Haptic-Auditory Rendering and Perception of Contact Stiffness

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Abstract. This paper presents an experiment on the relative contributions of haptic and auditory information to bimodal judgments of contact stiffness using a rigid probe. Haptic feedback is rendered via a Phantom® Omni™ device, while auditory stimuli are obtained using a physically-based audio model of impact, in which the colliding objects are described as modal resonators that interact through a non-linear impact force. The impact force can be controlled through a stiffness parameter, that influences the contact time of the impact. Previous studies have already indicated that this parameter has a major influence on the auditory perception of hardness/stiffness. In the experiment subjects had to tap on virtual surfaces, and were presented with audio-haptic feedback. In each condition the haptic stiffness had the same value while the acoustic stiffness was varied. Perceived stiffness was determined using an absolute magnitude-estimation procedure: subjects were asked to rate the surfaces on an ordered scale of verbal labels, based on their perceived stiffness. The results indicate that subjects consistently ranked the surfaces according to the auditory stimuli.

1 Introduction

The importance of multimodal feedback in computer graphics and interaction has been recognized for a long time [1] and is motivated by our daily interaction with the world. Streams of information coming from different channels complement and integrate each other, with some modality possibly dominating over the remaining ones, depending on the task [2, 3]. Research in ecological acoustics [4, 5] demonstrates that auditory feedback in particular can effectively convey information about a number of attributes of vibrating objects, such as material, shape, size, and so on.

Recent literature has shown that sound synthesis techniques based on physical models of sound generation mechanisms allow for high quality synthesis and for a high degree of interactivity, since the physical parameters of the sound models can be naturally controlled by the gestures and the actions of a user. Sounds from solids are especially interesting since auditory cues frequently occur when we touch or interact with objects. Sound models for impulsive and continuous contact have been proposed e.g. in [6, 7]. Physically-based sound models of contact have been shown to be effective in conveying information about e.g. material

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properties [8], and have been applied in [9] to the development of an audio-haptic interface for contact interactions.

Bi-modal perception in continuous contact interaction (i.e., scraping or sliding) has been studied by many authors. In a classic work Lederman [10] compared the effectiveness of tactile and auditory information in judging surface roughness, and showed that when both were present the tactile one played the strongest role in determining experimental performance. More recent research by Lederman et al. [11] has focused on bi-modal roughness perception when the surface is explored using a rigid probe rather than with the bare skin, and *vibratory* roughness perception occurs. The results showed that, although tactual dominance is still found, sound plays a more relevant role when using a probe than in the case of direct contact with bare fingers. Guest et al. [12] have also focused on audio-tactile interactions in roughness perception. In their experimental setup, participants were required to make forced-choice discrimination responses regarding the roughness of abrasive surfaces which they touched briefly. Texture sounds were captured by a microphone located close to the manipulated surface and subsequently filtered in various ways before being presented to the participants. The authors investigated how the filtering biased the subjects' judgements. McGee et al. [13] studied bi-modal perception of *virtual* roughness, i.e. roughness of synthetic haptic and auditory textures. The latter were synthesized from the same sinusoidal waveforms used to describe the profiles of the haptic textures, and therefore did not provide a veridical feedback. Nonetheless, experimental results indicated that the presence of auditory feedback affected the likelihood that different textures were successfully judged as different.

Bi-modal perception in impulsive contact (i.e., impact) is apparently less studied. DiFranco et al. [14] studied the effect of auditory feedback on haptic stiffness perception, through headphone reproduction of prerecorded contact sounds between several pairs of objects. Experimental results showed that contact sounds influenced the perception of object stiffness. However the sounds used in [14] were chosen on a purely subjective basis rather than on an analysis of what timbral dimensions are mostly related to auditory perception of contact stiffness. Useful indications about the auditory cues that are most relevant to stiffness/hardness perception come from studies in ecological acoustics [15, 16].

This paper investigates the effectiveness of synthetic impact sounds in modulating the haptic perception of stiffness experienced by a user. In Sect. 2 we present the sound physical model used in the remainder of the paper, and describe how the sound model is integrated into an architecture for audio-haptic rendering. Section 3 reports upon an experiment on bi-modal stiffness perception that makes use of this architecture. Results are discussed in Sect. 4.

2 Impact Sounds

2.1 A Physically-Based Sound Model

When a generic solid object engages in some external interactions (e.g. it is struck, scraped, and so on), the forces at the contact point cause deformations to propagate through the body, and consequently its surfaces to vibrate and emit sound waves. A physically-motivated model for the simulation of vibrating objects is modal synthesis [17, 18], which describes the object as bank of second order damped mechanical oscillators (the *normal modes*) excited by the interaction force. The frequencies and dampings of the oscillators depend on the geometry and the material of the object and the amount of energy transferred to each mode depends on the location of the force applied to the object. Under general hypothesis, and with appropriate boundary conditions, linear partial differential equations describing a vibrating system admit solutions described as superposition of vibration modes. In this sense modal synthesis is physically well motivated and widely applicable. Techniques based on modal synthesis have been exploited by many authors for real-time synthesis of realistic sound effects for interactive simulations (see e.g. [7, 19]).

We have developed a physically-based sound synthesis model of interacting objects, simulated through a modal description. The objects can be coupled through non-linear interaction forces that describe impulsive and continuous contact. While the models described in[7] are linear and based on feed-forward computation, our force models are non-linear and dynamic. These features allow improved interactivity and better quality, at the expense of higher computation loads. In this work we make use of a real-time implementation of the model, realized as a plugin to the open source real-time synthesis environment `pd` (Pure Data[1]). The full model and the implementation details are presented in [6]. Here we only discuss the impact force model.

The audio impact force model [20] is based on an extension of the Hertz theory of normal collision between elastic bodies [21]:

$$f_{\mathrm{A}}(x(t), v(t)) = \begin{cases} k_{\mathrm{A}} x(t)^{\alpha} + \lambda_{\mathrm{A}} x(t)^{\alpha} \cdot v(t) & x > 0, \\ 0 & x \leq 0, \end{cases} \tag{1}$$

where the compression x at the contact point is the difference between the displacements of the two bodies, and $v(t) = \dot{x}(t)$ is the compression velocity. The condition $x > 0$ states that there is actual compression, while the complementary condition says that the two objects are not in contact. The force model (1) includes both an elastic component $k_{\mathrm{A}} x^{\alpha}$ and a dissipative term $\lambda_{\mathrm{A}} x^{\alpha} v$. The latter accounts for viscoelastic losses during collision. The parameter k_{A} in (1) is the force *stiffness* and is in general a function of the mechanical properties of the two bodies, while λ_{A} is the force *damping weight*. Additionally a variable exponent α is introduced, whose value depends on the surface geometry of the contact (e.g., $\alpha = 3/2$ for the particular case of contacting spheres).

2.2 Force Stiffness, Contact Time, Spectral Centroid

In previous studies we have investigated the influence of the impact force parameters on the spectral centroid of the sound attack transient, and on the duration τ

[1] http://crca.ucsd.edu/~msp/

of the contact between the two objects during the stroke. In particular a power-law dependence of the contact time τ on the force stiffness was found [22]: $\tau(k_{\mathrm{A}}) \sim k_{\mathrm{A}}^{-1/\alpha+1}$. A study in [23] on synthetic impact sounds obtained from model (1) provided quantitative results that show a strong correlation between the spectral centroid of the attack transients and the contact time.

The spectral centroid of the attack transient is known to influence the auditory perception of stiffness. Freed [15] has investigated the abilitiy of listening subjects to estimate the hardness of hammers made of various materials, from the sound that they generated when striking metallic pans of varying sizes. His experiments showed that the useful information for mallet hardness rating is contained in the attack transients of the sounds, namely in the first 300 ms of the signals. Loudness and descriptors related to the spectral centroid (average value and temporal variability in the first 300 ms) were used as predictors in a multiple regression analysis, and were found to account for 75% of the variance of the hardness ratings.

Giordano [16] has also investigated auditory perception of collision hardness. He argues that the contact time τ has an influence on hardness perception, and that τ variations are likely to explain at least in part data from [15]. Specifically, an increase in τ determines a decrease in the loudness of the radiated signal, and in the amount of energy at high frequencies (and thus in the spectral centroid), since vibrational modes with a period higher than τ are minimally excited.

In summary, the studies reported in [22, 23] have shown that manipulation of the impact force parameters k_{A} affects in a predictable way the contact time and the average spectral centroid during the attack transient. These parameters in turn have a major influence on the perception of impact hardness. Examples of these effects are provided in Fig. 1.

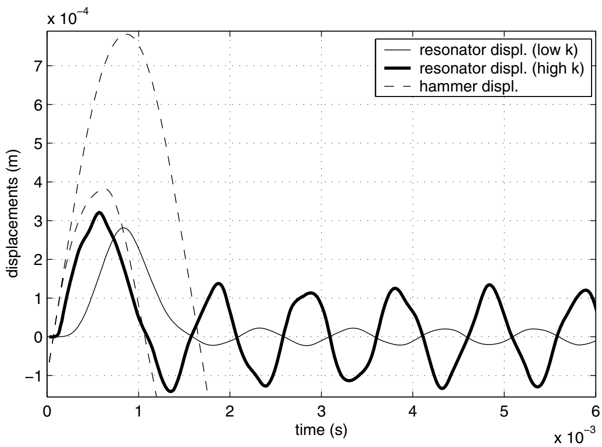

Fig. 1. Examples of transient attacks obtained from the impact model: short vs. long initial bumps, obtained by varying the force stiffness k_{A}

2.3 Audio-Haptic Rendering

The software experimental setup is composed of two processes which exchange information through a shared memory area (see Fig. 2). The first process renders graphics and the haptic freedback, and has been programmed with the Openhaptics™ Toolkit developed by Sensable. An event catching engine driven by a function callback model is adopted to monitor contact events. When such an event occurs, data needed for sound synthesis is copied on the shared memory area. The second process renders contact sounds according to the current physical/geometrical parameters read from the shared memory area, and has been programmed with `pd`.

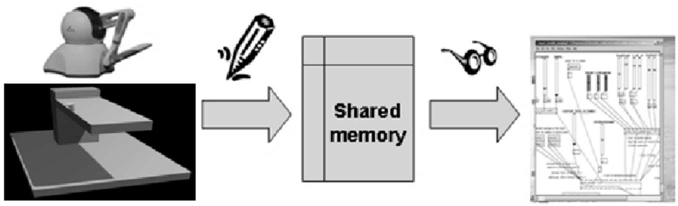

Fig. 2. The software architecture of the experimental setup

In order to achieve a realistic degree of interaction and unitary perception, the latency between the haptic, audio and visual feedback has to be very low. The comunication interface introduces some delay due to read/write access to shared memory. The code was heavily optimized so that the delay introduced by this process is negligible. We made many simulations of cyclic write/read access patterns, and found that in the worst case the delay introduced was in the order of μs, thus being negligible if compared to the latency due to haptic, sound and graphic rendering, which is in the order of some ms. During our experimental tests no subjects perceived any kind of noticeable intermodal latency.

Simulation of surface interaction in haptics is generally based on simple linear stiffness models [24]: a rigid immobile surface is modeled as a viscoelastic element, with a haptic stiffness k_{H} and a haptic viscosity coefficient λ_{H}, such that the haptic contact force is given as $f_{\mathrm{H}}(x(t), v(t)) = k_{\mathrm{H}}x(t) + \lambda_{\mathrm{H}}v(t)$, where x is the normal displacement relative to the surface. An ideally rigid wall should be simulated with k_{H} as high as possible. However limitations in the haptic sampling period T_{H} (typically ~ 1 kHz), and the spatial resolution of the device, limit the range for k_{H}: using values that are too high can cause the system to become unstable, i.e., to oscillate uncontrollably. Sufficient conditions for the stability of the interaction can be found by requiring the system to be *passive* [25]. In this work we have used this linear physical model haptic rendering of stiffness, since it is the one implemented in the Openhaptics™ Toolkit provided with the Phantom® device. This implies that k_{A} and k_{H} have different absolute values, because the physical models used for haptic and audio rendering are different.

3 Bi-modal Stiffness Perception

The architecture described in the previous section has been used to experimentally assess relative contributions of haptic and auditory information to bimodal judgments of contact stiffness using a rigid probe. More specifically, the experiment described in the remainder of this section is intended to assess the effectiveness of auditory feedback in modulating haptic perception of stiffness.

3.1 Participants

Sixteen subjects (between 19 and 30 years old) participated in the experiment. All participants reported themselves as being right-handed, and as having both normal hearing and normal tactual/motoric capabilities in their hands. All of them were naive as to the purposes and hypotheses of the test, and all of them volunteered. Some participants were musically trained.

3.2 Stimuli

The graphic display provided to subjects is shown in Fig. 3 (left). The small cone represents the position of the stylus. In every condition the haptic stiffness had the same value $k_{\rm H} = 400\,{\rm N/m}$. According to literature (see e.g. [26]) this can be considered an average value, with "soft" values being below 300 N/m and "hard" values starting above $600 - 700\,{\rm N/m}$. With this choice the haptic perception of stiffness is likely to be ambigous, and subjects are encouraged to rely on auditory judgement.

Auditory stiffness levels where obtained by varying the parameter $k_{\rm A}$, while all the remaining parameters of the physical sound model were held constant. The fundamental frequency of the struck object and the modal frequency distribution were chosen based on the equations for the ideal bar: with length $L = 20\,{\rm cm}$, height $h = 1\,{\rm cm}$, density $\rho \sim 1 \cdot 10^3\,{\rm Kg/m^3}$, and Young's modulus $E \sim 3 \cdot 10^{10}\,{\rm N/m^2}$ (in between typical values for wood and glass), the fundamental frequency is $f_0 = \frac{\pi h}{8\sqrt{12}L^2}\sqrt{\frac{E}{\rho}} \cdot 1.194^2 \sim 220\,{\rm Hz}$, and the modal distribution is given as $f_0 \cdot [1,\ 6.27,\ 17.54,\ 34.37,\ 56.81,\ 84.87, \ldots]$. With this choice of values the sixth modal frequency is close to the upper limit of the range of human hearing, therefore the first five modes were simulated.

The modal decay times were also chosen to match intermediate values between wood and glass. Impact force parameters other than $k_{\rm A}$ (see (1)) were also held constant. Given this set of parameter values, the interval of variability $[k_{\rm min}, k_{\rm max}] = [1 \cdot 10^3, 6.4 \cdot 10^5]\,{\rm N/m^\alpha}$ for the stiffness $k_{\rm A}$ was determined empirically as the largest interval outside of which further stiffness variations do not produce noticeable effects in the physical model behavior. Finally a series of exponentially spaced values $k_i = 2^i \cdot k_{\rm min}$ was sampled within this interval, resulting in a set of seven auditory stiffness values.

3.3 Procedure

Subjects were seated in front of a 15 in. wide computer monitor. The Phantom® Omni™ device was placed on their right-hand side, while a computer mouse was

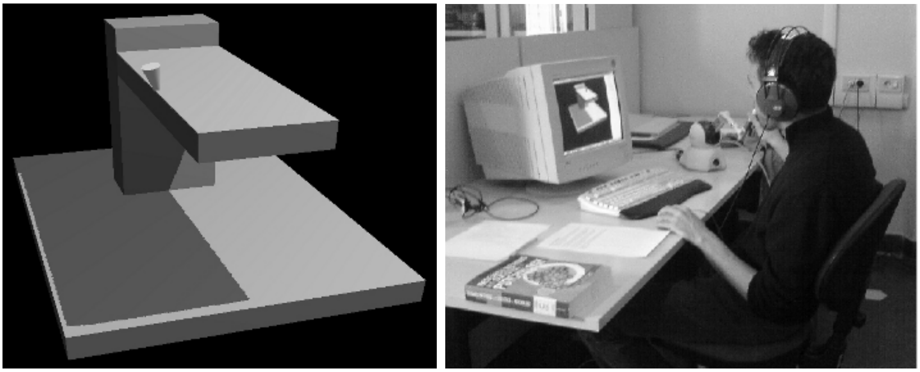

Fig. 3. Left: interactive graphic display presented to the subjects (the small cone represents the tip of the Phantom® stylus). Right: experimental setup.

placed on the left side. Auditory feedback was presented through headphones, connected to the output of a dedicated sound card. A picture of the experimental setup is provided in Fig. 3 (right).

Subjects were presented with the display depicted in Fig. 3 (left) and were instructed to judge the stiffness of the impact between a "hammer", represented by the device stylus, and the bar of the graphical display. Every object in the scene could be felt through the haptic device, but only touching the upper bar produced a sound. The graphic display did not change between conditions, and was intentionally composed of stylized objects, in order to limit as much as possible the amount of visual information delivered to subjects.

Perceived stiffness was determined through an absolute magnitude-estimation procedure (similarly to the procedure reported in [11]): participants were instructed to assign the non-zero, positive number that best described the magnitude of the perceived stiffness of the stimulus, along a scale ranging from 1 to 8. Verbal labels were associated to each point of the scale, ranging from "extremely soft" (1) to "extremely stiff" (8).

Participants did not receive any training before the experiment. During the experiment, auditory feedback conditions were presented with the following internal organization (not known to the subjects): first the seven stiffness levels were presented once each, then they were presented again three times each, and the 21 (level $\times$ repetition) combinations were randomized. In this way the first seven conditions provided participants with a minimal hidden training phase. The random order was different for each subject. Participants were allowed to interact with each condition as long as desired. Finally, in a post-experimental interview, subjects were asked the multiple-choice questions reported in Table 1.

3.4 Discussion

The experiment presented here has some similarities with the study conducted in [14]. However, there are some noticeable difference as well, specifically in the

Table 1. Post-experimental interview

1. In your opionion what was varying between each condition?
   [*haptics* | *audio* | *haptics and audio*]
2. In order to express your judgements, you relied mainly on...
   [*haptics* | *audio* | *haptics and audio*]
3. In your opinion the conditions simulate changes in the stiffness of...
   [*the bar* | *the hammer* | *both*]
4. In your opinion changes in the stiffness are due to changes of...
   [*bar material* | *hammer mat.* | *mat. of both* | *hammer shape* | *hamm. mat.&shape*]
5. Did the visual display influence your judgements? If yes, how?
   [*yes* | *no*]

design of the auditory feedback. Sounds for the experiments reported in [14] were real impact sounds, recorded by tapping various tools (the authors mention a pen and a screwdriver) against surfaces of various materials (styrofoam, metal plate, and so on). In the setup, a contact detection event in the haptic rendering pipeline triggered the playing of one of the sound files.

We argue that using recorded sounds as those in [14] has two main drawbacks. First, auditory stimuli produced by such a wide variety of interacting object are likely to be very easily discriminated, and may allow subjects to perform an *identification* task rather than a *rating* task. Second, the recorded sounds were obtained by varying not only the material of the striker, but also that of the struck object. Several studies (see e.g. [16]) support the hypothesis of a strong link between impact stiffness perception and material perception.

The synthetic stimuli used in this study differ only in the values of k_{A}, while the modal parameters associated to the struck object are constant. Therefore the two perceptual dimensions of impact stiffness and material of the struck object are decoupled. Moreover, as described above, the modal parameters of the struck object were chosen to lie between values typical for wood and glass, in order not to provide a clear perception of material to the subjects. As a result, the auditory cues associated to variations in stiffness are very subtle.

4 Results

4.1 Stiffness Scaling

After a preliminary analysis of collected data, three subjects were classified as "outliers" and discarded from the results. One of the outlier misunderstood the meaning of the perceptual scale, thus giving inconsistent answers. All the outliers provided contradictory answers to questions 1 and 2 of the post-experimental interview, confirming that they were very confused by the contrasting auditory and haptic cues received when strucking the object, with sounds that were not "appropriate" to the haptic sensation. One of the outlier reported troubles in assigning stiffness values because the perceptual scale was too sparse.

The hidden training phase (the first seven conditions of each series) was not included in the data analysis. Magnitude estimates were extracted using the following procedure: first, for each subject, estimates were averaged across stimulus repetitions; then, in order to compensate for differences in individual scales, the averaged estimates for each subject were normalized by dividing each score by the individual participant mean and multiplying by the grand mean (across participants). This procedure is resemblant of that used in [11].

One-way ANOVA was conducted on the mean magnitude estimates, and the effect of the auditory stiffness level was found to be statistically significant ($F = 122.87$, $p < 0.001$). A boxplot of the data is presented in Fig. 4. On average subjects identified the increase in stiffness with good accuracy, especially in the range 2000 – 32000 N/m. Near the extremal values the judgements are clearly less accurate. In particular the 1000 N/m value is on average perceived as stiffer than the next one. The boxplot shows that the range of responses for these two levels are very wide, confirming that subjects had difficulties in identifying the stimulus. Two mild outliers are plotted on the fourth column (8000 N/m). Note however that the interquartile range for this condition is extremely narrow, justifying to some extent the presence of these two mild outliers.

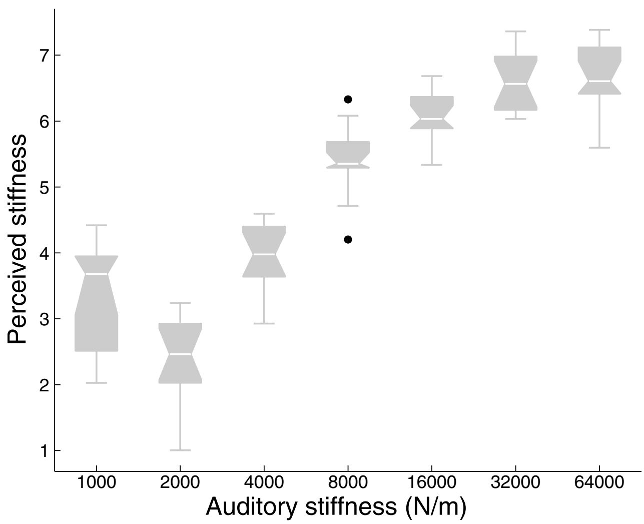

Fig. 4. Boxplots of perceived stiffness judgements (normalized magnitude estimates). For each box the central horizontal line represents the median, the top and the bottom represent the upper and lower quartiles, the vertical lines enclose the range of data.

4.2 Post-experimental Interview

The bar-plot in Fig. 5 shows the results of the post-experimental interview (see also Table 1). Question 1 and 2 were clearly related, although the first was mostly concerned with perception while the second asked about the strategy adopted by the subject in the rating task. Every subject's judgement was influenced, at least partially, by sound, but remarkably 5 out of 13 subjects perceived the haptic feedback changing together with audio and based their rating also on haptic feedback (although the haptic stiffness had the same value in all conditions, as explained in Sect. 3).

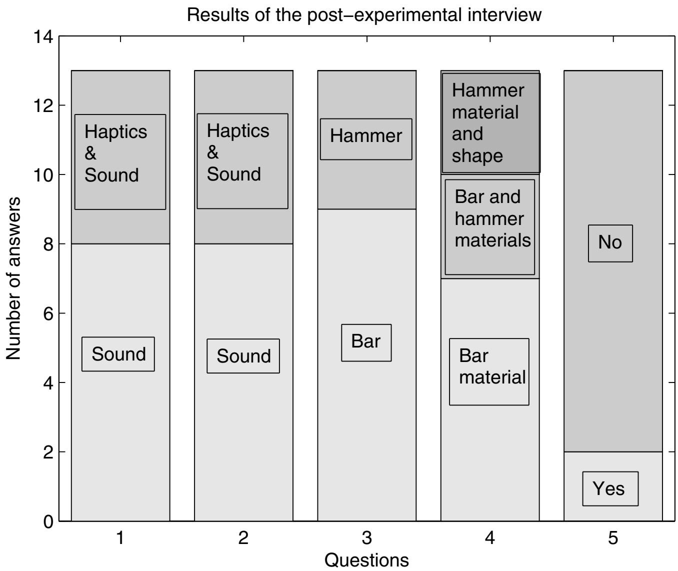

Fig. 5. Answers to the post–experimental interview. Each number corresponds to one question. The sixth question is discarded, since it was an open one.

Question 3 asked about which of the two objects underwent changes in stiffness. The "correct" answers would have been "both", since the stiffness value of the impact force relates to both the bar and the hammer properties. On the contrary, most of the subjects related different conditions to changes in bar stiffness, and a smaller percentage to changes in the hammer stiffness. This bias is in accordance with the findings by Giordano [16], although the graphic display may also have a role since the cone representing the hammer is a less veridical depiction that a parallelepiped for a bar. Since subjects are used to think about a cursor as a completely abstract representation of a position on a desktop, they may have associated the cone to a mouse cursor, and implicitly refused to give it a physical meaning.

This impression is confirmed by answers to question 4, since most of the subjects associated stiffness variations to changes in the material of the bar. The small fraction of subjects that related the change of stiffness to the hammer probably noticed that the acoustic properties of the bar did not change. In summary, listeners showed a somewhat limited ability to discriminate acoustically between hammer and struck object: this finding is compatible with the results reported by Giordano [16].

One of the strongest assumption in the experimental procedure was that the graphical display did not affect judgements: the answers to the fifth question clearly support this assumption. The two subjects that reported an influence of the graphical display commented that the graphics evoked some kind of hard material, like steel or thick wood.

5 Conclusion

The findings from the experiment reported in this paper support the effectiveness of auditory feedback in modulating haptic perception of stiffness. Magnitude estimates by the subjects provide clear indication that the perceived stiffness scales consistently with the physical parameter k_A which is varied in the auditory stimuli. Interestingly, a relevant portion (about 40%) of the subjects remarked in their answers to the post-experimental interview that they perceived variations in the haptic stiffness, although in every experimental condition the haptic stiffness had the same value.

The results suggest that auditory cues can be successfully used to augment and modulate the haptic display of stiffness, especially when the characteristics of the haptic system and the spatial resolution of the device, limit the range for surface stiffness rendering.

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References

1. Hahn, J.K., Fouad, H., Gritz, L., Lee, J.W.: Integrating sounds in virtual environments. Presence: Teleoperators and Virtual Environment **7**(1) (1998) 67–77
2. Welch, R.B., Warren, D.H.: Intersensory interactions. In Boff, K.R., Kaufman, L., Thomas, J.P., eds.: Handbook of Perception and Human Performance – Volume 1: Sensory processes and perception. John Wiley & Sons, New York (1986) 1–36
3. Ernst, M.O., Bülthoff, H.H.: Merging the senses into a robust percept. TRENDS in Cognitive Sciences **8**(4) (2004) 162–169
4. Gaver, W.W.: What in the world do we hear? an ecological approach to auditory event perception. Ecological Psychology **5**(1) (1993) 1–29
5. Gaver, W.W.: How do we hear in the world? explorations of ecological acoustics. Ecological Psychology **5**(4) (1993) 285–313
6. Avanzini, F., Rath, M., Rocchesso, D., Ottaviani, L.: Low-level sound models: resonators, interactions, surface textures. In Rocchesso, D., Fontana, F., eds.: The Sounding Object. Mondo Estremo, Firenze (2003) 137–172
7. van den Doel, K., Kry, P.G., Pai, D.K.: Foleyautomatic: Physically-based sound effects for interactive simulation and animation. In: Proc. ACM SIGGRAPH 2001, Los Angeles, CA (2001) 537–544
8. Klatzky, R.L., Pai, D.K., Krotkov, E.P.: Perception of material from contact sounds. Presence: Teleoperators and Virtual Environment **9**(4) (2000) 399–410

9. DiFilippo, D., Pai, D.K.: The AHI: An audio and haptic interface for contact interactions. In: Proc. ACM Symp. on User Interface Software and Technology (UIST'00), San Diego, CA (2000)
10. Lederman, S.J.: Auditory texture perception. Perception **8**(1) (1979) 93–103
11. Lederman, S.J., Klatzky, R.L., Morgan, T., Hamilton, C.: Integrating multimodal information about surface texture via a probe: Relative contribution of haptic and touch-produced sound sources. In: Proc. IEEE Symp. Haptic Interfaces for Virtual Environment and Teleoperator Systems (HAPTICS 2002), Orlando, FL (2002) 97–104
12. Guest, S., Catmur, C., Lloyd, D., Spence, C.: Audiotactile interactions in roughness perception. Exp. Brain Research **146**(2) (2002) 161–171
13. McGee, M.R., Gray, P., Brewster, S.: Mixed feelings: Multimodal perception of virtual roughness. In: Proc. Int. Conf. EuroHaptics, Edinburgh (2002) 47–52
14. DiFranco, D.E., Beauregard, G.L., Srinivasan, M.A.: The effect of auditory cues on the haptic perception of stiffness in virtual environments. Proceedings of the ASME Dynamic Systems and Control Division, (DSC-Vol.61) (1997)
15. Freed, D.J.: Auditory correlates of perceived mallet hardness for a set of recorded percussive events. J. Acoust. Soc. Am. **87**(1) (1990) 311–322
16. Giordano, B.: Sound source perception in impact sounds. PhD thesis, Department of General Psychology, University of Padova, Italy (2006)
17. Adrien, J.M.: The missing link: Modal synthesis. In De Poli, G., Piccialli, A., Roads, C., eds.: Representations of Musical Signals. MIT Press, Cambridge, MA (1991) 269–297
18. van den Doel, K., Pai, D.K.: Modal Synthesis for Vibrating Objects. In Greenebaum, K., ed.: Audio Anecdotes. AK Peters, Natick, MA (2004)
19. O'Brien, J.F., Shen, C., Gatchalian, C.M.: Synthesizing sounds from rigid-body simulations. In: Proc. ACM SIGGRAPH 2002, San Antonio, TX (2002) 175–181
20. Hunt, K.H., Crossley, F.R.E.: Coefficient of restitution interpreted as damping in vibroimpact. ASME J. Applied Mech. **42** (1975) 440–445
21. Timoshenko, S.P., Goodier, J.N.: . Mc Graw Hill, New York (1970)
22. Avanzini, F., Rocchesso, D.: Modeling Collision Sounds: Non-Linear Contact Force. In: Proc. COST-G6 Conf. Digital Audio Effects (DAFx01), Limerick (2001) 61–66
23. Avanzini, F., Rocchesso, D.: Physical modeling of impacts: theory and experiments on contact time and spectral centroid. In: Proc. Int. Conf. Sound and Music Computing (SMC06), Paris (2004) 287–293
24. Basdogan, C., Srinivasan, M.A.: Haptic rendering in virtual environments. In Stanney, K.M., ed.: Virtual Environments Handbook. Lawrence Erlbaum Associates (2002) 117–134
25. Colgate, J.E., Brown, J.M.: Factors Affecting the Z-Width of a Haptic Display. In: Proc. IEEE Int. Conf. on Robotics & Automation, San Diego (1994) 3205–3210
26. Kuchenbecker, K.J., Fiene, J., Niemeyer, G.: Improving Contact Realism through Event-Based Haptic Feedback. IEEE Trans. on Visualization and Comp. Graphics **13**(2) (2006) 219–230