A Method for Designing Flexible Neuro-fuzzy Systems

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Abstract. In the paper we develop a new method for designing and reduction of flexible neuro-fuzzy systems. The method allows to reduce number of discretization points in the defuzzifier, number of rules, number of inputs, and number of antecedents. The performance of our approach is illustrated on a typical benchmark.

1 Introduction

In the last decade various neuro-fuzzy systems have been developed (see e.g. [3], [5], [7], [12], [13]). They are characterized by natural language description and learning abilities. Typical applications include identification, pattern classification, prediction and control. Most of neuro-fuzzy systems are based on the Mamdani type reasoning described by a t-norm, e.g. product or min, applied to connect antecedents and consequences in the individual rules. Another approach is based on the logical method, e.g. an S-implication (see, e.g. [4], [6]) used to connect antecedents and consequences in the rule base. Flexible neuro-fuzzy systems have been developed in [1], [2], [8]-[10]. Such systems are characterized by various flexibility parameters incorporated into their construction. Moreover, they allow to combine the Mamdani type reasoning with the logical approach and to find a fuzzy reasoning (Mamdani or logical) in the process of learning. In this paper we continue to investigate flexible neuro-fuzzy systems and the goal is to develop a new method for their designing and complexity reduction.

In this paper we consider multi-input, single-output neuro-fuzzy system mapping $\mathbf{X} \rightarrow \mathbf{Y}$, where $\mathbf{X} \subset \mathbf{R}^n$ and $\mathbf{Y} \subset \mathbf{R}$. The fuzzy rule base of these systems consists of a collection of N fuzzy IF-THEN rules in the form

$$R^{(k)}: \quad \text{IF} \quad \mathbf{x} \quad \text{is} \quad \mathbf{A}^k \quad \text{THEN} \quad y \quad \text{is} \quad B^k, \tag{1}$$

where $\mathbf{x} = [x_1, \ldots, x_n] \in \mathbf{X}$, $y \in \mathbf{Y}$, $A_1^k, A_2^k, \ldots, A_n^k$ are fuzzy sets characterized by membership functions $\mu_{A_i^k}(x_i)$, $\mathbf{A}^k = A_1^k \times A_2^k \times \ldots \times A_n^k$, and B^k are fuzzy sets characterized by membership functions $\mu_{B^k}(y)$, respectively, $k = 1, \ldots, N$.

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Defuzzification in these systems is realised for example by COA (centre of area) method defined by the following formula

$$\bar{y} = \frac{\sum_{r=1}^{N} \bar{y}^r \cdot \mu_{B'}(\bar{y}^r)}{\sum_{r=1}^{N} \mu_{B'}(\bar{y}^r)} \tag{2}$$

where B' is the fuzzy set obtained from the linguistic model (1), using an appropriate fuzzy reasoning, and $\bar{y}^r$ denotes centres of the output membership functions $\mu_{B^r}(y)$, i.e. for $r = 1, \ldots, N$,

$$\mu_{B^r}(\bar{y}^r) = \max_{y \in \mathbf{Y}} \{\mu_{B^r}(y)\}. \tag{3}$$

2 New Flexible Neuro-fuzzy Systems

Neuro-fuzzy architectures developed so far in the literature are based on the formula (2) with the assumption that number of terms in both sums is equal to the number of rules N. In this paper we relax that assumption and replace formula (2) by

$$\bar{y} = \frac{\sum_{r=1}^{R} \bar{y}^r \cdot \mu_{B'}(\bar{y}^r)}{\sum_{r=1}^{R} \mu_{B'}(\bar{y}^r)}, \tag{4}$$

where $R \geq 1$. A great advantage of formula (4) over formula (2) is that an elimination of a single rule in (4) has no effect on number of discretization points.

For further investigations we choose flexible neuro-fuzzy systems of a logical type with an S-implication given by (for details see e.g. [8]-[10])

$$\bar{y} = \frac{\sum_{r=1}^{R} \bar{y}^r \cdot agr_r(\bar{\mathbf{x}}, \bar{y}^r)}{\sum_{r=1}^{R} agr_r(\bar{\mathbf{x}}, \bar{y}^r)}, \tag{5}$$

where

$$agr_r(\bar{\mathbf{x}}, \bar{y}^r) = \begin{pmatrix} (1-\alpha^{agr}) \operatorname{avg}(I_{1,r}(\bar{\mathbf{x}}, \bar{y}^r), \ldots, I_{N,r}(\bar{\mathbf{x}}, \bar{y}^r)) + \\ +\alpha^{agr} T^* \left\{ \begin{array}{c} I_{1,r}(\bar{\mathbf{x}}, \bar{y}^r), \ldots, I_{N,r}(\bar{\mathbf{x}}, \bar{y}^r); \\ w_1^{agr}, \ldots, w_N^{agr} \end{array} \right\} \end{pmatrix}, \tag{6}$$

$$I_{k,r}(\bar{\mathbf{x}}, \bar{y}^r) = \begin{pmatrix} \left(1-\alpha_k^I\right) \operatorname{avg}(1-\tau_k(\bar{\mathbf{x}}), \mu_{B^k}(\bar{y}^r)) + \\ +\alpha_k^I S\{1-\tau_k(\bar{\mathbf{x}}), \mu_{B^k}(\bar{y}^r)\} \end{pmatrix}, \tag{7}$$

and

$$\tau_k(\bar{\mathbf{x}}) = \left(\begin{array}{c} (1-\alpha_k^\tau) \operatorname{avg}\left(\mu_{A_1^k}(\bar{x}_1), \ldots, \mu_{A_n^k}(\bar{x}_n)\right) + \\ +\alpha_k^\tau T^* \left\{ \begin{array}{c} \mu_{A_1^k}(\bar{x}_1), \ldots, \mu_{A_n^k}(\bar{x}_n); \\ w_{1,k}^\tau, \ldots, w_{n,k}^\tau \end{array} \right\} \end{array} \right). \tag{8}$$

In formulas (6) and (8) we apply the weighted t-norm [8] defined by

$$T^* \{a_1, \ldots, a_n; w_1, \ldots, w_n\} = \overset{n}{\underset{i=1}{T}} \{1 - w_i (1 - a_i)\} \tag{9}$$

to connect the antecedents in each rule, $k = 1, \ldots, N$, and to aggregate the individual rules in the logical models, respectively. Observe that if $w_1 = 0$ then $T^* \{a_1, a_2; 0, w_2\} = T\{1, 1 - w_2(1 - a_2)\} = 1 - w_2(1 - a_2)$. Therefore, antecedent a_1 is discarded since its certainty is equal to 0.

We incorporate flexibility parameters [10] into construction of new neuro-fuzzy systems. These parameters have the following interpretation:

– weights in antecedents of the rules $w_{i,k}^\tau \in [0,1]$, $i = 1, \ldots, n$, $k = 1, \ldots, N$,
– weights in aggregation of the rules $w_k^{agr} \in [0,1]$, $k = 1, \ldots, N$,
– soft strength of firing controlled by parameter α_k^τ, $k = 1, \ldots, N$,
– soft implication controlled by parameter α_k^I, $k = 1, \ldots, N$,
– soft aggregation of rules controlled by parameter α^{agr}.

The general architecture (see e.g. [8]) of the above system is depicted in Fig. 1. It is easily seen that system (4) contains $N(3n + 5) + R + 1$ parameters to be determined in the process of learning.

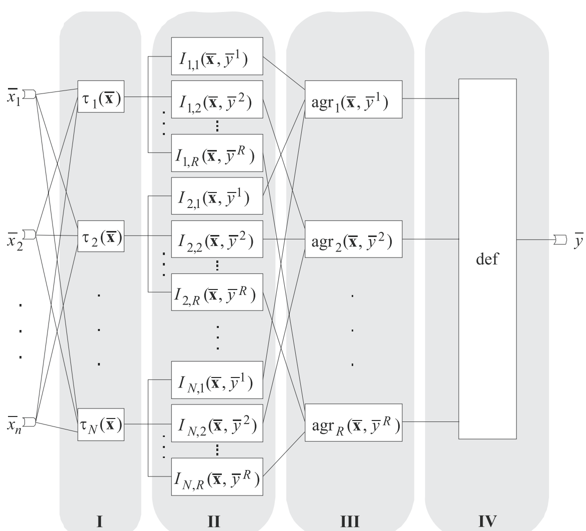

Fig. 1. The scheme of neuro-fuzzy system

3 Algorithm of Reduction of Neuro-fuzzy Systems

In this section we present an algorithm of reduction of neuro-fuzzy systems. The flowchart of the algorithm is depicted in Fig. 2.

It is assumed that the system under consideration works satisfactory after the learning process is finished. We apply the reduction procedure to that system in the following way:

- The initial system (structure and parameters) is saved before the reduction process starts.
- One parameter (discretization point in the defuzzifier, $r = 1, \ldots, R$, the whole rule, $k = 1, \ldots, N$, input, $i = 1, \ldots, n$, or antecedent, $i = 1, \ldots, n$, $k = 1, \ldots, N$) of the system is deleted.
- Learning by a single epoch is performed. Remaining parameters take over activity of the eliminated parameter.
- Performance of a reduced system is determined. If it is acceptable the reduced system is saved. Otherwise, the initial system is restored.

4 Simulation Results

The neuro-fuzzy system is simulated on Glass Identification problem [11]. The Glass Identification problem contains 214 instances and each instance is described by nine attributes (RI: refractive index, Na: sodium, Mg: magnesium, Al: aluminium, Si: silicon, K: potassium, Ca: calcium, Ba: barium, Fe: iron). All attributes are continuous. There are two classes: the window glass and the non-window glass. In our experiments, all sets are divided into a learning sequence (171 sets) and a testing sequence (43 sets). The study of the classification of the types of glass was motivated by criminological investigation. At the scene of the crime, the glass left can be used as evidence if it is correctly identified.

The experimental results for the Glass Identification problem are depicted in tables 1, 2, 3, 4, 5 and figures 3, 4. In Table 1 we show the percentage of mistakes in the learning and testing sequences before and after reduction, e.g. for $N = 2$ and $R = 3$ we have 3.51%/2.34% for the learning sequence before and after reduction and 2.33%/2.33% for the testing sequence before and after reduction. In Table 2 we present number of inputs, number of rules, number of discretization points in the defuzzifier, number of antecedents and number of parameters before and after reduction. In Table 3 we show degree of learning time reduction [%] for a reduced system. In Table 4 we present reduced inputs, antecedents, rules and discretization points in the defuzzifier. In Table 5 we depict percentage of neuro-fuzzy systems having a particular input (attribute) after the reduction process and percentage of inputs (attributes) corresponding to a particular neuro-fuzzy system after the reduction process. In Fig. 3a we show degree of parameter number reduction [%], in Fig. 3b degree of learning time reduction [%], in Fig. 4a percentage of neuro-fuzzy systems having a particular input (attribute) after the reduction process, in Fig. 4b percentage of inputs (attributes) corresponding to a particular neuro-fuzzy system after the reduction process.

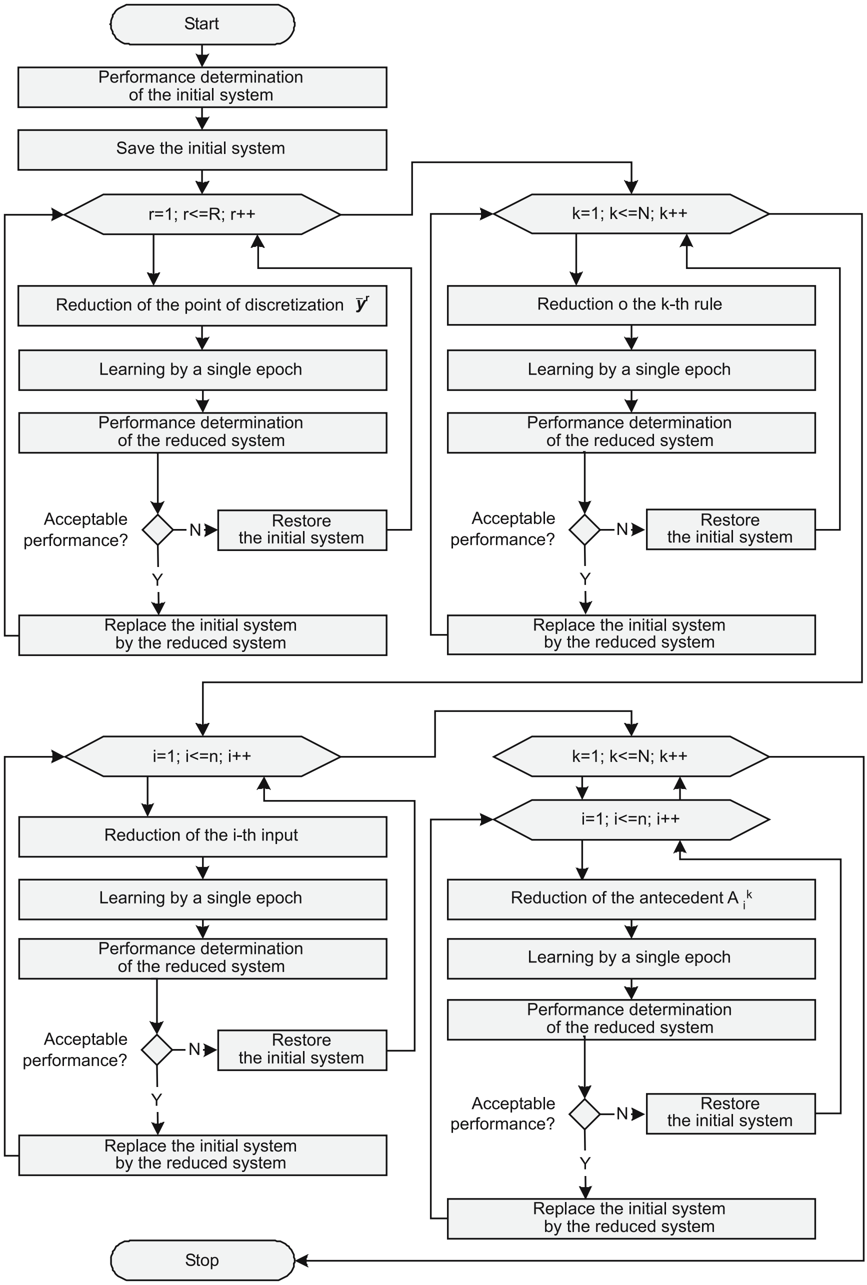

Fig. 2. The algorithm for reduction of flexible neuro-fuzzy systems

Table 1. Simulation results

Glass identification problem				
R	N			
	1	2	3	4
2	6.43%/5.85% 9.30%/9.30%	2.34%/2.34% 2.33%/2.33%	2.92%/2.92% 0.00%/0.00%	2.34%/2.34% 0.00%/0.00%
3	6.43%/5.85% 9.30%/9.30%	3.51%/2.34% 2.33%/2.33%	2.34%/2.34% 0.00%/0.00%	2.34%/2.34% 0.00%/0.00%
4	6.43%/6.43% 9.30%/9.30%	2.34%/2.34% 2.33%/2.33%	2.34%/2.34% 0.00%/0.00%	2.34%/2.34% 0.00%/0.00%

Table 2. Simulation results

Glass identification problem				
R	N			
	1	2	3	4
2	9/1/2/9/35 2/1/2/2/14	9/2/2/18/67 5/2/2/8/37	9/3/2/27/99 5/3/2/13/57	9/4/2/36/131 6/3/2/16/66
3	9/1/3/9/36 2/1/2/2/14	9/2/3/18/68 4/2/2/4/25	9/3/3/27/100 6/3/3/10/49	9/4/3/36/132 6/4/3/13/63
4	9/1/4/9/37 2/1/2/2/14	9/2/4/18/69 6/2/4/12/51	9/3/4/27/101 7/3/4/17/71	9/4/4/36/133 4/3/3/11/52

Table 3. Simulation results

Glass identification problem				
R	N			
	1	2	3	4
2	61%	58%	57%	57%
3	65%	70%	51%	48%
4	87%	50%	52%	67%

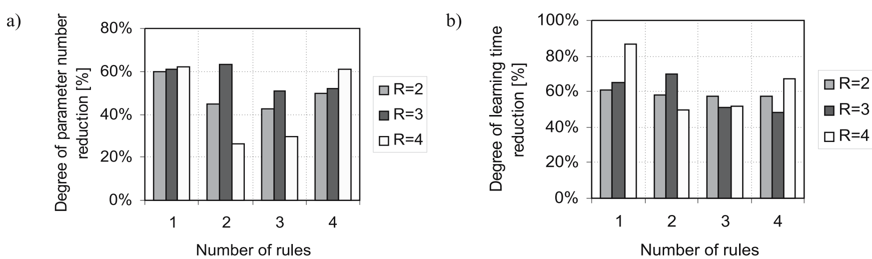

Fig. 3. Degree of a) parameter number reduction [%], b) learning time reduction [%]

Table 4. Simulation results

Glass identification problem				
R	N			
	1	2	3	4
2	$\bar{x}_1$, $\bar{x}_2$, $\bar{x}_4$, $\bar{x}_5$, $\bar{x}_6$, $\bar{x}_7$, $\bar{x}_9$	$\bar{x}_1$, $\bar{x}_2$, $\bar{x}_4$, $\bar{x}_5$, A_3^1, A_6^2	$\bar{x}_1$, $\bar{x}_5$, $\bar{x}_6$, $\bar{x}_7$, A_2^1, A_3^2	$\bar{x}_2$, $\bar{x}_6$, $\bar{x}_7$, A_1^1, A_5^2, $rule_4$
3	$\bar{x}_1$, $\bar{x}_2$, $\bar{x}_4$, $\bar{x}_5$, $\bar{x}_64$, $\bar{x}_7$, $\bar{x}_9$, $\bar{y}^1$	$\bar{x}_1$, $\bar{x}_2$, $\bar{x}_4$, $\bar{x}_5$, $\bar{x}_6$, A_3^1, A_9^1, A_7^2, A_8^2, $\bar{y}^1$	$\bar{x}_2$, $\bar{x}_5$, $\bar{x}_8$, A_1^1, A_4^1, A_9^1, A_3^2, A_9^2, A_3^3, A_4^3, A_6^3	$\bar{x}_2$, $\bar{x}_5$, $\bar{x}_7$, A_1^1, A_4^1, A_9^1, A_1^2, A_3^2, A_4^2, A_6^2, A_9^2, A_3^3, A_8^3, A_1^4
4	$\bar{x}_1$, $\bar{x}_2$, $\bar{x}_4$, $\bar{x}_5$, $\bar{x}_6$, $\bar{x}_7$, $\bar{x}_9$, $\bar{y}^1$, $\bar{y}^2$	$\bar{x}_1$, $\bar{x}_5$, $\bar{x}_7$	$\bar{x}_2$, $\bar{x}_4$, A_1^1, A_5^1, A_9^1, A_1^2	$\bar{x}_1$, $\bar{x}_2$, $\bar{x}_4$, $\bar{x}_5$, $\bar{x}_6$, A_7^1, $rule_4$

Table 5. Simulation results

Glass identification problem													
N	1	1	1	2	2	2	3	3	3	4	4	4	
R	2	3	4	2	3	4	2	3	4	2	3	4	
$\bar{x}_1$								v	v	v	v		33%
$\bar{x}_2$						v	v						17%
$\bar{x}_3$	v	v	v	v	v	v	v	v	v	v	v	v	100%
$\bar{x}_4$						v	v	v		v	v		42%
$\bar{x}_5$									v	v			17%
$\bar{x}_6$				v		v		v	v		v		42%
$\bar{x}_7$				v	v			v	v			v	42%
$\bar{x}_8$	v	v	v	v	v	v	v		v	v	v	v	92%
$\bar{x}_9$				v	v	v	v	v	v	v	v	v	75%
	22%	22%	22%	56%	44%	67%	56%	67%	78%	67%	67%	44%	

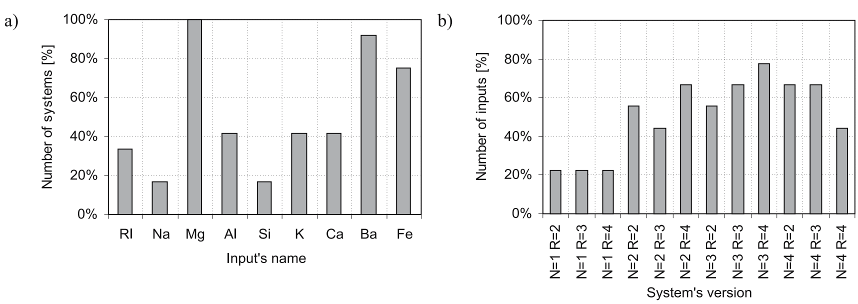

Fig. 4. Percentage of a) neuro-fuzzy systems having a particular input (attribute) after the reduction process, b) inputs (attributes) corresponding to a particular neuro-fuzzy system after the reduction process

5 Conclusions

In the paper we described a new method for designing and reduction of flexible neuro-fuzzy systems. From simulations it follows that the reduction process of neuro-fuzzy structures based on weighted triangular norms do not worsen the performance of these structures. The method allows to reduce number of discretization points in the defuzzifier, number of rules, number of inputs, and number of antecedents. It should be noted that our method allows to the decrease the learning time and to detect important features.

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References

1. K. Cpalka, L. Rutkowski, Neuro-Fuzzy Structures for Pattern Classification, WSEAS Trans. on Computers, Issue 7, vol.4, July 2005, pp. 697 688.
2. K. Cpalka, L. Rutkowski, Flexible Takagi-Sugeno Neuro-Fuzzy Structures for Nonlinear Approximation, WSEAS Trans. on Computers, Issue 9, vol.4, September 2005, pp. 1450-1458.
3. E. Czogala, J. Leski, Fuzzy and Neuro-Fuzzy Intelligent Systems, Physica Verlag, A Springer Verlag Company, Heidelberg, New York, 2000.
4. C. Fodor, On fuzzy implication, Fuzzy Sets and Systems, vol. 42, pp. 293 300, 1991.
5. M. Gorzalczany, Computational Intelligence Systems and Applications: Neuro-Fuzzy and Fuzzy Neural Synergisms. Springer Verlag 2002.
6. E. P. Klement, R. Mesiar, E. Pap, Triangular Norms, Kluwer Academic Publishers, Netherlands 2000.
7. D. Nauck and R. Kruse. Neuro-fuzzy systems for function approximation, Fuzzy Sets and Systems, vol. 101, pp. 261 271, 1999.
8. L. Rutkowski, Flexible Neuro-Fuzzy Systems. Kluwer Academic Publishers, 2004.
9. L. Rutkowski and K. Cpalka, Designing and learning of adjustable quasi triangular norms with applications to neuro-fuzzy systems, IEEE Trans. on Fuzzy Systems, vol. 13, Feb 2005, pp. 140 151.
10. L. Rutkowski, K. Cpalka, Flexible neuro-fuzzy systems, IEEE Transactions on Neural Networks, vol. 14, May 2003, pp. 554 574.
11. UCI respository of machine learning databases, Available online: http://ftp.ics.uci.edu/pub/machine learning databases/.
12. J. S. Wang and C. S. G. Lee. Self adaptive neuro-fuzzy inference systems for classification applications, IEEE Trans. on Fuzzy Systems, vol. 10, pp. 790 802, December 2002.
13. R. R. Yager, D. P. Filev, Essentials of fuzzy modelling and control. John Wiley & Sons, 1994.