# **On the Security of HMAC and NMAC Based on HAVAL, MD4, MD5, SHA-0 and SHA-1 (Extended Abstract)***-*

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**Abstract.** HMAC is a widely used message authentication code and a pseudorandom function generator based on cryptographic hash functions such as MD5 and SHA-1. It has been standardized by ANSI, IETF, ISO and NIST. HMAC is proved to be secure as long as the compression function of the underlying hash function is a pseudorandom function. In this paper we devise two new distinguishers of the structure of HMAC, called differential and rectangle distinguishers, and use them to discuss the security of HMAC based on HAVAL, MD4, MD5, SHA-0 and SHA-1. We show how to distinguish HMAC with reduced or full versions of these cryptographic hash functions from a random function or from HMAC with a random function. We also show how to use our differential distinguisher to devise a forgery attack on HMAC. Our distinguishing and forgery attacks can also be mounted on NMAC based on HAVAL, MD4, MD5, SHA-0 and SHA-1.

## **1 Introduction**

HMAC, which was designed by Bellare, Canetti and Krawczyk, is a standardized hash-based MAC algorithm that is widely used as a MAC algorithm and as a pseudorandom function generator [\[2\]](#page-13-0). HMAC takes a message of an arbitrary

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bit-length and hashes it with one secret key. For the same length of the message it calls the compression function of the underlying hash function additionally three more times than the iterated hash construction, i.e., the MD construction. For long messages, its efficiency is thus almost the same as the MD construction. Furthermore, cryptographic hash functions such as MD5 and SHA-1 can be used in HMAC, which are more efficient in software than block ciphers, and thus HMAC is typically faster than block cipher based MACs. HMAC is proved to be a pseudorandom function under the assumption that the compression function of the un-derlying hash function is a pseudorandom function [\[1\]](#page-13-2) (note that the security proof of pseudorandomness provides the MAC security [\[3\]](#page-13-3)). However, this does not guarantee the security of HMAC if it is instantiated with a specific cryptographic hash function such as MD5 or SHA-1. The recent attacks of Wang et al. [\[14](#page-14-0)[,15](#page-14-1)[,16](#page-14-2)[,17](#page-14-3)[,18\]](#page-14-4) and Biham et al. [\[5,](#page-13-4)[6\]](#page-13-5) have undermined the confidence in the most popular collision resistant hash functions such as MD5 and SHA-1. However, it is widely assumed that these attacks have no impact on the security of MAC algorithms based on these hash functions such as HMAC since they use a keyed initial value.

This paper is the first work which presents a detailed analysis of distinguishing and forgery attacks on HMAC based on MD5, SHA-1 and other MDx-type hash functions. Our results allow to quantify to which extent the vulnerabilities of these hash functions carry over to the HMAC construction. This is achieved by the introduction of two novel distinguishers of the general structure of HMAC. We use a message pair which induces a collision in its corresponding MAC pair for designing a differential distinguisher of HMAC and also use a message quartet which induces two collisions in its corresponding MAC quartet for designing a rectangle distinguisher of HMAC. With these two distinguishers we discuss the security of HMAC based on HAVAL [\[19\]](#page-14-5), MD4 [\[12\]](#page-14-6), MD5 [\[13\]](#page-14-7), SHA-0 [\[20\]](#page-14-8) and SHA-1 [\[21\]](#page-14-9).

First, we construct new differentials of the full 3-pass HAVAL and reduced MD5 to form rectangle distinguishers of HMAC, and we use them to distinguish HMAC with the full 3-pass HAVAL and reduced MD5 from HMAC with a random function. Second, we investigate how effectively the differentials of MD4, SHA-0 and SHA-1 found by Wang et al. [\[14](#page-14-0)[,15](#page-14-1)[,16](#page-14-2)[,17,](#page-14-3)[18\]](#page-14-4) and Biham et al. [\[5](#page-13-4)[,6\]](#page-13-5) are applied to our differential and rectangle distinguishers in HMAC. After converting their differentials into our differential and rectangle distinguishers, we devise distinguishing and forgery attacks on HMAC based on reduced or full versions of MD4, SHA-0 and SHA-1. In particular, we show how to distinguish HMAC with the full SHA-0 and MD4 from HMAC with a random function and present a forgery attack on HMAC with the full MD4. See for details of the results Table [2](#page-11-0) in Sect. 6 (the function  $h_2$  and the probabilities  $\hat{p}$  and q in Table [2](#page-11-0) will be defined in the following sections). Our distinguishing and forgery attacks can be mounted on NMAC based on HAVAL, MD4, MD5, SHA-0 and SHA-1 with the same complexity.

## **2 Description of HMAC**

HMAC [\[2\]](#page-13-0) applies in both its inner and outer parts the iterated MD construction of a hash function H given a compression function h,  $H(IV, M) =$ 

 $h(\cdots h(h(IV, M^1), M^2) \cdots, M^n)$ , where IV is a l-bit fixed initial value and M is an arbitrary-length message which is padded to a multiple of b-bit and divided into n b-bit blocks  $M^1||M^2|| \cdots ||M^n$  (note that the outputs of functions h and  $H$  are *l*-bit strings).

$$
\mathbf{HMAC}(K, M) = H(IV, (K \oplus opad)||H(IV, (K \oplus ipad)||M))
$$
  
=  $h(h(IV, (K \oplus opad)), H(h(IV, (K \oplus ipad)), M)),$  (1)

<span id="page-2-1"></span>where K is the secret key, opad, ipad are constants and  $|K \oplus opad| = |K \oplus ipad|$ b. If HMAC takes a one-block message  $M$ , it can be expressed as

$$
\mathbf{HMAC}(K, M) = h(h(IV, (K \oplus opad)), h(h(IV, (K \oplus ipad)), M)).
$$
 (2)

<span id="page-2-0"></span>In order to facilitate the description of our analysis of HMAC we denote the four compression functions h in [\(2\)](#page-2-0) by  $h_1$ ,  $h_2$ ,  $h_3$  and  $h_4$ , and the four functions in [\(1\)](#page-2-1) by  $h_1$ ,  $H_2$ ,  $h_3$  and  $h_4$ . See Fig. [1](#page-2-2) for a schematic description of HMAC with this notation. Note that the outputs of  $H_2$  and  $h_2$  are padded to a b-bit string to be inserted into  $h_4$ .



<span id="page-2-2"></span>**Fig. 1.** A schematic description of HMAC

In practice the function  $h$  can be replaced by the compression function of cryptographic hash functions such as HAVAL [\[19\]](#page-14-5), MD4 [\[12\]](#page-14-6), MD5 [\[13\]](#page-14-7), SHA-0 [\[20\]](#page-14-8), SHA-1 [\[21\]](#page-14-9) and so on.

# **3 Some General Attacks on HMAC**

Using the birthday paradox we can induce a general distinguishing attack on HMAC as follows [\[11\]](#page-14-10):

1. Collect  $2^{l/2}$  randomly chosen messages with a b-bit length, denoted  $M_i$ , and ask for their MAC values, denoted  $C_i$ .

- 2. Find message pairs  $M_j$  and  $M_k$  such that  $C_j = C_k$ .
- 3. For each of  $(M_j, M_k)$  pairs such that  $C_j = C_k$ , ask for a MAC pair of  $M_j||P$ and  $M_k||P$ , where P is some non-empty string. If there is at least one MAC pair that collides in this step, output the MAC algorithm = HMAC.

This attack requires about  $2^{l/2}$  messages and works with a probability of 0.63 by the birthday paradox when the MAC algorithm is HMAC (this is due to the fact that if there exists at least one message pair  $(M_i, M_k)$  such that their outputs of  $h_2$  or  $H_2$  are same, this attack always works). This attack can also easily be converted into a general forgery attack on HMAC. Once we get a MAC pair that collides in Step 3, we again ask for the corresponding MAC of  $M_j||P||P'$ , denoted C, where  $P'$  is some non-empty string. We can then construct a forgery, i.e., a new message  $M_k||P||P'$  with a valid MAC, i.e., C with the same success rate.

These general attacks make distinguishing and forgery attacks on HMAC which require more than  $2^{l/2}$  message queries have not much advantage. We thus consider attacks of distinguishing HMAC from a random function, and forgery attacks on HMAC which work with a data complexity of less than  $2^{l/2}$ messages. In addition to these two kinds of attacks, we also consider attacks of distinguishing instantiated HMAC (by existing hash functions) from HMAC with a random function. In these attacks it does not matter whether or not they require more than  $2^{l/2}$  message queries, since there does not exist a general attack based on the birthday paradox which can distinguish HMAC with existing hash functions from HMAC with a random function. For the clarification we denote the first and second distinguishing attacks by distinguishing-R and *distinguishing-H attacks*, respectively. The distinguishing- $R$  attack is useful when the cryptanalyst wants to check whether output strings are produced from HMAC (in this case, the cryptanalyst does not know whether the output producing algorithm is  $HMAC$ ), while the distinguishing- $H$  attack is useful when the cryptanalyst wants to check which cryptographic hash function is embedded in HMAC (in this case, the cryptanalyst somehow already knew that the output producing algorithm is HMAC, for instance, by the distinguishing- $R$  attack, but does not know the underlying hash function in HMAC).

### **4 Distinguishers of HMAC**

In this section we present two distinguishers of the general structure of HMAC, which can lead to distinguishing or forgery attacks if HMAC is instantiated with some cryptographic hash function with a low difference propagation. These two distinguishers, called *differential* and *rectangle distinguishers*, are both built based on internal collisions. We focus on HMAC with one-block messages, which is the main target in our attacks.

#### **4.1 Differential Distinguisher of HMAC**

By using MAC collisions we construct a differential distinguisher of HMAC. It works as follows:

- $-$  Choose a message  $M_i$  at random and compute another message  $M'_i = M_i \oplus \alpha$ , where  $M_i$  has the same length as  $\alpha \neq 0$ .
- With a chosen message attack, obtain the MAC values  $C_i = \text{HMAC}(K, M_i)$ and  $C_i' = \text{HMAC}(K, M_i').$
- $-$  Check if  $C_i \oplus C'_i = 0$ .

Assuming that the values  $h_1(IV, K \oplus ipad)$  are uniformly distributed for a given key K, the last test holds with a probability<sup>[1](#page-4-0)</sup> of approximately q, where  $q =$  $Pr_{X,I} [h_2(I, X) \oplus h_2(I, X \oplus \alpha) = 0].$  On the other hand, for a random function or  $HMAC$  with a random function<sup>[2](#page-4-1)</sup>, the last test holds with a probability of approximately 2−<sup>l</sup> . Hence, we have the following differential distinguisher of HMAC.

**Proposition 1.** [A Differential Distinguisher of HMAC] Assume that the output values of the function  $h_1$  are distributed uniformly at random. Then HMAC can be distinguished from a random function and from HMAC with a random function if  $q > 2^{-l}$ , where  $q = Pr_{X,I} [h_2(I, X) \oplus h_2(I, X \oplus \alpha) = 0].$ 

In order for this differential distinguisher to be used in distinguishing- $R$  and forgery attacks, the probability q should be larger than  $2^{-l/2}$ , which makes possible for those attacks to work with less than  $2^{l/2}$  message queries (details are described in Sect. 6).

### **4.2 Rectangle Distinguisher of HMAC**

The rectangle distinguisher of HMAC can be built by the rectangle attack which is widely used in analyzing block ciphers [\[4\]](#page-13-6). In block ciphers the rectangle attack can be mounted based on their bijectivity. However, in MACs it can exploit the non-bijectivity, i.e., two different messages may correspond to a same MAC value or a same intermediate value (an internal collision). We use this nonbijective property to devise our rectangle distinguisher of HMAC. Our rectangle distinguisher of HMAC works as follows (refer to Fig. [2\)](#page-5-0):

- Choose two messages  $M_i$  and  $M_j$  at random and compute two other messages  $M'_i = M_i \oplus \alpha$  and  $M'_j = M_j \oplus \alpha$ , where  $M_i$  and  $M_j$  both have the same length as  $\alpha \neq 0$ .
- With a chosen message attack, obtain the MAC values  $C_i = \text{HMAC}(K, M_i)$ ,  $C_i' = \text{HMAC}(K, M_i'), C_j = \text{HMAC}(K, M_j)$  and  $C_j' = \text{HMAC}(K, M_j').$
- $-$  Check if  $C_i ⊕ C_j = C'_i ⊕ C'_j = 0$  or  $C_i ⊕ C'_j = C'_i ⊕ C_j = 0$ .

We denote by  $X_i$ ,  $X'_i$ ,  $X_j$  and  $X'_j$  the outputs of  $h_2 \circ h_1$  for the messages  $M_i$ ,  $M'_i$ ,  $M_j$  and  $M'_j$ , respectively (see Fig. [2\)](#page-5-0). Note that in Fig. [2](#page-5-0)  $K \oplus ipad$ and  $K \oplus opad$  are inserted into the message parts of the functions  $h_1$  and  $h_3$ ,

<span id="page-4-0"></span><sup>&</sup>lt;sup>1</sup> In fact, the last test holds with a probability of approximately  $q + (1 - q) \cdot 2^{-l}$ . Because even if the  $M_i$  and  $M'_i$  do not cause a collision after the function  $h_2$ , their MAC values can still have a same value. However, in the computation of a probability for our differential distinguisher we do not consider this case.

<span id="page-4-1"></span> $^2$  From  $[1]$  we know that HMAC with a random function behaves like a random function.



**Fig. 2.** A Rectangle Distinguisher of HMAC  $(M_i \oplus M'_i = M_j \oplus M'_j = \alpha)$ 

<span id="page-5-0"></span>respectively. In order to compute the probability to satisfy the last test we should consider the following probabilities:  $p = Pr_{X,I} [h_2(I, X) \oplus h_2(I, X \oplus \alpha) = \beta]$  and  $\hat{p} = \sqrt{\sum_{\beta}(p^2)}$ .

Assuming that the values  $h_1(IV, K \oplus ipad)$  are uniformly distributed for a given key K, we get  $X_i \oplus X'_i = X_j \oplus X'_j = \beta$  with probability  $p^2$ . Since the function  $h_2$  is not a permutation (here, the domain of  $h_2$  is the message space and its co-domain is the space of hash values), we expect  $X_i \oplus X_j = 0$  with probability  $2^{-l}$  under the assumption that the output values of  $h_2$  are distributed uniformly at random. Once we get  $X_i \oplus X'_i = X_j \oplus X'_j = \beta$  and  $X_i \oplus X_j = 0$ , we have the following equation:

$$
X'_i \oplus X'_j = (X_i \oplus \beta) \oplus (X_j \oplus \beta) = X_i \oplus X_j = 0
$$

These equations allow us to get  $C_i \oplus C_j = C'_i \oplus C'_j = 0$  and thus the probability<sup>[3](#page-5-1)</sup> of satisfying  $C_i \oplus C_j = C'_i \oplus C'_j = 0$  is approximately

<span id="page-5-1"></span><sup>&</sup>lt;sup>3</sup> Note that the probability of satisfying  $C_i \oplus C_j = C'_i \oplus C'_j = 0$  is slightly larger than  $\hat{p}^2 \cdot 2^{-l}$ . Because even if the  $X_i$  and  $X_j$  (or the  $X'_i$  and  $X'_j$ ) are not the same, still there is a chance to have  $C_i \oplus C_j = C'_i \oplus C'_j = 0$ . However, we believe that a simplified analysis is sufficient for the computation of the probability for our rectangle distinguisher.

$$
\sum_{\beta} p^2 \cdot 2^{-l} = \hat{p}^2 \cdot 2^{-l}.
$$

Similarly, we get  $X_i \oplus X'_j = 0$  with a probability of  $2^{-l}$  and thus  $C_i \oplus C'_j =$  $C_i' \oplus C_j = 0$  holds with the same probability  $\hat{p}^2 \cdot 2^{-l}$ .

On the other hand, for a random function or HMAC with a random function,  $C_i \oplus C_j = C'_i \oplus C'_j = 0$  and  $C_i \oplus C'_j = C'_i \oplus C_j = 0$  hold with a probability of approximately  $2^{-2l}$ , respectively, since each requires a 2l-bit restriction to be satisfied. Hence, we have the following rectangle distinguisher of HMAC.

**Proposition 2.** [A Rectangle Distinguisher of HMAC] Assume that the output values of the functions  $h_1$  and  $H_2$  are distributed uniformly at random. Then  $HMAC\ can\ be\ distinguished\ from\ a\ random\ function\ and\ from\ HMAC\ with\ a$ random function if  $\hat{p}^2 \cdot 2^{-l} > 2^{-2l}$ , i.e.,  $\hat{p} > 2^{-l/2}$ , where  $\hat{p} = \sqrt{\sum_{\beta}(p^2)}$  and  $p = Pr_{X,I} [h_2(I, X) \oplus h_2(I, X \oplus \alpha) = \beta].$ 

Our rectangle distinguisher cannot be used in distinguishing- $R$  and forgery attacks, since its required data complexity is always larger than  $2^{l/2}$  messages (details are described in Sect. 6). This is due to the fact that the rectangle probability is always less than or equal to  $2^{-l}$ .

Unlike the differential distinguisher of HMAC, the rectangle distinguisher uses a number of differentials without any restriction for output differences, while its requirement to work is more expensive than that of the differential distinguisher, i.e., it uses probability  $2^{-l/2}$  instead of  $2^{-l}$  for its comparison. If it is easy to get some nonzero output difference from the compression function of the underlying hash function, but it is difficult to get a zero output difference, i.e., a collision, then this rectangle distinguisher would be useful.

The success of our two distinguishers for HMAC depends significantly on the strength of  $h_2$ , which means the distinguishers do not depend strongly on the properties of  $h_1$ ,  $h_3$  and  $h_4$ . Even if  $h_1$ ,  $h_3$  and  $h_4$  employs cryptographically strong compression functions (even iterated hash functions), our distinguishers can still work if  $h_2$  has a low difference propagation.

# **5 Differentials on HAVAL, MD4, MD5, SHA-0, SHA-1**

First, we check how many rounds of the compression functions of HAVAL, MD4, MD5, SHA-0 and SHA-1 can be used for  $h_2$  in our rectangle distinguisher, i.e., we investigate for how many rounds of each compression function  $\hat{p} > 2^{-l/2}$ holds. Second, we present differential distinguishers of MD4, SHA-0 and SHA-1 with probabilities q such that  $q > 2^{-l}$  or  $q > 2^{-l/2}$ . Here, we do not take into account multi-block differentials, for they are inferior to one-block differentials in HMAC. See the full version of this paper [\[9\]](#page-13-7) for more details of multi-block differentials.

#### **5.1 Differentials for Rectangle Distinguishers**

In order to compute the number of rounds for each compression function such that  $\hat{p} > 2^{-l/2}$ , we investigate a differential with probability p from which the probability  $\hat{p}$  can be estimated. We first consider the compression function of 3-pass HAVAL.

In the compression function of HAVAL we insert a one-bit difference to two message words to produce a collision after the first pass with a high probability. This enables us to get probability-one differentials through many rounds in the first and second passes. In more detail, if we denote by  $r_1$ ,  $r_2$ ,  $r_3$ ,  $r_4$ ,  $r_5$  and  $r<sub>6</sub>$  the round numbers involved in two such message words in the three passes where  $r_1 < r_2 < \cdots < r_6$ , we can construct a 96-round differential with the following probability: for the rounds  $0 \sim r_1$  probability 1, for each of the rounds  $(r_1 + 1) \sim r_2$  probability  $2^{-1}$ , for the rounds  $(r_2 + 1) \sim (r_3 - 1)$  probability 1, for each of the rounds  $r_3 \sim r_4$  probability  $2^{-1}$ , for each of the rounds  $(r_4 + 1) \sim (r_5 - 1)$  probability  $2^{-2}$ , for each of the rounds  $r_5 \sim r_6$  probability  $2^{-3}$  and for each of the rounds  $(r_6+1) \sim 95$  probability  $2^{-4}$  (this can be done by the computation of differential probabilities derived from the differential distributions of Boolean functions and the use of both XOR and modular additions). These probabilities may be slightly different according to in which message word between the two a difference  $0x80000000$  is given. But the total probability is the same:  $2^{-(r_2-r_1+r_4-r_3+1+2(r_5-r_4-1)+3(r_6-r_5+1)+4(95-r_6))}$ .

As a result of an exhaustive search<sup>[4](#page-7-0)</sup>, inserting a one-bit difference to the third and eleventh message words provides the best probability  $p = 2^{-102}$ . See Table [1](#page-8-0) for more details. In Table 1  $e_i$  represents a 32-bit word that has  $0's$  in all bit positions except for bit i and  $e_{i_1,\dots,i_k}$  represents  $e_{i_1} \oplus \dots \oplus e_{i_k}$  (in our notation the leftmost bit is referred to as the 31-th bit, i.e., the most significant bit). Note that we use the XOR difference as the measure of difference and in the computation of the probability  $p$  in Table [1](#page-8-0) the modular additions of the unknown initial value and the last output value are considered. In our analysis we take into account the probability that the last output difference is preserved through the final modular additions.

In order to calculate  $\hat{p}$  we should sum the square of the probability of all differentials with message difference  $\alpha$ . However, it is computationally infeasible and thus we have carried out experiments on the last three rounds (rounds 93  $\sim$ 95) to estimate a lower bound for  $\hat{p}$  (our simulation is based on the assumption that chosen message pairs follow the first 93-round differential in Table [1\)](#page-8-0). For this work, we have randomly chosen a number of  $IVs$  with  $2^{28}$  message pairs  $M_i$ ,  $M_i^*$  and  $2^{28}$  input pairs of round 93  $I_i$ ,  $I_i^*$  each and computed  $M_i = M_i \oplus \alpha$ ,  $M_i^{*'} = M_i^* \oplus \alpha$  and  $I_i' = I_i \oplus \delta$  and  $I_i^{*'} = I_i^* \oplus \delta$ , where  $\alpha$  is the message difference and  $\delta$  is the input difference of round 93 in Table [1.](#page-8-0) We have then encrypted through rounds 93 ~ 95  $I_i$ ,  $I'_i$ ,  $I_i^*$  and  $I_i^{*'}$  with  $M_i$ ,  $M'_i$ ,  $M_i^*$  and  $M_i^{*'}$  to obtain outputs  $O_i$ ,  $O'_i$ ,  $O_i^*$  and  $O_i^{*'}$ . Finally, we have checked if  $(O_i + IV) \oplus (O_i^{'} + IV) =$ 

<span id="page-7-0"></span><sup>&</sup>lt;sup>4</sup> The exhaustive search has experimentally been done by considering all possible  $r_1$ ,  $r_2$ ,  $r_3$ ,  $r_4$ ,  $r_5$  and  $r_6$  which can produce a collision after the first pass.

<span id="page-8-0"></span>

Round (i)	$\Delta A^i$	$\Delta \vec{B}^i$	$\Delta C^i$	$\Delta D$	$\Delta E^i$	$\Delta \vec{F}^i$	$\varDelta G^i$	$\Delta H^i$	$\Delta m$	Prob.
Ō	0	0	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	0	ī
ī $\overline{2}$	$\overline{0}$ $\mathbf 0$	$\overline{0}$ 0	$\overline{0}$ $\mathbf 0$	0 o	0 o	$\overline{0}$ 0	0 O	$\overline{0}$ 0	$\overline{0}$ $\Delta m^2$	ī $\mathbf{1}$
3	0	0	0	0	0	$\mathbf 0$	o	e31	$^{e_{31}}$ $\mathbf 0$	$\overline{2}$
$\overline{\bf 4}$	0	0	0	0	0	0	e <sub>31</sub>	$\bf{0}$	0	1 $\overline{2}$
5	0	0	0	0	0	$e_{31}$	0	0	0	$\overline{2}$
6	0	0	0	0	e31	0	0	0	$\bf{0}$	$\overline{2}$
7	0	0	0	$e_{31}$	0	$\mathbf 0$	0	0	$\overline{O}$	$\overline{2}$
8 9	$\overline{0}$ $\overline{0}$	$\overline{0}$	e31 $\overline{0}$	0 $\overline{0}$	$\overline{0}$ 0	$\overline{0}$ $\overline{0}$	$\overline{0}$ Ō	$\overline{0}$ $\overline{0}$	$\overline{0}$ $\overline{0}$	$\overline{2}$ $\overline{2}$
$^{10}$		e3. 0	0	0	0	0	0	0	$\Delta m^{10}$ $e_{20}(=$	ı $\overline{2}$
$\overline{11}$	$\frac{e_3}{0}$	0	$\overline{0}$	0	0	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	ī
Ì	$\ddot{\phantom{a}}$	t	$\ddot{\cdot}$	t	t	t	t	Ì	$\ddot{\cdot}$	$\ddot{\phantom{a}}$
44	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	ī -1
45 46	$\mathbf 0$ $\mathbf 0$	o $\mathbf 0$	$\overline{0}$ $\mathbf 0$	$\overline{0}$ o	$\overline{\mathbf{0}}$ o	$\mathbf 0$ 0	Ō o	$\overline{\mathbf{0}}$	$\Delta m^{10}$ $_{e_{20}}$ o	$\overline{2}$ Л $\overline{2}$
47	0	0	0	0	0	0	e <sub>20</sub>	<u>e20</u> 0	0	ı $\overline{2}$
48	0	0	0	0	0	$_{20}$	0	0	0	$\overline{2}$
49	0	0	0	0	$e_{20}$	0	0	0	0	$\overline{2}$ ٦
50	0	0	0	$e_{20}$	0	0	0	0	O	$\overline{2}$ ı
$\overline{51}$	$\overline{0}$	$\overline{0}$	$e_{20}$	0	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{2}$ I
52	$\overline{0}$	e20	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{2}$ т I
53 54	$e_{20}$ O	0 O	$\overline{0}$ 0	0 0	0 0	0 0	$\overline{0}$ o	0	$\overline{0}$ O	$\overline{2}$ ı $\overline{a}$
55	0	0	0	0	0	0	eg	e9 0	0	ı $\overline{2}$
56	$\mathbf 0$	$\mathbf 0$	O	O	0	eg	$\mathbf 0$	$\mathbf 0$	$\mathbf 0$	1 $\overline{2}$
57	0	0	0	0	eg	0	0	0	0	I $\overline{2}$
58	0	$\overline{0}$	$\overline{0}$	eg	0	$\overline{0}$	$\overline{0}$	0	$\overline{0}$	٦ $\overline{2}$
59	$\overline{0}$	$\overline{0}$	eg	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	л $\overline{2}$ τ
60	$\overline{O}$	еg	$\overline{O}$	$\overline{O}$	$\overline{0}$	$\overline{O}$	$\overline{0}$	$\overline{0}$	2. $\Delta m$ e31	$\overline{2}$ 2
61 62	e9 $\overline{0}$	0 $\overline{0}$	$\overline{0}$ $\overline{0}$	$\overline{0}$ $\overline{0}$	$\overline{0}$ $\overline{0}$	$\overline{0}$ $\overline{0}$	0	$^{e_{31}}$	0 0	2 2 $\overline{2}$
63	0	0	0	0	0	e31	e31 e30	$\overline{e_{30}}$ 0	$\mathbf 0$	$\overline{2}$
64	0	0	0	0	e31	e <sub>30</sub>	0	0	O	$\overline{2}$
65	0	0	0	$^{e_3}$	e30	0	O	O	O	$\overline{2}$
66	0	0	$e_{31}$	$e_{30}$	0	0	O	0	$\mathbf 0$	$\overline{2}$
67	$\overline{0}$	$^{e_{31}}$	$e_{30}$	0	0	$\overline{0}$	0	$\overline{0}$	0	$\overline{2}$
68	$e_{31}$	$e_{30}$	0	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{2}$
69 70	e30 O	0 0	0 0	0 0	0 0	$\overline{0}$ O	Ō	$_{e_{20}}$	$\Omega$ 0	$\overline{2}$ $\overline{a}$
71	0	0	0	0	0	$_{\rm e_{20}}$	$\epsilon_{20}$ e <sub>19</sub>	$e_{19}$ $\mathbf 0$	0	$\overline{2}$
72	0	0	0	$\overline{0}$	$^{e_{20}}$	$^{e_{19}}$	0	0	0	D $\overline{2}$
73	0	0	0	$^{e_{20}}$	$_{19}$	0	0	0	0	$\overline{\phantom{0}}$
74	0	$\overline{0}$	$_{e_{20}}$	$e_{19}$	0	$\mathbf 0$	$\overline{0}$	0	O	$\overline{2}$
75	$\overline{0}$	$_{e_{20}}$	e <sub>19</sub>	0	$\overline{0}$	$\mathbf 0$	o	$\mathbf 0$	$\mathbf 0$	D $\overline{2}$ Ð
76	$e_{20}$	$_{19}$	$\overline{0}$ $\overline{0}$	$\overline{0}$ $\overline{0}$	Ō 0	$\overline{0}$	Ō	$\overline{0}$	$\overline{0}$	$\overline{2}$ D $\overline{2}$
77 78	e <sub>19</sub> 0	0 0	0	0	0	0 0	0 e9	. eg	0 0	2 $\overline{2}$
79	0	0	0	0	0	$_{eg}$	$^{e_8}$	$\frac{e_8}{}$ 0	0	2 $\overline{2}$
80	0	0	O	0	eg	$^{e_8}$	O	O	O	2 $\overline{2}$
81	O	0	O	eg	eg	0	O	O	O	2 $\overline{2}$
82	0	0	e9	$\frac{e}{3}$	0	$\mathbf 0$	0	0	0	2 $\overline{2}$ ۰,
$\overline{83}$	$\overline{0}$	$e_9$	$^{e_8}$	$\Omega$	$\overline{0}$	$\mathbf 0$	0	0	$\overline{0}$	$\overline{2}$ 2
84 85	e9	e8 $\overline{O}$	0 $\mathbf 0$	$\overline{0}$ $\mathbf 0$	$\overline{0}$ o	$\mathbf 0$ $\mathbf 0$	$\overline{0}$ $\mathbf 0$	$\overline{0}$	0 $\overline{O}$	$\overline{2}$ 2 $\overline{2}$
86	e8 0	0	0	0	0	0	e <sub>30</sub>	e30 $e_{29}$	0	2 $\overline{2}$
87	0	0	0	0	0	$^{e_{30}}$	e <sub>29</sub>	0	0	2 $\overline{2}$
88	0	0	$\overline{0}$	$\overline{0}$	e30	e <sub>29</sub>	0	0	0	2 $\overline{2}$
89	0	0	0	e30	e29	0	0	0	0	2 $\overline{2}$
90	$\mathbf 0$	$\mathbf 0$	e30	$_{\rm eqc}$	0	$\mathbf 0$	$\mathbf 0$	$\mathbf 0$	$\mathbf 0$	2 $\overline{2}$ ×
91	0	$_{\rm e_{30}}$	$e_{29}$	$\mathbf 0$	0	$\mathbf 0$	$\mathbf 0$	o	$\Delta m^{10}$ $e_{20}(=$	$\overline{2}$ 3
92 93	$^{e_{30}}$	$e_{29}$ 0	$\overline{0}$ 0	$\overline{0}$ 0	$\overline{0}$ 0	$\overline{0}$ 0	$\overline{0}$	$^{e_{20}}$ $e_{19}$	$\overline{0}$ $\overline{0}$	$\overline{2}$ 3 $\overline{2}$
94	e <sub>29</sub> 0	0	0	0	0	$_{20}$	$e_{20}$ e <sub>19</sub>	$^{e_{18}}$	0	3 $\mathbf{2}^{\mathbf{1}}$
95	0	0	0	0	$_{\rm e_{20}}$	<u>e19</u>	<u>e18</u>	$\mathbf 0$	$\Delta m^2$ $e_{31}(=$	3 $\mathbf{2}^{\bullet}$
96	0	0	0	$^{e_{20}}$	$^{e}19$	<u>e 18</u>	0	$^{e_{31}}$	0	4 $\overline{2}$
$0 \sim 95$			$\mathcal{D}$ $\boldsymbol{p}$ =	102	$\overline{2}$ p	99.1			HAVAL) $(3$ pass	
97	$\overline{0}$	$\overline{0}$	$e_{20}$	$e_{19}$	$^{e_{18}}$	$\overline{0}$	$e_{31}$	$\overline{0}$	$\overline{0}$	4 $\overline{2}$
98	$\overline{0}$	$^{e_{20}}$	$e_{19}$	$^{e_{18}}$	0	e31	0	$\overline{0}$	$\overline{0}$	4 $\overline{2}$
99	$_{\rm e_{20}}$	e <sub>19</sub>	e18	$\overline{\mathbf{0}}$	. 831	$\overline{\mathbf{0}}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{2}$
100	$_{e_{19}}$	$_{28}$	$\mathbf 0$	e31	0	$\mathbf 0$	$\mathbf 0$	eg	$_{e31}$ (: $\Delta m$	4 $\overline{a}$
101	<u>e18</u>	0	<u>e31</u>	0	0	0	e9	e <sub>8.31</sub>	0	5 $\mathbf{2}$
102	0	$^{e_{31}}$	0	0	0	e9	$^{e_{8,31}}$	$^{\circ}$		
101 $0 \sim$	124.4 127 $\overline{2}$ $\overline{2}$ HAVAL) $\hat{p}$ (reduced 4 pass $\boldsymbol{p}$									

**Table 1.** A Differential of HAVAL

 $(O_i^* + IV) \oplus (O_i^{*'} + IV)$ . In our experiments we have observed that the number of such quartets was ranging 320  $\sim$  2130 for each IV. This simulation result suggests that the square of the probability  $\hat{p}$  for rounds 93  $\sim$  95 is approximately  $2^{-18.2}$  and thus we can estimate the probability  $\hat{p} \approx 2^{-9.1} \cdot 2^{-90} = 2^{-99.1}$  since the differential probability for rounds  $0 \sim 92$  in Table [1](#page-8-0) is  $2^{-90}$ . Furthermore, we can extend this differential up to [1](#page-8-0)01 rounds such that  $\hat{p} > 2^{-128}$ . See Table 1 for this extension. We have also performed a series of simulations on the last two rounds and from the simulation result we can estimate  $\hat{p} \approx 2^{-124.4}$  for rounds  $0 \sim 101$ .

Similarly, we have investigated differentials on the compression function of MD5 with high probabilities by inserting a one-bit difference in two or three message words to produce a collision after the first pass. As a result, we can construct a 33-round differential on MD5 with probability  $2^{-69}$ , which can be used to construct differentials with probability  $\hat{p}$ . See [\[9\]](#page-13-7) for details of our reduced MD5 differential. Our investigations on HAVAL and MD5 have started from the assumption that low-weight differentials work out best when we can not use neutral bits and message modifications. However, still there is a possibility that HAVAL and MD5 have stronger differentials which can be derived by other methods.

For MD4, SHA-0 and SHA-1, we have used the previous differentials in our distinguishers, i.e., a 48-round differential on MD4 with probability  $2^{-56}$  in [\[18\]](#page-14-4), a 65-round differential on SHA-0 with probability 2−<sup>78</sup> in [\[5,](#page-13-4)[6\]](#page-13-5) and a 43-round differential on SHA-1 with probability 2−<sup>80</sup> in [\[6\]](#page-13-5). The 43-round differential on SHA-1 is an extended one for the 34-round differential described in [\[6\]](#page-13-5), and the computations of differential probabilities on SHA-0 and SHA-1 are recomputed[5](#page-9-0). See [\[9\]](#page-13-7) for the recomputed differentials of SHA-0 and SHA-1. We have also carried out the same experiments on the last few rounds to estimate each  $\hat{p}$  and from our simulations we can estimate  $\hat{p} \approx 2^{-56}$ ,  $2^{-60.6}$ ,  $2^{-78}$  and  $2^{-73.4}$  for 48-round MD4, 33-round MD5, 65-round SHA-0 and 43-round SHA-1, respectively.

#### **5.2 Differentials for Differential Distinguishers**

As stated above, our differential distinguisher works based on a differential which causes a zero difference, i.e., a collision, after the function  $h_2$ . We use the foregoing differentials or the previously known differentials on MD4, SHA-0 and SHA-1 in our distinguishing and forgery attacks:

**–** For SHA-0, the 65-round differential with probability 2−<sup>78</sup> of Table 5 in [\[9\]](#page-13-7) can be extended into a 82-round differential with probability  $2^{-98}$  ( $\approx q$ ), which causes a collision (this extended differential has appeared in [\[5\]](#page-13-4), but the differential probability is lower than that in [\[5\]](#page-13-4) since we cannot use neutral bits.)

<span id="page-9-0"></span> $5$  The main difference of the computations of differential probabilities between [\[5](#page-13-4)[,6\]](#page-13-5) and this paper is the use of neutral bits. In the SHA-0 and SHA-1 initial values are known, which enables us to use neutral bits on message pairs to improve differential probabilities. However, in our analysis of HMAC initial values are determined by a secret key  $K$ , which implies they are unknown.

- **–** For SHA-1, the first 34-round differential with probability 2−<sup>52</sup> of Table 6 in [\[9\]](#page-13-7) can be used as our differential distinguisher.
- For the full MD4, there exists a differential with probability  $2^{-56}$  (≈ q), which causes a zero output difference from an unknown initial value [\[18\]](#page-14-4).
- **–** For the full SHA-0, there exists a differential with probability 2−<sup>107</sup> (≈ q), which causes a zero output difference from an unknown initial value [\[15](#page-14-1)[,17\]](#page-14-3).

# **6 Distinguishing and Forgery Attacks on HMAC**

We use the probabilities  $\hat{p}$  and q to show two distinguishing and a forgery attacks on the HMAC construction, and apply these attacks to HMAC based on HAVAL, MD4, MD5, SHA-0 and SHA-1.

Our first distinguishing attack on HMAC using  $\hat{p}$  and a rectangle distinguisher is described as follows:

- 1. Collect  $2^{(l+1)/2} \cdot \hat{p}^{-1}$  message pairs  $(M_i, M'_i)$  with difference  $\alpha$ , where all the  $M_i$  and  $M'_i$  have the same bit-length t.
- 2. With a chosen message attack scenario, ask for MAC pairs of all the  $(M_i, M'_i)$ . We denote the corresponding MAC pairs by  $(C_i, C'_i)$ . (We assume that the MAC algorithm is either an instantiated HMAC or a random function (or HMAC with a random function) which maps from  $t$  bits to  $l$  bits.)
- 3. Check if  $C_i \oplus C_j = C'_i \oplus C'_j = 0$  or  $C_i \oplus C'_j = C'_i \oplus C_j = 0$  for all i, j such that  $1 \leq i < j \leq 2^{(l+1)/2} \cdot \hat{p}^{-1}$ . If there is at least one MAC quartet that satisfies this test, output the MAC algorithm = HMAC, otherwise, output the MAC algorithm = a random function (or HMAC with a random function).

The data complexity of this attack is  $2^{1+((l+1)/2)} \cdot \hat{p}^{-1}$  chosen messages and this attack requires a memory of  $2^{1+(l+1)/2} \cdot \hat{p}^{-1}$  l-bit blocks for storing all the MAC values. The time complexity of this attack is dominated by Step 1 (the data collection time) and Step 3, which seeks colliding MAC quartets. Since it can be done efficiently by sorting the MAC pairs  $(C_i, C'_i)$ 's by  $C_i$ 's, the time complexity of this attack is thus a fraction of the time required to compute the MAC values for the chosen messages (Step 1).

We now analyze the success rate of this attack. In Step 1 the  $2^{(l+1)/2} \cdot \hat{p}^{-1}$ message pairs form  $2^l \cdot \hat{p}^{-2}$  message quartets  $((M_i, M'_i), (M_j, M'_j))$  corresponding to MAC quartets  $((C_i, C'_i), (C_j, C'_j))$  for  $1 \leq i < j \leq 2^{(l+1)/2} \cdot \hat{p}^{-1}$ . Since for HMAC  $C_i \oplus C_j = C'_i \oplus C'_j = 0$  holds with a probability of  $2^{-l} \cdot \hat{p}^2$ , and  $C_i \oplus C'_j =$  $C_i' \oplus C_j = 0$  also holds with the same probability (this probability has been computed in Sect. 4), the expected number of MAC quartets satisfying the last test is 2 (=  $(2^l \cdot \hat{p}^{-2}) \cdot (2^{-l} \cdot \hat{p}^2) + (2^l \cdot \hat{p}^{-2}) \cdot (2^{-l} \cdot \hat{p}^2)$ ). On the other hand, for a random function (or HMAC with a random function),  $C_i \oplus C_j = C'_i \oplus C'_j = 0$ holds with a probability of  $2^{-2l}$ , and  $C_i \oplus C'_j = C'_i \oplus C_j = 0$  also holds with the same probability and thus the expectation of satisfying the test is  $2^{-l+1} \cdot (\hat{p}^{-2}) (=$  $2^{-2l} \cdot (2^l \cdot \hat{p}^{-2}) + 2^{-2l} \cdot (2^l \cdot \hat{p}^{-2})$ . Hence, the success rate of this attack is

$$
\frac{1 - (1 - 2^{-l} \cdot \hat{p}^2)^{2^{l+1} \cdot \hat{p}^{-2}}}{2} + \frac{(1 - 2^{-2l})^{2^{l+1} \cdot \hat{p}^{-2}}}{2} \approx \frac{1 - e^{-2}}{2} + \frac{e^{-2^{-l+1} \cdot \hat{p}^{-2}}}{2}.
$$

Hash Function	Type of Distinguisher	Type of Attack	h <sub>2</sub> #R	Probability of Distinguisher	Data Complexity	<b>Success</b> Rate
3-pass HAVAL (96 rounds)	$\mathbf{R}^\dagger$	Distinguishing	96	$\hat{\mathbf{p}} = 2^{-99.1}$	228.6	0.93
4-pass HAVAL $(128 \text{ rounds})$	$_{\rm R}$	Distinguishing	102	$\hat{p} = 2^{-124.4}$	$2^{253.9}$	0.93
MD4 48 rounds)	$\mathbf{R}^\dagger$ $\mathbf{D}^\dagger$	Distinguishing Forgery	48 48	$\mathbf{\hat{p}}=2^{-56}$ ${\bf q}={\bf 2}^{-56}$	$2^{121.5}$ $2^{58}$	0.93 0.93
MD5 $(64$ rounds)	R	Distinguishing	33	$\hat{p} = 2^{-60.\overline{6}}$	2126.1	0.92
$SHA-0$ (80 rounds)	$_{\rm R}$ $\mathbf{D}^\dagger$ $\mathbf{D}^\dagger$ D D	Distinguishing Distinguishing Distinguishing Forgery Forgery	65 82 80 54 65	$\hat{p} = 2^{-78}$ ${\bf q} = {\bf 2}^{-98}$ $\mathbf{q}=\mathbf{2}^{-107}$ $q = 2^{-61}$ $q = 2^{-78}$	$2^{159.5}$ $2^{100}$ $2^{109}$ $2^{63}$ $2^{80}$	0.87 0.93 0.93 0.93 0.93
$SHA-1$ 80 rounds)	R D	Distinguishing Forgery	43 34	$\hat{p} = 2^{-73.4}$ $q = 2^{-51}$	$2^{154.9}$ $2^{53}$	0.93 0.93

<span id="page-11-0"></span>**Table 2.** Distinguishing and forgery attacks on HMAC with HAVAL, MD4, MD5, SHA-0 and SHA-1

 $\dagger$ : the attacks can work on HMAC based on full-round (or extended-round) hash functions. R: Rectangle, D: Differential, #R: the number of rounds

Data complexity is the amount of chosen messages

In the rectangle attacks, memory complexity is the same as data complexity

Distinguishing attack is to distinguish instantiated HMAC from HMAC with a random function

Here, the first term is approximately 0.43. Our second distinguishing attack on HMAC using  $q$  and a differential distinguisher is described as follows:

- 1. Collect  $2 \cdot q^{-1}$  message pairs  $(M_i, M'_i)$  with difference  $\alpha$ , where all the  $M_i$ and  $M'_i$  have the same bit-length t.
- 2. With a chosen message attack scenario, ask for MAC pairs of all the  $(M_i, M'_i)$ . We denote the corresponding MAC pairs by  $(C_i, C'_i)$ . We assume that the MAC algorithm is either an instantiated HMAC or a random function (or HMAC with a random function) which maps  $t$  bits to  $l$  bits.
- 3. Check if  $C_i \oplus C'_i = 0$ . If there is at least one MAC pair that satisfies this test, output the MAC algorithm  $=$  HMAC, otherwise, output the MAC algorithm  $=$  a random function (or HMAC with a random function).

The data complexity of this attack is  $2^2 \cdot q^{-1}$  chosen messages and this attack does not require any storage, and the time complexity of this attack itself is a fraction of the time required to compute the MAC values for the chosen messages. Similarly, the success rate of this attack is computed as follows:

$$
\frac{1 - (1 - q)^{2 \cdot q^{-1}}}{2} + \frac{(1 - 2^{-l})^{2 \cdot q^{-1}}}{2} \approx \frac{1 - e^{-2}}{2} + \frac{e^{-2^{-l+1} \cdot q^{-1}}}{2}.
$$

Finally, our forgery attack on HMAC using  $q$  and a differential distinguisher is described as follows:

- 1. Run Step 1 in the second distinguishing attack.
- 2. Run Step 2 in the second distinguishing attack, but we assume that the MAC algorithm is an instantiated HMAC.
- 3. Check if  $C_i \oplus C'_i = 0$  and ask for the MAC pair of  $M_i||P$  and  $M'_i||P$ , where  $M_i$  and  $M'_i$  have a same MAC value and P is some non-empty string. If the obtained MAC pair collides, again ask for the MAC value of  $M_i||P||P',$ where  $P'$  is some non-empty string. We denote this obtained MAC value by C. Output C as the MAC value of  $M_i^{\prime}||P||P^{\prime}$ . Otherwise, restart this step until we check all MAC pairs  $(C_i, C'_i)$ .

It is easy to see that this forgery attack works with (almost) the same data complexity and the same success rate as our second distinguishing attack.

We can easily apply these three attacks to HMAC based on HAVAL, MD4, MD5, SHA-0 and SHA-1 by using their probabilities  $\hat{p}$  and q. Table [2](#page-11-0) shows the results of distinguishing and forgery attacks on those instantiations of HMAC[6](#page-12-0). In Table [2](#page-11-0) forgery attacks also imply distinguishing- $R$  and distinguishing- $H$ attacks.

**Note:** Our distinguishing and forgery attacks are also applicable to HMAC in which the four components  $h_1$ ,  $h_2$ ,  $h_3$ ,  $h_4$  are instantiated with different compression functions (see for example the pseudorandom functions of SSL 3.0). For example, if HMAC employs full-round MD-5, full-round MD-4, full-round MD5 and full-round MD5 for  $h_1$ ,  $h_2$ ,  $h_3$  and  $h_4$ , respectively, it can be forged with a data complexity of  $2^{58}$  chosen messages. This is due to the fact that our distinguishing and forgery attacks depend only on the function  $h_2$ . Furthermore, the distinguishing and forgery attacks in Table [2](#page-11-0) also work on NMAC based on HAVAL, MD4, MD5, SHA-0 and SHA-1.

# **7 Conclusions**

We have presented differential and rectangle distinguishers on HMAC, which are derived from its structural property. They allow to present distinguishing and forgery attacks on HMAC that can be mounted when HMAC employs hash functions with slow difference propagations. With these distinguishing and forgery attacks we have shown that HMAC with the full versionsof 3-pass HAVAL and SHA-0 can be distinguished from HMAC with a random function, and HMAC with the full version of MD4 can be forged. These distinguishing and forgery attacks have also been applied to HMAC based on reduced versions of MD5 and SHA-1. We have also shown that our distinguishing and forgery attacks can be mounted on NMAC (which is a generalized version of HMAC) with the same complexity. Furthermore, we have shown that our differential and rectangle distinguishers can lead to second-preimage attacks on HMAC and NMAC. All these attacks do not

<span id="page-12-0"></span><sup>6</sup> These attacks are mounted under the assumption that the output values of the functions  $h_1$  and  $h_2$  distribute uniformly over all possible values when K and  $M_i$ are chosen uniformly at random (differential distinguishers are independent of the distributions of the output values of the functions  $h_2$  and  $H_2$ ).

contradict the security proof of HMAC, but they improve our understanding of the security of HMAC based on existing cryptographic hash functions.

Our differential distinguisher on HMAC works only if the underlying hash function has a differential with a zero output difference with probability larger than 2−|hash value<sup>|</sup> . Our rectangle distinguisher on HMAC works only if the underlying hash function has differentials such that the sum of the square of their probabilities is larger than 2<sup>-|hash value|</sup>. Unlike the previous attacks on hash functions, our analysis on the hash function embedded in HMAC should be done under an unknown fixed initial value (which is determined by a secret key). This fact makes difficult to use the recently proposed message modification technique (Wang et al.'s attacks) and neutral-bit technique (Biham et al.'s attacks) in analyzing HMAC based on specific cryptographic hash functions. However, it is interesting to investigate if their methods can be applied to HMAC with some new other techniques when HMAC is instantiated with a specific cryptographic hash function. We expect that the method developed in this paper would be useful for the further analysis of HMAC.

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