Parameter Optimisation of a Linear Tetrahedral Mass Tensor Model for a Maxillofacial Soft Tissue Simulator

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Abstract. We present an extensive validation of the prediction accuracy of our soft tissue simulator for maxillofacial surgery planning using a linear Tetrahedral Mass Tensor Model (MTM). Prediction accuracy is quantified by measuring distances between the predicted data and the actual post-operative CT data for a database containing 10 patients who underwent maxillofacial surgery. Two different setups are considered. First two important parameters of a homogeneous MTM are optimised, namely the material's Poisson Ratio ν and the number of tetrahedra contained by the mesh. Optimal results were achieved with $\nu \approx 0.46$ and $N_{tetra} \geq 30.000$. Moreover the average simulation time could be reduced to less than 2.5 seconds. In the second setup an inhomogeneous MTM that differentiate between biomechanical properties for fat and muscle tissue is introduced. Simulation results show to be independent of Young's Moduli and optimal results were achieved for $\nu_{fat} = 0.485$ and $\nu_{muscle} = 0.43$. Moreover it turned out that using such an inhomogeneous model doesn't improve simulation accuracy significantly when compared to the homogeneous model.

1 Introduction

Maxillofacial surgery treats abnormalities of the skeleton of the head. Skull remodelling implies osteotomies, bone fragment repositioning, restoration of bone defects and inserting implants. Since the human face plays a key role in interpersonal relationships, people are very sensitive to changes to their outlook. Therefore planning of the operation and reliable prediction of the facial changes are very important.

This simulation of the deformation of the facial soft tissues due to bone movement, demands a mathematical model that is able to imitate the behavior of the facial tissues. We presented in the past [1] the usage of a linear Mass Tensor Model (MTM) as biomechanical model. This model tries to combine the advantages of Mass Spring Models, that have an easy architecture and short simulation times, and Finite Element Models that are considered to be very biomechanically relevant, which results in accurate soft tissue predictions.

M. Harders and G. Székely (Eds.): ISBMS 2006, LNCS 4072, pp. 159–168, 2006.

The original MTM was introduced by Cotin *et al.* [2]. In the MTM the modelled object is discretized into a tetrahedral mesh. Inside every tetrahedron T_i, the displacement field is defined by a linear interpolation of the displacement vectors of the four vertices of T_i, as defined by the finite element theory. It is than shown that the total elastic force at vertex j, after displacement of some of the mesh vertices, is given by:

$$F_j = [K_{jj}]u_j + \sum_{k \in N(j)} [K_{jk}]u_k \tag{1}$$

where u_k is the displacement of vertex k, $N(j)$ is the collection of all vertices neighbouring to vertex j and $[K_{jk}]$ are the global stiffness tensors for vertex j.

These stiffness tensors are directly proportional to the material's Modulus of Young and are dependant on the material's Poisson Ratio ν and the initial mesh configuration [2]. When calculating the soft tissue deformations due to maxillofacial surgery we first displace a subset of soft tissue points over a predefined distance derived from the bone related planning. Next the new position of all the other soft tissue points is found by demanding that the total elastic force in each of these soft tissue points should be zero when the point is in rest. This equals to solving following equation for all these points:

$$F_j(u_j^{new}) = 0 \rightarrow [K_{jj}]u_j^{new} + \sum_{k \in N(j)} [K_{jk}]u_k^{new} = 0 \tag{2}$$

which is solved using an iterative local steepest gradient approach.

In this work we investigate the influence of different parameters of the MTM on the accuracy of the prediction result. Since visualisation of the new facial outlook is the goal of our simulator, we validate prediction accuracy by measuring distances between the predicted and actual post-operative data. We refer to [3] for more detail about this validation work flow.

Two different setups are studied. First the facial soft tissues are considered to be a homogeneous material. For 10 data sets, containing pre-operative and post-operative patient's CT data, we derive the optimal biomechanical constants, i.e. the Poisson Ratio and Young's Modulus, and the optimal mesh size. In the second setup the homogeneous MTM is extended to an inhomogeneous model, in which we differentiate between fat and muscle tissue. We briefly summarize how the model is built and investigate the effect of varying the value of the biomechanical constants for fat and muscle tissue. Optimal parameters are derived. Results are presented and discussed in the third and fourth section.

2 Material and Methods

2.1 The Homogeneous Tetrahedral Mass Tensor Model

In a first setup we approximate the facial tissues as a homogeneous linear elastic material. The same value of the biomechanical constants, i.e. the Poisson Ratio

and Young's Modulus, will be assigned to all the soft tissue points. Since the global stiffness constants are directly proportional to Young's Modulus in the MTM, it can easily be seen that we can substitute equation 2 by:

$$\frac{[K_{jj}]}{E}u_j + \sum_{k \in N(j)} \frac{[K_{jk}]}{E}u_k = 0 \rightarrow [K^*_{jj}]u_j + \sum_{k \in N(j)} [K^*_{jk}]u_k = 0 \qquad (3)$$

with E Young's Modulus and $[K^*_{jk}]$ the new global stiffness tensors, which are independent of the Young's Modulus. Because only calculation of the new rest position of the soft tissue points is relevant for our application,i.e. solving the above equation, just two parameters need to be further defined: the mesh topology (more specific the mesh size was tested) and the Poisson Ratio.

The Poisson Ratio. The Poisson Ratio is defined as the ratio of the contraction strain normal to the applied load to the extension strain in the direction of the applied load. This ratio can vary between 0.0 and 0.5, where a value of 0.5 corresponds to a perfectly incompressible material but is not achievable with a linear elastic model. We therefor we varied in our experiment the ratio between 0 and 0.495. We generated for each data set a tetrahedral mesh including all facial soft tissues and containing on average 80.000 tetrahedra. For each value assigned to the Poisson Ratio, we calculated the new facial outlook and measured distances between corresponding points of the predicted and post-operative data, using a validation framework as discussed in [3]. Next we calculated the 50%, 90% and 95% percentiles of the generated distance maps. These statistics give a good indication of the prediction accuracy.

The Mesh Size. In [3] we suggest to map deformations calculated on a volumetric tetrahedral mesh, including only the facial soft tissues that will deform during simulation, to a dense surface representation of the whole skin for computational efficiency and a nice visualisation result. When using this method we can easily investigate the influence of the mesh size on the final prediction result, since deformations calculated on different meshes are always mapped to the same skin surface. The tetrahedral mesh is built out of the pre-operative CT data. This meshing includes three steps. First the facial soft tissues are semi-automatically segmented using a levelset approach. Next a triangular mesh that envelops these segmented facial soft tissues, is constructed with the Amira software (Amira, TGS, France). Finally, starting from this triangular surface, we assemble a tetrahedral mesh using the Netgen package [4].

To control the number of tetrahedra used, we generate triangular meshes containing $[500, 2500, ..., 50000, 75000]$ triangles. For each triangular mesh a tetrahedral mesh is constructed and the new facial outlook is calculated. During these simulations the Poisson Ratio was set to 0.46. Afterwards the simulated deformations were mapped to the facial skin surface and distances between the deformed skin surface and the co-registered post-operative skin surface were calculated. The 50%, 90% and 95% percentiles of the distance distributions were determined.

2.2 The Inhomogeneous Tetrahedral Mass Tensor Model

To improve simulation results, we investigate the effect of assigning different biomechanical properties to each tissue type. We distinguish between two tissue types: fat tissue and muscle tissue.

Model Building. To distinguish between the different tissue types we look at the intensity values in the pre-operative CT data and use a so-called Gaussian mixture model in combination with the EM-algorithm [5]. This model tries to approximate the intensity histogram as a combination of Gaussian distributions, where each distribution corresponds to a specific tissue type (see figure 1).

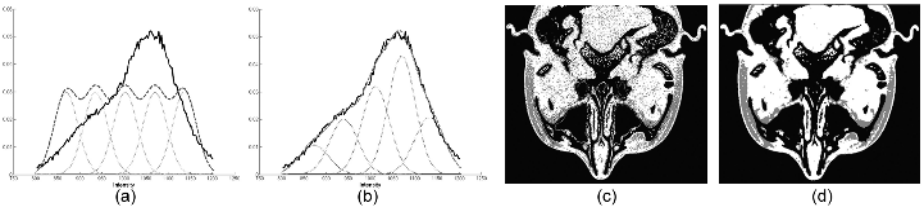


Fig. 1. (a) 5 Gaussian distributions are used to initialise the mixture model. The black line shows the intensity histogram of the CT slice, the initial Gaussians are drawn in grey dots and the dark grey dashed line indicates the initial approximation of the histogram. (b) Gaussian distributions after EM optimisation. (c)The segmented CT slice after application of the calculated tresholds. Grey labels correspond to fat tissue, while white labels correspond to muscle tissue. (d) The segmented CT slice after median filtering.

The Gaussian mixture model allows optimal threshold definition in every CT slice, i.e. the value for which the probability that a certain intensity value is labelled as fat, equals the probability to be labelled as muscle tissue. After applying these optimal tresholds for each CT slice (figure 1 (c)), we use a 5 by 5 median filter to remove noisy classifications. Figure 1 (d) shows the final segmentation map. The orange labels correspond to fat tissue, while white labels are defined as muscle tissue.

According to the Finite Element theory, a tetrahedral mesh should now be built where each of the tetrahedra contains only muscle or fat tissue. This approach can lead to a rather tedious and error-prone meshing step. Therefore we suggest to first build a very dense tetrahedral mesh, containing all facial tissues that will deform during simulation, based on the homogeneous segmented data. Typically such a tetrahedral mesh contains 150.000 tetrahedra (average tetrahedron volume smaller than 2.0 mm^3). Next we define for each tetrahedron the Young's Modulus and Poisson Ratio as the average Young's Modulus and Poisson Ratio over all voxels that lie inside this tetrahedron.

Parameters. Corresponding to the homogeneous model, since we only want to define the displacement of each soft tissue point after applying a fixed displacement to a subset of the points, we can again rewrite formula 2:

$$\frac{[K_{jj}]}{E_{fat}}u_j + \sum_{k \in N(j)} \frac{[K_{jk}]}{E_{fat}}u_k = 0 \rightarrow [K^{'}_{jj}]u_j + \sum_{k \in N(j)} [K^{'}_{jk}]u_k = 0 \qquad (4)$$

where E_{fat} is the Young's Modulus of fat tissue and $[K^{'}_{jk}]$ are the new global stiffness tensors. These tensors are now proportional to the ratio E_{muscle}/E_{fat}, ν_{muscle}, ν_{fat} and the initial mesh topology.

We investigate for this inhomogeneous model the influence of only the first 3 parameters on the final prediction result. ν_{muscle} and ν_{fat} were separately varied between 0.0 and 0.495. For E_{fat} and E_{muscle} many values have been reported in the literature [6,7,8]. Because of to the wide range of these reported values, we decided to vary E_{muscle}/E_{fat} between 1 and 64, imposing that muscle tissue must be stiffer than fat tissue. For all parameter settings distances between the simulated facial outlook and the actual post-operative outcome were measured as discussed in [3]. The 50%, 90% and 95% percentiles of the distance distributions were determined.

3 Results

We acquired a data set of 10 patients who underwent a maxillofacial procedure, including pre-operative and post-operative CT data. The average voxelsize of the CT data measured $0.35 \times 0.35 \times 0.5\ mm$. The patient group counts 2 Class III and 8 Class II patients [9]. For all patients the work flow as presented in [3], was used to generate the tetrahedral mesh and proper boundary conditions, that serve as input to the MTM.

3.1 Poisson Coefficient

Figure 2 summarizes the prediction error behavior in function of the Poisson Ratio for all 10 patients. On the left hand side the 90% percentile prediction errors for all 10 patients are shown. On each graph a cross indicates where the minimal prediction error was reached. We note that for different patients this minimum was achieved at different Poisson Ratio's. Moreover, it was noted for some patients that the recorded optimal Poisson Ratio, i.e. when the inspected error statistic becomes minimal, was dependent of this statistic. To find some sort of global mean over all data sets, we averaged the 50%, 90% and 95% percentile prediction error over all patients. As can be seen in figure 2, the prediction error was minimised for all three statistics when $0.45 \leq \nu \leq 0.46$. This value corresponds nicely to reported values on the Poisson Ratio of soft tissues [10].

We calculated for all 10 patients the difference in mm between the minimal 90% percentile and the 90% percentile prediction error, obtained when the Poisson Ratio was set to 0.46, which was found to be the global minimum. Only

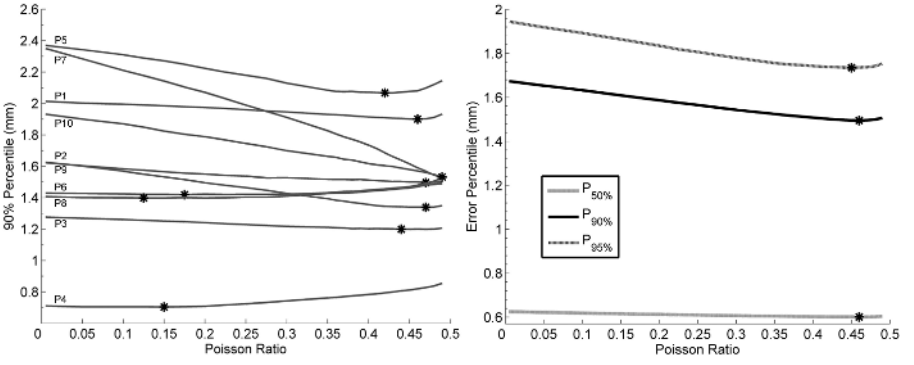


Fig. 2. (a) the 90% percentile prediction error (mm) in function of the Poisson Ratio of the homogeneous Mass Tensor Model for all 10 patients. (b) At the right hand side the average 50%, 90% and 95% percentiles are shown.

for patient 4 the difference measured more than 0.07 mm ($D^{P4}_{90\%} = 0.116\ mm$) Moreover the average difference between both percentiles over all data sets was found to be smaller than 0.05 mm. This difference is clearly negligible to the absolute accuracy of the predictions ($\approx 1.5\ mm$).

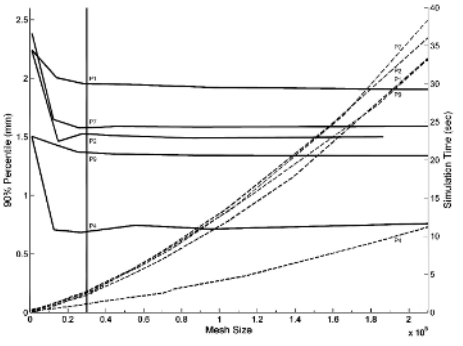
3.2 The Mesh Size

For 5 of the 10 patients we calculated the prediction accuracy in function of the mesh size. In figure 3 we present the results of these simulations. Accuracy becomes more or less constant when a 'sufficient' number of tetrahedra was used. For one data set, patient 4, even a slight decrease in accuracy was observed when the mesh size was increased. Probably this effect is induced by the volume/surface mapping technique [3] which includes some kind of Gaussian smoothing. When a less dense tetrahedral mesh is chosen, smoothing is larger and this may result in a better prediction of the new facial outlook.

When the mesh contains approximately 30.000 tetrahedra, the average difference between the 90% percentile achieved for this mesh and the result obtained whit a very dense mesh ($N_{tetra} > 200.000$), becomes smaller than $0.05mm$. This difference is negligible compared to the absolute accuracy of the predictions. Moreover figure 3 shows that simulation time varies more or less linear to the mesh size. As a consequence simulation time could be reduced to an average value of 2.2 *sec*. These very fast simulation times are a great benefit, when using the soft tissue simulator in daily clinical practice.

3.3 Inhomogeneous Model Parameters

We calculated the new facial outlook of all 10 patients, varying E_{muscle}/E_{fat} between 1 and 64 and ν_{muscle} and ν_{fat} between 0 and 0.495, as discussed in



Patient	N_{tetra}	T_{sim}	$D_{90\%}$
P1	31943	2.8	0.013
P2	27390	2.1	0.033
P4	31238	1.1	-0.068
P7	30094	2.7	-0.059
P9	28681	2.1	0.031
Average	29869	2.2	0.048

Fig. 3. On the left hand side the 90% percentiles (full lines) and simulation times (dotted lines) in function of the mesh size are shown. The black vertical line indicates when the mesh contains more than 30.000 tetrahedra. In the table on the right hand side, the third column shows the simulation time needed in seconds for a mesh containing approximately 30.000 tetrahedra. The differences in mm between the 90% percentile when using more than 200.000 and the 90% percentile obtained when using only 30.000 tetrahedra, are listed in the last column.

section 2.2. Figure 4 (a) lists simulation results for one typical data set, patient 9. For different ratio's of the Young's Moduli (E_{muscle}/E_{fat}) the minimal and maximal 50% and 90% percentiles over all possible values for ν_{fat} and ν_{muscle}, that were achieved, are calculated. As shown, the effect of the Young's Moduli is negligible compared to the absolute accuracy of the predictions ($\approx 1.5\ mm$), while the dependance on the Poisson Ratio's is in the order of $0.2\ mm$. This conclusion was enforced after processing the other data sets. For patient 9 the optimal prediction accuracy ($P_{90\%} = 1.382\ mm$) was achieved with $\nu_{fat} = 0.4$ and $\nu_{muscle} = 0.485$.

	Min		Max	
E_m/E_f	$P_{50\%}$	$P_{90\%}$	$P_{50\%}$	$P_{90\%}$
1	0.378	1.382	0.448	1.590
2	0.377	1.384	0.449	1.591
8	0.380	1.385	0.449	1.588
16	0.384	1.386	0.450	1.588
32	0.386	1.387	0.450	1.588
64	0.389	1.388	0.451	1.589

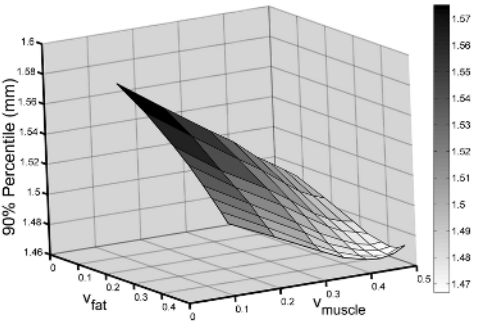


Fig. 4. The left table presents for data set 9, the minimum and maximum 50% and 90% percentiles over all possible values for ν_{fat} and ν_{muscle} for different ratio's of the Young's Moduli E_{muscle} and E_{fat}. At the right hand side, the average 90% percentile over all data sets is shown for $E_{muscle}/E_{fat} = 1$.

Similar to the homogeneous model, not all data sets reached a minimum for the same set of parameters. To find a global minimum, the 90% percentiles were averaged over all data sets and the minimum was defined. As expected these averaged values are independent of the ratio of Young's Moduli. The resulting graph for $E_{fat}/E_{muscle} = 1$ (figure 4) shows the prediction accuracy in function of the Poisson Ratio for fat and muscle. An optimal accuracy was reached for $\nu_{fat} = 0.485$ and $\nu_{muscle} = 0.43$.

4 Discussion and Conclusion

In this work we tried to optimise the biomechanical constants, i.e. the Young's Modulus and the Poisson Ratio, and the tetrahedral mesh size for a linear Mass Tensor Model (MTM) used to simulate the new facial outlook after maxillofacial surgery. Since the MTM is a variant of the Finite Element Model (FEM), results are applicable to all biomechanical FEM-based models. Two different setups were considered: a homogeneous and inhomogeneous model.

For the homogeneous setup, we showed that optimal simulation results are achieved with $0.45 \leq \nu \leq 0.46$. Moreover mesh size could be reduced to 30.000 tetrahedra, which lowered the average simulation time to 2.2 seconds, but did not affect the prediction accuracy. Since in our application no external force are calculated the influence of Young's Modulus is null. For the inhomogeneous model we showed that simulation results were almost independent of the ratio of Young's Moduli and highest accuracy was achieved with $\nu_{fat} \approx 0.485$ and $\nu_{muscle} \approx 0.43$. When comparing the homogeneous and inhomogeneous model, the simulation results were not found to be significantly better. Consequently we conclude that there is no net improvement when using an inhomogeneous tissue model.

Recently Zachow *et al.* [11], reported on a quantitative evaluation of 3D soft tissue predictions for maxillofacial surgery on a single data set. In this work they also investigated the influence of the Poisson Ratio and the usage of a inhomogeneous Finite Element Model, on the final prediction result of this one data set. They concluded that an optimal accuracy was achieved with $0.43 \leq \nu \leq 0.45$, but the observed variations in prediction accuracy were rather small ($\approx 0.02mm$). We however conclude, based on a more extensive validation as it includes 10 patients, that for some patients variations are indeed rather small (see figure 2), but for other data sets (patient 5,7,9 and 10) an appropriate choice of the Poisson Ratio ν clearly influences the final prediction result. For this last group, accuracy clearly improved when the Poisson Ratio was increased. Consequently the difference between the best prediction, i.e. when the 90% percentile was minimal, and the facial prediction when ν was set to 0.46, was smaller than 0.1 mm for al data sets. This difference is still negligible compared to the actual prediction accuracy ($P_{90\%} \approx 1.5\ mm$).

For some patients maximal accuracy is achieved in different facial regions for different parameter settings. This may result in a rather flat slope of the global prediction accuracy in function of the Poisson Ratio. To verify this hypothesis, we

defined for each facial tissue point the optimal Poisson Ratio, i.e. the value that minimises the 90% error percentile, and visualised these optimal values as a color code onto the facial skin surface. Figure 5 (a) shows the result for patient 2. When inspecting this image, one should also keep in mind the variation in accuracy that was obtained by setting ν to this optimal value. Therefore figure 5 (b) shows the measured standard deviation of the 90% percentile over all Poisson Ratio's, by means of a color code. The color code ranges from 0 mm to 0.3 mm. As can be seen for the cheek and the Labiomental Fold region, optimal accuracy is obtained when the Poisson Ratio is set to a high value ($\nu \approx 0.47$), while for the lip, tip of the nose and the Gonion region the most accurate result is obtained with $\nu \approx 0.1$. This difference probably causes the quite flat behavior of the global 90% error percentile in function of the Poisson Ratio. Similar conclusions could be made for the other data sets.

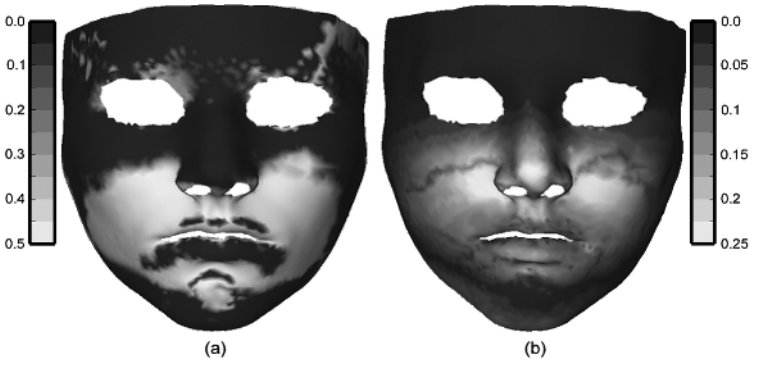


Fig. 5. For each facial tissue point the optimal Poisson Ratio (a) and standard deviation of the 90% error percentile in function of the Poisson Ratio (b), were calculated and visualised by means of a color code

Future research is required to define these different facial regions, based on the pre-operative CT data and planning data. These differences in optimal Poisson Ratio, may also arise from difference in lip posture between the pre-operative an post-operative acquisition or incorrectly defined boundary conditions like already suggested in [3]. In the near future, we hope to be able to define some correlation between the planned procedure and the biomechanical behavior of certain facial regions by extending and statistical analysing our validation database.

Acknowledgements

This work is part of the Flemish government IWT GBOU 020195 project on Realistic image-based facial modelling for forensic reconstruction and surgery simulation and K.U.Leuven/OF/GOA/2004/05.

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