathbf{V}_j^h contain elements of the observed time series $\{v_t^i\}$ for each actuator (the spatial dimension), “converting” or “reconstructing” the dynamical information in two-dimensional data to information in the $d_s d_t$ -dimensional embedding space [20]. More precisely, we use spatiotemporal delay vectors $\mathbf{V}_k^i = (v_k^i, v_k^{i+1}, v_k^{i+2}, \dots, v_k^{i+d_s-1})$, whose elements are time-delay vectors $\mathbf{v}_k^i = (v_k^i, v_{k+1}^i, v_{k+2}^i, \dots, v_{k+d_t-1}^i)$, and the spatial index i is fixed [14]. The norm $\|\mathbf{V}_l^g - \mathbf{V}_j^h\|$ is the distance between the vectors in the $d_s d_t$ -dimensional space, e.g., the maximum norm:

$$\|\mathbf{V}_l^g - \mathbf{V}_j^h\| = \max_{\sigma=0}^{d_s-1} \max_{\tau=0}^{d_t-1} (v_{l+\tau}^{g+\sigma} - v_{j+\tau}^{h+\sigma})$$

Put simply, correlation integral $C_{d_s d_t}(S, T, r)$ computes the fraction of pairs of vectors in the $d_s d_t$ -dimensional embedding space that are separated by a distance less than or equal to r . In order to eliminate auto-correlation effects, the vectors in equation (6) should be chosen to satisfy $|l - j| > L$, for an integer L , and $|g - h| > M$, for an integer M , in order to exclude auto-correlation effects among temporally close delays or closely coupled segments [23]. The standard temporal delay reconstruction [20] is recovered by setting $d_s = 1$ [4].

The correlation entropy K_2 (the generalized entropy rate) measures the irregularity or unpredictability of the system. A complementary quantity is the *excess entropy* E [8,5] — it may be viewed as a measure of the apparent memory or structure in the system. The generalized excess entropy E_2 is defined by considering how the finite-template (finite-delay and finite-extent) entropy rate estimates $K_2^{d_s d_t}(S, T, r)$ (equation (5)), converge to their asymptotic values K_2 (equation (4)). It is estimated for a fixed spatial extent D_s and a given time range D_t as:

$$E_2(D_s, D_t, S, T, r) = \sum_{d_s=1}^{D_s} \sum_{d_t=1}^{D_t} (K_2^{d_s d_t}(S, T, r) - K_2). \quad (7)$$

For regular locomotion the asymptotic values should be zero (while non-zero entropies would indicate non-periodicity, i.e. deterministic chaos). It was shown that the excess entropy also measures the amount of historical information stored in the present that is communicated to the future [5,8]. In other words, it can be represented as asymptotic mutual information between two adjacent $d_s d_t$ -dimensional half-planes

$$\lim_{d_s, d_t \rightarrow \infty} I(\mathbf{V}_{-d_t}^g; \mathbf{V}_0^h) = \lim_{d_s, d_t \rightarrow \infty} I((v_{-d_t}^g, v_{-d_t}^{g+1}, v_{-d_t}^{g+2}, \dots, v_{-d_t}^{g+d_s-1}); (v_0^h, v_0^{h+1}, v_0^{h+2}, \dots, v_0^{h+d_s-1}))$$

where $\mathbf{v}_k^i = (v_k^i, v_{k+1}^i, v_{k+2}^i, \dots, v_{k+d_t-1}^i)$. This alternative representation establishes that the proposed measure may estimate information transfer within the space of actuators: the more information between the spatiotemporal past and the spatiotemporal future is transferred, the more coordination is achieved. If $g = h$ in the last expression, the transfer is purely between the temporal past and the temporal future. Otherwise, if $g \neq h$, we are concerned with how much information contained in the past of one group of actuators is injected into the future of another group of actuators.

When dealing with non-zero entropy rates K_2 , one may consider *relative excess entropy*:

$$e_2(D_s, D_t, S, T, r) = \sum_{d_s=1}^{D_s} \sum_{d_t=1}^{D_t} \frac{K_2^{d_s d_t}(S, T, r) - K_2}{K_2 + \epsilon} \tag{8}$$

where ϵ is a small constant (e.g., $\epsilon = 0.03$), balancing the relative excess entropy e_2 for very small entropy rates K_2 . The relative excess entropy e_2 attempts to “reward” the structure (coupling) in the locomotion and “penalise” its non-regularity.

4 Results

In this section we present experimental results of Snakebot’s evolution based on estimates of the excess entropy E_2 (equation (7)) and the relative excess entropy e_2 (equation (8)). The Genetic Programming (GP) techniques employed in the evolution are described elsewhere [22,21]. In particular, the genotype is associated with two algebraic expressions, which represent the temporal patterns of desired turning angles of both the horizontal and vertical actuators of each morphological segment. Because locomotion gaits, by definition, are periodical, we include the periodic functions \sin and \cos in the function set of GP in addition to the basic algebraic functions. The selection is based on a binary tournament with selection ratio of 0.1 and reproduction ratio of 0.9. The mutation operator is the random subtree mutation with ratio of 0.01. Snakebots evolve within a population of 200 individuals, and the best performers are selected according to the excess entropy values, over a number of generations.

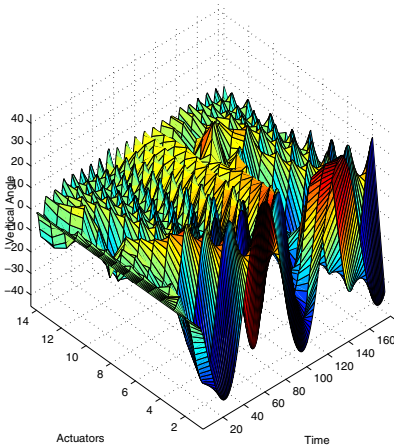


Fig. 3. First offspring: actuator angles

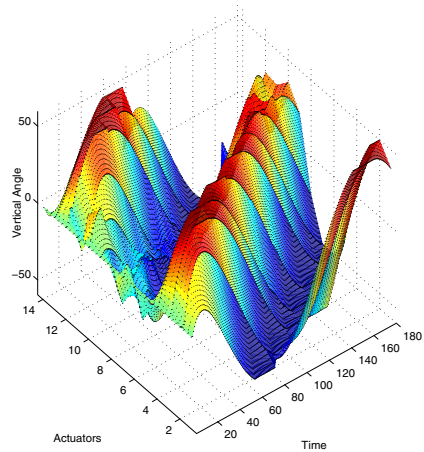


Fig. 4. Evolved solution: actuator angles

Figures 3 and 4 contrast (for vertical actuators) actual angles used by the first offspring and the final generation. Similarly, Figures 5 and 6 contrast the spatiotemporal correlation entropies produced by the first offspring and the evolved solution. It can be easily observed that more regular angle dynamics of the evolved solution manifests

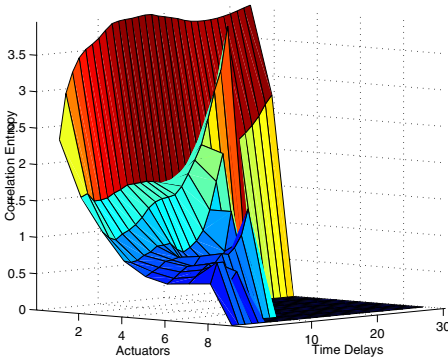


Fig. 5. First offspring: correlation entropy

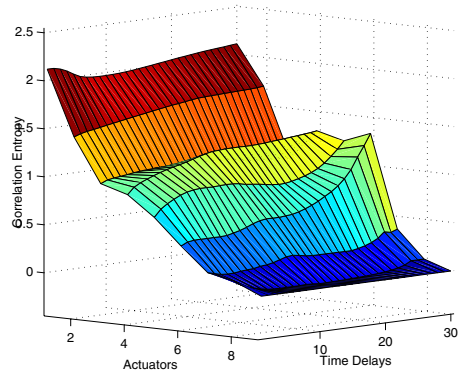


Fig. 6. Evolved solution: correlation entropy

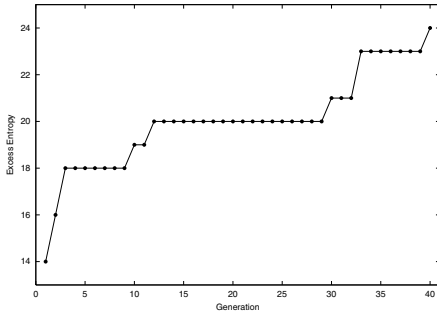


Fig. 7. Snakebot fitness over time: the best performer in each generation, using excess entropy

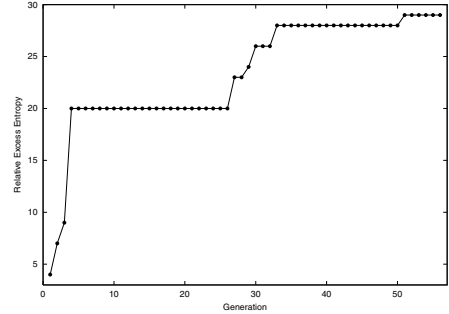


Fig. 8. Snakebot fitness over time: the best performer in each generation, using relative excess entropy

itself as more significant excess entropy. Figures 7 and 8 show typical fitness growth towards higher excess entropies estimated as E_2 (equation (7)) and the relative excess entropies e_2 (equation (8)), for two different experiments. It should be noted that there are well-coordinated Snakebots which are moving not as quickly as the Snakebots evolved according to the direct velocity-based measure, i.e. the set of fast solutions is contained within the set of well-coordinated solutions. This means that the obtained approximation of the direct fitness function by the information-theoretic selection pressure towards regularity is sound but not complete.

In certain circumstances, a fitness function rewarding coordination may be more suitable than a direct velocity-based measure: a Snakebot trapped by obstacles may need to employ a locomotion gait with highly coordinated actuators but near-zero absolute velocity. In fact, the obtained solutions exhibit reasonable robustness to challenging terrains, trading-off some velocity for resilience to obstacles. In particular, the evolved Snakebot shown in Figure 9 is able to traverse ragged terrains with obstacles three times as high as the segment diameter, move through a narrow corridor (only twice as wide as the segment diameter), and overcome various extended barriers. In addition, the

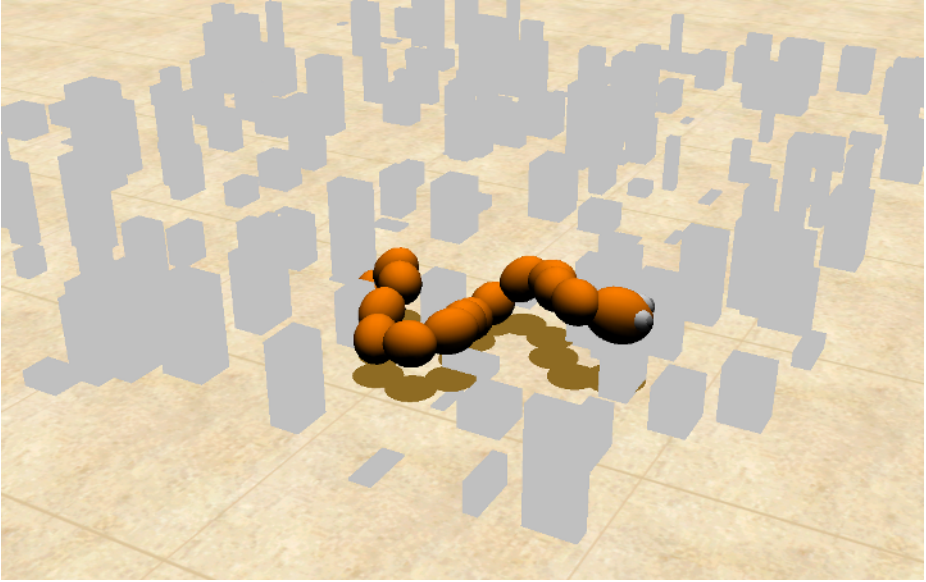


Fig. 9. Snakebot negotiating a terrain with obstacles

Snakebot is robust to failures of individual segments: e.g., it is able to move even when every third segment is completely incapacitated, albeit with only a half of the normal speed. Interestingly enough, the relative excess entropy is increased in partially damaged Snakebots, as the amount of transferred information in the coupled locomotion has to increase. Moreover, there appears to be a strong correlation between the number of damaged (evenly spread) segments s and the resulting relative excess entropy $e_2^s \approx \beta s$, where the coefficient β of the linear fit is approximately equal to the relative excess entropy of a non-damaged Snakebot e_2^0 . This observation opens a way for Snakebot's self-diagnostics and adaptation: the run-time value of e_2 may identify the number of damaged segments, enabling a more appropriate response.

5 Conclusions

We modelled a specific step towards a theory of information-driven evolutionary design, using information-theoretic measures of spatiotemporal coordination in a modular robotic system (Snakebot). These measures estimate the generalized correlation entropies K_2 computed over a time series of actuators' states and the spatiotemporal excess entropies E_2 . As expected, increased coordination of actuators is achieved by agents with faster locomotion. However, the set of fast solutions is a subset of the set of well-coordinated solutions. A more precise approximation of fast locomotion is a subject of future work. In parallel, we are investigating other tasks adaptation to which may require a high degree of actuators' coordination : e.g., rugged terrain traversal, energy-efficient locomotion, etc. Both directions essentially require identification of channels through which the information transfer among system's components is optimized. We

believe that development of adequate information-theoretic criteria, such as the measure of spatiotemporal coordination of distributed actuators, will contribute to design guidelines for co-evolving multi-agent systems.

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