Flow Graphs and Decision Tables with Fuzzy Attributes

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Abstract. This paper is concerned with the issue of design and analysis of fuzzy decision systems, basing on recorded process data. A concept of fuzzy flow graphs is proposed to allow representation of decision tables with fuzzy attributes. Basic notions of the crisp flow graph approach are generalized. Satisfaction of flow graph properties, with respect to fuzzy connectives used in calculations, is taken into account. Alternative definitions of the path's certainty and strength are introduced. In an illustrative example a decision table with fuzzy attributes is analyzed and interpreted in terms of flow graphs.

1 Introduction

An appropriate utilization of recorded process data and decision examples, for creating a set of relevant fuzzy decision rules, is an important problem in applications of fuzzy inference systems [3,9]. The used data can be conveniently represented in the form of a decision table with fuzzy attributes. It might be advantageous to carry out an analysis of this kind of decision table, by applying the fuzzy rough set model [4].

A hybrid approach to decision algorithms, proposed by Pawlak [5,6,7], combines the idea of flow graphs with the crisp rough set model. Greco, Pawlak and Słowiński [1] proved that relaxation of mutual exclusion of decision rules does not violate basic properties of flow graphs, and every decision algorithm can be associated with a flow graph.

The main goal of this paper consists in developing a fuzzy flow graph approach, which is suitable for representing and analyzing fuzzy decision systems. First of all, we want to address crucial issues of the generalized flow graph approach, concerning especially its connection to fuzzy inference systems. We concentrate on the aspect of representing and selecting fuzzy decision rules with the help of flow graphs. The problem of a correct choice of fuzzy connectives is considered with the aim to retain the flow conservation equations. Furthermore, we give new definitions of the path's certainty and strength, by respecting only the relevant part of the flow, i.e by disregarding the flow components which come from other paths.

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2 Decision Tables with Fuzzy Attributes

In further discussion, we apply fuzzy decision tables in the form introduced in [4]. We consider a finite universe U with N elements: $U = \{x_1, x_2, \ldots, x_N\}$. Every element x of the universe U will be described with the help of fuzzy attributes, which are divided into a subset of n condition attributes $C = \{c_1, c_2, \ldots, c_n\}$, and a subset of m decision attributes $D = \{d_1, d_2, \ldots, d_m\}$.

We assign a set of linguistic values to every fuzzy attribute. Let us denote by $V_i = \{V_{i1}, V_{i2}, \ldots, V_{in_i}\}$ the family of linguistic values of the condition attribute c_i, and by $W_j = \{W_{j1}, W_{j2}, \ldots, W_{jm_j}\}$ the family of linguistic values of the decision attribute d_j, where n_i and m_j is the number of the linguistic values of the i-th condition and the j-th decision attribute, respectively, $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.

For any element $x \in U$, its membership degrees in all linguistic values of the condition attribute c_i (or decision attribute d_j) should be determined. This is accomplished during the fuzzification stage, basing on the recorded crisp value of a particular attribute of x. The value of an attribute for a given element x is a fuzzy set on the domain of all linguistic values of that attribute.

We denote by $V_i(x)$ the fuzzy value of the condition attribute c_i for any x, as a fuzzy set on the domain of the linguistic values of c_i

$$V_i(x) = \{\mu_{V_{i1}}(x)/V_{i1},\, \mu_{V_{i2}}(x)/V_{i2},\, \ldots,\, \mu_{V_{in_i}}(x)/V_{in_i}\}\,. \tag{1}$$

$W_j(x)$ denotes the fuzzy value of the decision attribute d_j for any x, as a fuzzy set on the domain of the linguistic values of d_j

$$W_j(x) = \{\mu_{W_{j1}}(x)/W_{j1},\, \mu_{W_{j2}}(x)/W_{j2},\, \ldots,\, \mu_{W_{jm_j}}(x)/W_{jm_j}\}\,. \tag{2}$$

If the linguistic values of an attribute have the form of singletons or disjoint intervals, with membership degree equal to 1 on the original domain of the attribute, then only one linguistic value can be assigned to that attribute. In that case we get a classical crisp decision table. In general, we obtain a non-zero membership of x to more than one linguistic value of an attribute.

Table 1. Decision table with fuzzy attributes

	c_1	c_2	$\cdots$	c_n	d_1	d_2	$\cdots$	d_m
x_1	$V_1(x_1)$	$V_2(x_1)$	$\cdots$	$V_n(x_1)$	$W_1(x_1)$	$W_2(x_1)$	$\cdots$	$W_m(x_1)$
x_2	$V_1(x_2)$	$V_2(x_2)$	$\cdots$	$V_n(x_2)$	$W_1(x_2)$	$W_2(x_2)$	$\cdots$	$W_m(x_2)$
				$\cdots$				
x_N	$V_1(x_N)$	$V_2(x_N)$	$\cdots$	$V_n(x_N)$	$W_1(x_N)$	$W_2(x_N)$	$\cdots$	$W_m(x_N)$

Furthermore, we assume that for any element $x \in U$, all linguistic values $V_i(x)$ and $W_j(x)$ $(i = 1, 2, \ldots, n\,,\ \ j = 1, 2, \ldots m)$ satisfy the requirments

$$\mathrm{power}(V_i(x)) = \sum_{k=1}^{n_i} \mu_{V_{ik}}(x) = 1\,, \qquad \mathrm{power}(W_j(x)) = \sum_{k=1}^{m_j} \mu_{W_{jk}}(x) = 1\,. \tag{3}$$

This assumption allows us to generalize the flow graph approach and use it for analysis of fuzzy information system.

Decision tables with fuzzy values of attributes will be applied for examining all possible decision rules generated by using the Cartesian product of sets of the linguistic values.

Let us denote by R_k the k-th decision rule from the set consisting of r possible decision rules ($r = \prod_{i=1}^{n} n_i \prod_{j=1}^{m} m_j$)

$$\begin{array}{ll} R_k\colon & \text{IF } c_1 \text{ is } V_1^k \text{ AND } c_2 \text{ is } V_2^k \ \ldots \text{ AND } c_n \text{ is } V_n^k \\ & \text{THEN } d_1 \text{ is } W_1^k \text{ AND } d_2 \text{ is } W_2^k \ \ldots \text{ AND } d_m \text{ is } W_m^k \end{array} \tag{4}$$

where $k = 1, 2, \ldots, r\,,\ V_i^k \in V_i\,,\ i = 1, 2, \ldots n\,,\ W_j^k \in W_j\,,\ j = 1, 2, \ldots, m.$

When we use the fuzzy Cartesian products $C^k = V_1^k \times V_2^k \ldots \times V_n^k$ and $D^k = W_1^k \times W_2^k \ldots \times W_m^k$, the k-th decision rule can be written in the form of a fuzzy implication $C^k \to D^k$.

We need to select a subset of those decision rules which are relevant to the considered decision process. To this end, we determine to what degree any element $x \in U$, corresponding to a single row of the decision table, confirms particular decision rules. We should calculate the truth value of the decision rule's antecedent and the truth value of the decision rule's consequent, by determining the conjunction of the respective membership degrees of x in the linguistic values of attributes.

In the case of a decision table with crisp attributes, a decision rule is confirmed for some x, if the result of conjunction is equal to 1, both for the rule's premise and the rule's conclusion. Otherwise, the element x does not confirm the considered decision rule. The set of those elements $x \in U$, which confirm a decision rule, is called the support of the decision rule.

In order to determine the confirmation degree of fuzzy decision rules, in the case of decision tables with fuzzy attributes, we need to apply a T-norm operator. By $\mathrm{cd}(x, k)$, we denote the confirmation degree of the k-th decision rule by the element $x \in U$

$$\mathrm{cd}(x, k) = \mathrm{T}(\mathrm{cda}(x, k), \mathrm{cdc}(x, k))\,, \tag{5}$$

where $\mathrm{cda}(x, k)$ denotes the confirmation degree of the decision rule's antecedent

$$\mathrm{cda}(x, k) = \mathrm{T}(\mu_{V_1^k}(x), \mu_{V_2^k}(x), \ldots, \mu_{V_n^k}(x))\,, \tag{6}$$

and $\mathrm{cdc}(x, k)$ the confirmation degree of the decision rule's consequent

$$\mathrm{cdc}(x, k) = \mathrm{T}(\mu_{W_1^k}(x), \mu_{W_2^k}(x), \ldots, \mu_{W_m^k}(x))\,. \tag{7}$$

By determining the confirmation degrees (6), (7) and (5), we get the following fuzzy sets on the domain U:
the support of the decision rule's antecedent

$$\mathrm{support}(\mathrm{cda}(x, k)) = \{\mathrm{cda}(x_1, k)/x_1,\ \mathrm{cda}(x_2, k)/x_2,\ \ldots,\ \mathrm{cda}(x_N, k)/x_N\}\,, \tag{8}$$

the support of the decision rule's consequent

$$\text{support}(\text{cdc}(x,k)) = \{\text{cdc}(x_1,k)/x_1,\ \text{cdc}(x_2,k)/x_2,\ \ldots,\ \text{cda}(x_N,k)/x_N\}\,, \quad (9)$$

and the support of the decision rule R_k, respectively

$$\text{support}(R_k) = \{\text{cd}(x_1,k)/x_1,\ \text{cd}(x_2,k)/x_2,\ \ldots,\ \text{cd}(x_N,k)/x_N\}\,. \quad (10)$$

The introduced notions will be used in the next section to define strength, certainty, and coverage factors of a decision rule.

3 Flow Graphs

The idea of using flow graphs in the framework of rough sets, for discovering the statistical properties of crisp decision algorithms, was proposed by Pawlak [5,6,7]. We want to utilize and extend this concept with the aim of applying flow graphs to analysis of fuzzy information systems. First, we recall basic notions of the crisp flow graph approach.

A flow graph is given in the form of directed acyclic final graph $G = (\mathcal{N}, \mathcal{B}, \varphi)$, where $\mathcal{N}$ is a set of nodes, $\mathcal{B} \subseteq \mathcal{N} \times \mathcal{N}$, is a set of directed branches, φ: $\mathcal{B} \rightarrow \mathrm{R}^+$ is a flow function with values in the set of non-negative reals R^+.

For any $(X, Y) \in \mathcal{B}$, X is an input of Y and Y is an output of X. The quantity $\varphi(X, Y)$ is called the throughflow from X to Y.

$I(X)$ and $O(X)$ denote an input and an output of X, respectively. The input $I(G)$ and output $O(G)$ of a graph G are defined by

$$I(G) = \{X \in \mathcal{N}\colon\ I(X) = \emptyset\}\,, \qquad O(G) = \{X \in \mathcal{N}\colon\ O(X) = \emptyset\}\,. \quad (11)$$

Every node $X \in \mathcal{N}$ of a flow graph G is characterized by its inflow

$$\varphi_+(X) = \sum_{Y \in I(X)} \varphi(Y, X)\,, \quad (12)$$

and by its outflow

$$\varphi_-(X) = \sum_{Y \in O(X)} \varphi(X, Y)\,. \quad (13)$$

For any internal node X, the equality $\varphi_+(X) = \varphi_-(X) = \varphi(X)$ is satisfied. The quantity $\varphi(X)$ is called the flow of the node X.

The flow for the whole graph G is defined by

$$\varphi(G) = \sum_{x \in I(G)} \varphi_-(X) = \sum_{x \in O(G)} \varphi_+(X)\,. \quad (14)$$

By using the flow $\varphi(G)$, the normalized troughflow $\sigma(X, Y)$ and the normalized flow $\sigma(X)$ are determined as follows

$$\sigma(X, Y) = \frac{\varphi(X, Y)}{\varphi(G)}\,, \qquad \sigma(X) = \frac{\varphi(X)}{\varphi(G)}\,. \quad (15)$$

For every branch of a flow graph G the certainty factor is defined by

$$\mathrm{cer}(X,Y) = \frac{\sigma(X,Y)}{\sigma(X)} \,. \tag{16}$$

The coverage factor for every branch of a flow graph G is defined by

$$\mathrm{cov}(X,Y) = \frac{\sigma(X,Y)}{\sigma(Y)} \,. \tag{17}$$

The certainty and coverage factors satisfy the following properties

$$\sum_{Y \in O(X)} \mathrm{cer}(X,Y) = 1\,, \qquad \sum_{X \in I(Y)} \mathrm{cov}(X,Y) = 1\,. \tag{18}$$

The measures (16) and (17) are useful for analysis of decision algorithms [2].

Let us now consider the possibility of applying flow graphs in the case of fuzzy decision algorithms. Any decision table with fuzzy attributes can be conveniently expressed as a flow graph. We can assume, without losing the generality of consideration, that only one decision attribute will be used. Each attribute is represented by a layer of nodes. The nodes of the input and hidden layers correspond to linguistic values of the condition attributes, whereas the output layer nodes correspond to linguistic values of the decision attribute.

Let us denote by $\tilde{X}$ a fuzzy set on the universe U, which describes membership degree of particular elements $x \in U$ in the linguistic value represented by X. The membership degrees of all x in the set $\tilde{X}$ can be found in a respective column of the considered decision table.

The flow $\varphi(X,Y)$ for the branch (X,Y) is equal to power (fuzzy cardinality) of the product of fuzzy sets $\tilde{X}$ and $\tilde{Y}$. However, the T-norm operator `prod` should be used for determining the product of sets, in order to satisfy the following equation for the input and internal layer nodes

$$\varphi_{-}(X) = \mathrm{power}(\tilde{X}) = \sum_{Y \in O(X)} \varphi(X,Y) = \sum_{Y \in O(X)} \mathrm{power}(\tilde{X} \cap \tilde{Y})\,. \tag{19}$$

An analogous equation can be given for the output and internal layer nodes

$$\varphi_{+}(X) = \mathrm{power}(\tilde{X}) = \sum_{Y \in I(X)} \varphi(Y,X) = \sum_{Y \in I(X)} \mathrm{power}(\tilde{X} \cap \tilde{Y})\,. \tag{20}$$

Similarly, the equality $\varphi_{+}(X) = \varphi_{-}(X) = \varphi(X)$ is satisfied for any internal node X, when the T-norm operator `prod` is used. The above equations do not hold in general, if we use another T-norm operator, e.g. `min`. This is because the total normalized inflow (outflow) of each layer does depend on the form of T-norm operator used in calculations. In order to satisfy (19) and (20), the total normalized inflow (outflow) of a layer should be equal to 1. By applying the property (3), we can show that this is fulfilled, when we choose the T-norm operator `prod`.

In the special case of crisp decision tables, the formulae (19) and (20) become equivalent to (12) and (13).

The input and hidden layers of the flow graph can be merged into a single layer, which contains nodes representing all possible combinations of linguistic values of the condition attributes. Let us denote by X^* a node of the resulting layer. The node X^* corresponds to antecedent of a certain decision rule R_k. Support of the antecedent of the decision rule R_k is determined by using (8).

The decision rule R_k is represented by a branch (X^*,Y), where Y denotes a node of the output layer.

Power of the support of the rule R_k, defined by (10), is equal to the flow between the nodes X^* and Y

$$\varphi(X^*, Y) = \mathrm{power}(\mathrm{support}(R_k)) \,. \tag{21}$$

By using the formulae (8), (9) and (10), we can determine, for every decision rule R_k, the certainty factor $\mathrm{cer}(X^*, Y)$, coverage factor $\mathrm{cov}(X^*, Y)$, and strength of the rule $\sigma(X^*, Y)$

$$\mathrm{cer}(X^*, Y) = \mathrm{cer}(R_k) = \frac{\mathrm{power}(\mathrm{support}(R_k))}{\mathrm{power}(\mathrm{support}(\mathrm{cda}(x, k)))} \,, \tag{22}$$

$$\mathrm{cov}(X^*, Y) = \mathrm{cov}(R_k) = \frac{\mathrm{power}(\mathrm{support}(R_k))}{\mathrm{power}(\mathrm{support}(\mathrm{cdc}(x, k)))} \,, \tag{23}$$

$$\sigma(X^*, Y) = \mathrm{strength}(R_k) = \frac{\mathrm{power}(\mathrm{support}(R_k))}{\mathrm{card}(U)} \,. \tag{24}$$

Every decision rule can be represented by a sequence of nodes $[X_1 \ldots X_n]$, i.e. by a path from the 1-th to the n-th layer of the flow graph G. For a given path $[X_1 \ldots X_n]$, the resulting certainty and strength can be defined. In contrast to the definitions presented in [5,6,7], in which the statistical properties of flow are taken into account, we introduce an alternative form of the path's certainty and strength

$$\mathrm{cer}[X_1 \ldots X_n] = \prod_{i=1}^{n-1} \mathrm{cer}(X_1 \ldots X_i, X_{i+1}) \,, \tag{25}$$

$$\sigma[X_1 \ldots X_n] = \sigma(X_1)\, \mathrm{cer}[X_1 \ldots X_n] \,, \tag{26}$$

where

$$\mathrm{cer}(X_1 \ldots X_i, X_{i+1}) = \frac{\mathrm{power}(\tilde{X}_1 \cap \tilde{X}_2 \cap \ldots \cap \tilde{X}_{i+1})}{\mathrm{power}(\tilde{X}_1 \cap \tilde{X}_2 \cap \ldots \cap \tilde{X}_i)} \,. \tag{27}$$

With the help of equation (25), we can determine what part of the flow of the starting node X_1 reaches the final node X_n, passing through all nodes of the path $[X_1 \ldots X_n]$.

4 Example

We consider a decision table with fuzzy attributes (Table 2). There are two condition attributes A and B, and one decision attribute D. All attributes have three linguistic values. We use the same labels for both the linguistic values of the attributes and the corresponding nodes of the flow graph. As we stated in previous section, we use the T-norm operator `prod` in our calculations.

Table 2. Decision table with fuzzy attributes

	A			B			D		
	A_1	A_2	A_3	B_1	B_2	B_3	D_1	D_2	D_3
x_1	0.2	0.8	0.0	0.0	0.9	0.1	0.0	0.9	0.1
x_2	0.9	0.1	0.0	1.0	0.0	0.0	0.0	0.1	0.9
x_3	0.0	0.2	0.8	0.0	0.1	0.9	0.9	0.1	0.0
x_4	0.2	0.8	0.0	0.0	0.8	0.2	0.0	1.0	0.0
x_5	0.0	0.8	0.2	0.9	0.1	0.0	0.0	0.1	0.9
x_6	0.9	0.1	0.0	0.0	0.2	0.8	1.0	0.0	0.0
x_7	0.1	0.9	0.0	0.0	0.9	0.1	0.1	0.9	0.0
x_8	0.0	0.1	0.9	0.8	0.2	0.0	0.0	0.0	1.0
x_9	0.0	0.1	0.9	0.0	0.1	0.9	0.9	0.1	0.0
x_{10}	0.1	0.9	0.0	0.2	0.8	0.0	0.0	1.0	0.0

The values of normalized flow, for nodes representing condition attributes, are given in Table 3. We can easy check that the flow conservation equations (19) and (20) are satisfied, for example,

$$\sigma_-(A_1) = \frac{\text{power}(\widetilde{A_1})}{\text{card}(U)} = \sum_{i=1}^{3} \sigma(A_1, B_i) = 0.240\,,$$

$$\sigma_+(B_1) = \frac{\text{power}(\widetilde{B_1})}{\text{card}(U)} = \sum_{i=1}^{3} \sigma(A_i, B_1) = 0.290\,.$$

In the next step, we merge the layers corresponding to condition attributes into a resulting layer, which represents all possible linguistic values in the antecedences of decision rules. We determine the degrees of satisfaction of the rules' antecedences for particular elements $x \in U$. For the antecedence represented by A_1B_1, we get:

$$\widetilde{A_1B_1} = \tilde{A}_1 \cap \tilde{B}_1 = \{\ 0.00/x_1,\ 0.90/x_2,\ 0.00/x_3,\ 0.00/x_4,\ 0.00/x_5,\ 0.00/x_6,\ 0.00/x_7,\ 0.00/x_8,\ 0.00/x_9,\ 0.02/x_{10}\},$$

$$\varphi(A_1, B_1) = \text{power}(\widetilde{A_1B_1}) = 0.92, \quad \sigma(A_1, B_1) = \tfrac{\varphi(A_1,B_1)}{\text{card}U} = 0.092.$$

Table 3. Normalized flow between nodes of condition attributes' layers

$\sigma(A_i, B_j)$				
	B_1	B_2	B_3	Σ
A_1	0.092	0.069	0.079	0.240
A_2	0.108	0.304	0.068	0.480
A_3	0.090	0.037	0.153	0.280
Σ	0.290	0.410	0.300	1.000

Table 4. Normalized flow between resulting and output layer

$\sigma(A_iB_j, D_k)$				
	D_1	D_2	D_3	Σ
A_1B_1	0.0000	0.0110	0.0810	0.0920
A_1B_2	0.0189	0.0483	0.0018	0.0690
A_1B_3	0.0721	0.0067	0.0002	0.0790
A_2B_1	0.0000	0.0262	0.0818	0.1080
A_2B_2	0.0128	0.2748	0.0164	0.3040
A_2B_3	0.0332	0.0340	0.0008	0.0680
A_3B_1	0.0000	0.0018	0.0882	0.0900
A_3B_2	0.0153	0.0019	0.0198	0.0370
A_3B_3	0.1377	0.0153	0.0000	0.1530
Σ	0.2900	0.4200	0.2900	1.0000

Table 5. Certainty factor for branches between resulting and output layer

$\text{cer}(A_iB_j, D_k)$				
	D_1	D_2	D_3	Σ
A_1B_1	0.0000	0.1196	0.8804	1.00
A_1B_2	0.2740	0.7000	0.0260	1.00
A_1B_3	0.9127	0.0848	0.0025	1.00
A_2B_1	0.0000	0.2426	0.7574	1.00
A_2B_2	0.0421	0.9039	0.0539	1.00
A_2B_3	0.4882	0.5000	0.0118	1.00
A_3B_1	0.0000	0.0200	0.9800	1.00
A_3B_2	0.4140	0.0510	0.5350	1.00
A_3B_3	0.9000	0.1000	0.0000	1.00

The results of calculation of normalized flow between nodes of the resulting layer and nodes of the output layer are given in Table 4. The values of normalized outflow $\sigma_-(A_iB_j)$, $i, j \in \{1, 2, 3\}$, (column $\sum$ in Table 4) are equal to the respective values of normalized troughflow $\sigma(A_i, B_j)$, given in Table 3, e.g.,

Table 6. Coverage factor for branches between resulting and output layer

	$\text{cov}(A_iB_j, D_k)$		
	D_1	D_2	D_3
A_1B_1	0.0000	0.0262	0.2793
A_1B_2	0.0652	0.1150	0.0062
A_1B_3	0.2486	0.0159	0.0007
A_2B_1	0.0000	0.0624	0.2821
A_2B_2	0.0441	0.6543	0.0566
A_2B_3	0.1145	0.0810	0.0028
A_3B_1	0.0000	0.0043	0.3040
A_3B_2	0.0528	0.0045	0.0683
A_3B_3	0.4748	0.0364	0.0000
Σ	1.0000	1.0000	1.0000

Table 7. Decision rules with the largest value of certainty factor

decision rule	certainty	coverage	strength [%]
$A_1B_1 \to D_3$	0.8804	0.2793	8.10
$A_1B_2 \to D_2$	0.7000	0.1150	4.83
$A_1B_3 \to D_1$	0.9127	0.2486	7.21
$A_2B_1 \to D_3$	0.7574	0.2821	8.18
$A_2B_2 \to D_2$	0.9039	0.6543	27.48
$A_3B_1 \to D_3$	0.9800	0.3040	8.82
$A_3B_3 \to D_1$	0.9000	0.4748	13.77

$$\sigma_-(A_1B_1) = \sigma(A_1, B_1) = \sum_{j=1}^{3} \sigma(A_1B_1, D_j) = 0.0920\,.$$

Thus, the flow conservation equations are satisfied. This is due to applying the T-norm operator `prod`.

For branches connecting the resulting and output layers, the certainty and coverage factors are determined according to (16) and (17). The results are given in Tables 5 and 6. They correspond to certainty and coverage factors of the decision rules $A_iB_j \to D_k,\ \ i, j, k \in \{1, 2, 3\}$, expressed by the formulae (22) and (23). For example, $\text{cer}(A_1B_1, D_3) = 0.8804$ means that 88.04% of outflow from the node A_1B_1 reaches the decision node D_3, $\text{cov}(A_1B_1, D_3) = 0.2793$ means that 27.93% of inflow to the decision node D_3 comes from the node A_1B_1.

Another important measure is the strength of decision rule expressed by (24). For example, the strength of the rule $A_1B_1 \to D_3$ is equal to 8.1%. We can say that the troughflow (A_1B_1, D_3) constitutes 8.1% of the total flow of the considered graph.

Fuzzy decision rules with the largest values of certainty factor (Table 7) can be included in the final fuzzy inference system. The respective values of coverage factor are useful for explaining these decision rules.

5 Conclusions

The proposed approach to fuzzy flow graphs is suitable for representing and analyzing decision tables with fuzzy attributes. Every layer of a flow graph corresponds to a particular attribute, and all nodes of a layer correspond to linguistic values of the attribute. For calculating the flow between nodes, the T-norm operator `prod` was chosen in order to satisfy the flow conservation equations. New definitions of the path's certainty and strength were given with the aim to correctly determine the change of the original flow along the paths. Future work should consider the problem of generating optimal flow graphs, by taking into account the properties of crisp or fuzzy decision tables (e.g. significance of attributes). This can be done by applying the methods of the rough sets theory. In particular, the variable precision fuzzy rough sets model seems to be a promising tool for reduction of fuzzy flow graphs.

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