New Methods for Evaluating the Answerscripts of Students Using Fuzzy Sets

Hui-Yu Wang[1] and Shyi-Ming Chen[2]

[1] Department of Education, National Chengchi University, Taipei, Taiwan, R.O.C.
94152514@nccu.edu.tw
[2] Department of Computer Science and Information Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan, R.O.C.
smchen@et.ntust.edu.tw

Abstract. In this paper, we present new methods for evaluating the answerscripts of students, where the evaluating values are represented by fuzzy numbers, and an optimism index λ determined by the evaluator is used to indicate the degree of optimism of the evaluator for evaluating the answerscripts of students, where the value of λ is between zero and one. The universe of discourse is formed by a set of satisfaction levels. The fuzzy mark awarded to the answer of each question of the answerscript of a student is represented by a type-2 fuzzy set. The proposed methods can overcome the drawbacks of the existing methods. It can evaluate the answerscripts of students in a more flexible and more intelligent manner.

1 Introduction

In recent years, some methods have been presented for students' evaluation [1], [2], [5], [7]-[10], [13]-[17]. In [5], Chen and Lee have pointed out that the methods presented in [1] have the following drawbacks: (1) Because they used a matching function to measure the degrees of similarity between the standard fuzzy sets and the fuzzy marks of the questions, it will take a large amount of time to perform the matching operations. (2) Two different fuzzy marks may be translated into the same awarded grade and it is unfair for students' evaluation. Therefore, they presented two new methods for applying fuzzy sets in students' answerscripts evaluation to overcome the drawbacks of the ones presented in [1]. However, the drawbacks of the methods presented in [5] are that they can not deal with the situation that the evaluating values are represented by fuzzy numbers and they don't consider the degree of optimism of the evaluator in evaluating students' answerscripts. If we can allow the evaluating values to be represented by fuzzy numbers and can consider the degree of optimism of the evaluator in evaluating the students' answerscripts, then there is room for more flexibility.

In this paper, we present new methods for evaluating students' answerscripts, where the evaluating values are represented by fuzzy numbers and an optimism index λ[7] determined by the evaluator is used to indicate the degree of optimism of the evaluator for evaluating students' answerscripts, where $0 \leq \lambda \leq 1$. The fuzzy mark awarded to the answer of each question of a student's answerscript is represented by a

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type-2 fuzzy set. The proposed methods can overcome the drawbacks of the methods presented in [1] and [5]. It can evaluate students' answerscripts in a more flexible and more intelligent manner.

2 Basic Concepts of Fuzzy Sets and Fuzzy Numbers

In the following, we briefly review basic concepts of fuzzy sets and fuzzy numbers from [4], [12], [19] and [20]. Let X be the universe of discourse, $X = \{x_1, x_2, \cdots, x_n\}$. A fuzzy set A of the universe of discourse X can be represented as follows:

$$A = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), \cdots, (x_n, \mu_A(x_n))\}, \tag{1}$$

where μ_A is the membership function of the fuzzy set A, $\mu_A(x_i)$ denotes the grade of membership of x_i belonging to the fuzzy set A, and $\mu_A(x_i) \in [0, 1]$. A fuzzy set A of the universe of discourse U is convex if and only if for all u_1, u_2 in X,

$$\mu_A(\lambda x_1 + (1 - \lambda) x_2) \geq \text{Min}(\mu_A(x_1), (\mu_A(x_2)), \tag{2}$$

where $0 \leq \lambda \leq 1$. If $\exists\, x_i \in X$, such that $\mu_A(x_i) = 1$, then the fuzzy set A is called a normal fuzzy set. A fuzzy number is a fuzzy set in the universe of discourse X that is both convex and normal. A triangular fuzzy number A of the universe of discourse X can be characterized by a triangular membership function parameterized by a triplet (a, b, c) as shown in Fig. 1. According to [4] and [12], the defuzzified value *DEF(A)* of the trapezoidal fuzzy number A shown in Fig. 1 is as follows:

$$DEF(A) = \frac{a+b+b+c}{4}. \tag{3}$$

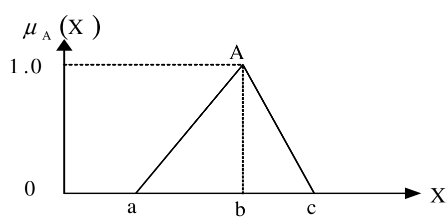

Fig. 1. A triangular fuzzy number A

3 A New Method for Student's Evaluation Using Fuzzy Numbers

In this section, we present a new method for students' answerscripts evaluation, where the evaluating values are represented by fuzzy numbers and an optimism index λ [7] determined by the evaluator is used to indicate the degree of optimism of the evaluator for evaluating students' answerscripts, where $\lambda \in [0, 1]$. If $0 \leq \lambda < 0.5$, then the evaluator is a pessimistic evaluator. If $\lambda = 0.5$, then the evaluator is a normal evaluator. If $0.5 < \lambda \leq 1.0$, then the evaluator is an optimistic evaluator. Eleven satisfaction levels shown in Table 1 [5] are used to evaluate the students' answerscripts regarding a question of a test/examination, where the corresponding degrees of satisfaction of the eleven satisfaction levels are shown in Table 1.

Table 1. Satisfaction Levels and Their Corresponding Degrees of Satisfaction [5]

Satisfaction Levels	Degrees of Satisfaction
extremely good (EG)	100% (i.e., 1.00)
very very good (VVG)	91%-99% (i.e., 0.91-0.99)
very good (VG)	81%-90% (i.e., 0.81-0.90)
good (G)	71%-80% (i.e., 0.71-0.80)
more or less good (MG)	61%-70% (i.e., 0.61-0.70)
fair (F)	51%-60% (i.e., 0.51-0.60)
more or less bad (MB)	41%-50% (i.e., 0.41-0.50)
bad (B)	25%-40% (i.e., 0.25-0.40)
very bad (VB)	10%-24% (i.e., 0.10-0.24)
very very bad (VVB)	1%-9% (i.e., 0.01-0.09)
extremely bad (EB)	0% (i.e., 0)

Let T and L be two mapping functions to map a satisfaction level to its maximum degree of satisfaction and its minimum degree of satisfaction shown in Table 1, respectively. Therefore, from Table 1, we can see that

$$
\begin{aligned}
&\text{T(extremely good)} = 1.00 \text{ (i.e., T(EG)} = 1.00\text{)},\\
&\text{L(extremely good)} = 1.00 \text{ (i.e., L(EG)} = 1.00\text{)},\\
&\text{T(very very good)} = 0.99 \text{ (i.e., T(VVG)} = 0.99\text{)},\\
&\text{L(very very good)} = 0.91 \text{ (i.e., L(VVG)} = 0.91\text{)},\\
&\text{T(very good)} = 0.90 \text{ (i.e., T(VG)} = 0.90\text{)},\\
&\text{L(very good)} = 0.81 \text{ (i.e., L(VG)} = 0.81\text{)},\\
&\text{T(good)} = 0.80 \text{ (i.e., T(G)} = 0.80\text{)},\\
&\text{L(good)} = 0.71 \text{ (i.e., L(G)} = 0.71\text{)},\\
&\text{T(more or less good)} = 0.70 \text{ (i.e., T(MG)} = 0.70\text{)},\\
&\text{L(more or less good)} = 0.61 \text{ (i.e., L(MG)} = 0.61\text{)},\\
&\text{T(fair)} = 0.60 \text{ (i.e., T(F)} = 0.60\text{)},\\
&\text{L(fair)} = 0.51 \text{ (i.e., L(F)} = 0.51\text{)},\\
&\text{T(more or less bad)} = 0.50 \text{ (i.e., T(MB)} = 0.50\text{)},\\
&\text{L(more or less bad)} = 0.41 \text{ (i.e., L(MB)} = 0.41\text{)},\\
&\text{T(bad)} = 0.40 \text{ (i.e., T(B)} = 0.40\text{)},\\
&\text{L(bad)} = 0.25 \text{ (i.e., L(B)} = 0.25\text{)},\\
&\text{T(very bad)} = 0.24 \text{ (i.e., T(VB)} = 0.24\text{)},\\
&\text{L(very bad)} = 0.10 \text{ (i.e., L(VB)} = 0.10\text{)},\\
&\text{T(very very bad)} = 0.09 \text{ (i.e., T(VVB)} = 0.09\text{)},\\
&\text{L(very very bad)} = 0.01 \text{ (i.e., L(VVB)} = 0.01\text{)},\\
&\text{T(extremely bad)} = 0 \text{ (i.e., T(EB)} = 0\text{)}.
\end{aligned}
\tag{4}
$$

Assume that an evaluator evaluates the students' answerscripts by using an extended fuzzy grade sheet as shown in Table 2, where $\tilde{y}_i$ denotes a fuzzy number defined in [0, 1] and $1 \le i \le 11$. In any row of Table 4, the columns from the second to the twelfth indicate the fuzzy mark awarded to the answer to the corresponding question in the first column, where the fuzzy mark is represented as a type-2 fuzzy set. The last (i.e., the thirteenth) column of the extended fuzzy grade sheet shown in Table 2 indicates the degree of satisfaction evaluated by the proposed method awarded to each question. The box at the bottom of the extended fuzzy grade sheet shown in Table 2 indicates the total mark awarded to the student. Let $\tilde{0}$, $\tilde{0}$, $0.\tilde{1}$, $0.\tilde{2}$, $0.\tilde{3}$, $0.\tilde{4}$, $0.\tilde{5}$, $0.\tilde{6}$, $0.\tilde{7}$, $0.\tilde{8}$, $0.\tilde{9}$ and $1.\tilde{0}$ be triangular fuzzy numbers, where

$$\begin{aligned}
&\tilde{0} = (0, 0, 0), && 0.\tilde{1} = (0, 0.1, 0.2), && 0.\tilde{2} = (0.1, 0.2, 0.3),\\
&0.\tilde{3} = (0.2, 0.3, 0.4), && 0.\tilde{4} = (0.3, 0.4, 0.5), && 0.\tilde{5} = (0.4, 0.5, 0.6),\\
&0.\tilde{6} = (0.5, 0.6, 0.7), && 0.\tilde{7} = (0.6, 0.7, 0.8), && 0.\tilde{8} = (0.7, 0.8, 0.9),\\
&0.\tilde{9} = (0.8, 0.9, 1.0), && 1.\tilde{0} = (1.0, 1.0, 1.0).
\end{aligned} \tag{5}$$

Table 2. Fuzzy Mark Represented by Fuzzy Numbers of Question Q.*i* in An Extended Fuzzy Grade Sheet

Question No.	Satisfaction Levels											Degree of Satisfaction
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Q.*i*	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{y}_5$	$\tilde{y}_6$	$\tilde{y}_7$	$\tilde{y}_8$	$\tilde{y}_9$	$\tilde{y}_{10}$	$\tilde{y}_{11}$	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
												Total Mark =

For example, assume that an evaluator uses an extended fuzzy grade sheet to evaluate the fuzzy mark of the first question (i.e., Q.1) of a test/examination of a student, shown as follows:

$$\begin{aligned}
FN_{Q.1} = \{&(\text{EG}, \tilde{0}), (\text{VVG}, 0.\tilde{9}), (\text{VG}, 0.\tilde{8}), (\text{G}, 0.\tilde{5}), (\text{MG}, \tilde{0}),\\
&(\text{V}, \tilde{0}), (\text{MB}, \tilde{0}), (\text{B}, \tilde{0}), (\text{VB}, \tilde{0}), (\text{VVB}, \tilde{0}), (\text{EB}, \tilde{0})\}.
\end{aligned}$$

For convenience, the fuzzy set $FN_{Q.1}$ can also be abbreviated into

$$FN_{Q.1} = \{(\text{VVG}, 0.\tilde{9}), (\text{VG}, 0.\tilde{8}), (\text{G}, 0.\tilde{5})\}.$$

It indicates that the satisfaction level of the student's answerscript with respect to the first question is: about 90% very very good, about 80% very good and about 50% good.

Assume that the fuzzy mark of the question Q.*i* of a student's answerscript evaluated by an evaluator is as shown in Table 2, where $\tilde{y}_i$ is a fuzzy number in the universe of discourse [0, 1] and $1 \leq i \leq 11$. Assume that the degree of optimism of the evaluator determined by the evaluator for evaluating students' answerscript is λ, where $\lambda \in [0, 1]$. If $0 \leq \lambda < 0.5$, then the evaluator is a pessimistic evaluator. If $\lambda = 0.5$, then the evaluator is a normal evaluator. If $0.5 < \lambda \leq 1.0$, then the evaluator is an optimistic evaluator. The proposed method for students' answerscripts evaluation based on fuzzy numbers is now presented as follows:

Step 1: Based on formula (3), defuzzify each fuzzy number $\tilde{y}_i$ in the extended fuzzy grade sheet shown in Table 2 into a crisp value DEF($\tilde{y}_i$), where DEF($\tilde{y}_i$)$\in [0, 1]$ and $1 \leq i \leq 11$, as shown in Table 3.

Table 3. Defuzzified Values of Fuzzy Marks of Question Q.*i* of Table 2

Question No.	Satisfaction Levels											Degree of Satisfaction
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Q.*i*	DEF($\tilde{y}_1$)	DEF($\tilde{y}_1$)	DEF($\tilde{y}_1$)	DEF($\tilde{y}_1$)	DEF($\tilde{y}_1$)	DEF($\tilde{y}_1$)	DEF($\tilde{y}_1$)	DEF($\tilde{y}_1$)	DEF($\tilde{y}_1$)	DEF($\tilde{y}_1$)	DEF($\tilde{y}_1$)	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
												Total Mark =

Step 2: From formula (4), we can see that T(EG) = 1, L(EG) = 1, T(VVG) = 0.99, L(VVG) = 0.91, T(VG) = 0.90, L(VG) = 0.81, T(G) = 0.80, L(G) = 0.71, T(MG) = 0.70, L(MG) = 0.61, T(F) = 0.60, L(F) = 0.51, T(MB) = 0.50, L(MB) = 0.41, T(B) = 0.40, L(B) = 0.25, T(VB) = 0.24, L(VB) = 0.10, T(VVB) = 0.09, L(VVB) = 0.01, T(EB) = 0 and L(EB) = 0. In this case, the degree of satisfaction D(Q.*i*) of the question Q.*i* of the student's answerscript can be evaluated by the function D,

$$\mathrm{D(Q}.i) = \frac{DEF(\tilde{y}_1)\times[(1-\lambda)\mathrm{L(EG)}+\lambda T(EG)] + DEF(\tilde{y}_2)\times[(1-\lambda)\mathrm{L(VVG)}+\lambda T(VVG)] + \ldots + DEF(\tilde{y}_{11})\times[(1-\lambda)\mathrm{L(EB)}+\lambda T(EB)]}{DEF(\tilde{y}_1) + DEF(\tilde{y}_2) + \ldots + DEF(\tilde{y}_{11})}, \quad (6)$$

where DEF($\tilde{y}_j$) denotes the defuzzified value of the fuzzy number $\tilde{y}_j$, $1 \leq j \leq 11$, and $0 \leq$ D(Q.*i*) ≤ 1. The larger the value of D(Q.*i*), the higher the degree of satisfaction that the question Q.i of the student's answerscript satisfies the evaluator's opinion.

Step 3: Consider the situation that the total mark of a candidate's answerscript to a paper is 100 marks. Assume that there are *n* questions to be answered, i.e.,

TOTAL MARKS = 100,
Q.1 carries s_1 marks,
Q.2 carries s_2 marks,
⋮
Q.n carries s_n marks,

where $\sum_{i=1}^{n} s_i = 100$, $0 \leq s_i \leq 100$, and $1 \leq i \leq n$. Assume that the evaluated degree of satisfaction of the question Q.1, Q.2, ⋯, and Q.*n* are D(Q.1), D(Q. 2), ⋯, and D(Q.*n*), respectively, then the total mark of the student is evaluated as follows:

$$s_1 \times D(Q.1) + s_2 \times D(Q.2) + \cdots + s_n \times D(Q.n). \quad (7)$$

Put this total mark in the appropriate box at the bottom of the extended fuzzy grade sheet.

Example 3.1: Consider a candidate's answerscript to an examination of 100 marks. Assume that in total there are four questions to be answered:

TOTAL MARKS = 100,
Q.1 carries 20 marks,
Q.2 carries 30 marks,
Q.3 carries 25 marks,
Q.4 carries 25 marks.

Assume that an evaluator awards the students' answerscript by an extended fuzzy grade sheet as shown in Table 4 and assume that the optimism index λ of the evaluator is 0.6 (i.e., $\lambda = 0.6$).

Table 4. Extended Fuzzy Grade Sheet of Example 3.1

Question No.	Satisfaction Levels											Degree of Satisfaction
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
Q.1	$0.\tilde{8}$	$0.\tilde{9}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	
Q.2	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$0.\tilde{6}$	$0.\tilde{9}$	$0.\tilde{5}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	
Q.3	$\tilde{0}$	$\tilde{0}$	$0.\tilde{8}$	$0.\tilde{7}$	$0.\tilde{5}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	
Q.4	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$0.\tilde{5}$	$0.\tilde{9}$	$0.\tilde{2}$	$\tilde{0}$	
												Total Mark =

Step 1: From formula (5), we can see that $\tilde{0} = (0, 0, 0)$, $0.\tilde{2} = (0.1, 0.2, 0.3)$, $0.\tilde{5} = (0.4, 0.5, 0.6)$, $0.\tilde{6} = (0.5, 0.6, 0.7)$, $0.\tilde{7} = (0.6, 0.7, 0.8)$, $0.\tilde{8} = (0.7, 0.8, 0.9)$ and $0.\tilde{9} = (0.8, 0.9, 1.0)$. Based on formula (3), we can get $DEF(\tilde{0}) = 0$, $DEF(0.\tilde{2}) = 0.2$, $DEF(0.\tilde{5}) = 0.5$, $DEF(0.\tilde{6}) = 0.6$, $DEF(0.\tilde{8}) = 0.8$, and $DEF(0.\tilde{9}) = 0.9$. Therefore, the triangular fuzzy numbers shown in Table 4 can be defuzzified into crisp values as shown in Table 5.

Step 2: Because the optimism index λ of the evaluator is 0.6 (i.e., $\lambda = 0.6$), according to formula (4) and by applying formula (6), we can see that

$$D(Q.1) = \frac{DEF(0.\tilde{8}) \times [(1-0.6) \times L(EG) + 0.6 \times T(EG)] + DEF(0.\tilde{9}) \times [(1-0.6) \times L(VVG) + 0.6 \times T(VVG)]}{DEF(0.\tilde{8}) + DEF(0.\tilde{9})},$$

$$= \frac{0.8 \times [(1-0.6) \times L(EG) + 0.6 \times T(EG)] + 0.9 \times [(1-0.6) \times L(VVG) + 0.6 \times T(VVG)]}{0.8 + 0.9},$$

$$= \frac{0.8 \times (0.4 \times 1.0 + 0.6 \times 1.0) + 0.9 \times (0.4 \times 0.91 + 0.6 \times 0.99)}{0.8 + 0.9},$$

$$= 0.9778,$$

$$D(Q.2) = \frac{DEF(\tilde{0.6}) \times [(1-0.6) \times L(G) + 0.6 \times T(G)] + DEF(\tilde{0.9}) \times [(1-0.6) \times L(MG) + 0.6 \times T(MG)] + DEF(\tilde{0.5}) \times [(1-0.6) \times L(F) + 0.6 \times T(F)]}{DEF(\tilde{0.6}) + DEF(\tilde{0.9}) + DEF(\tilde{0.5})},$$

$$= \frac{0.6 \times [(1-0.6) \times L(G) + 0.6 \times T(G)] + 0.9 \times [(1-0.6) \times L(MG) + 0.6 \times T(MG)] + 0.5 \times [(1-0.6) \times L(F) + 0.6 \times T(F)]}{0.6 + 0.9 + 0.5},$$

$$= \frac{0.6 \times (0.4 \times 0.71 + 0.6 \times 0.80) + 0.9 \times (0.4 \times 0.61 + 0.6 \times 0.70) + 0.5 \times (0.4 \times 0.51 + 0.6 \times 0.60)}{0.6 + 0.9 + 0.5},$$

$$= 0.6690,$$

$$D(Q.3) = \frac{DEF(\tilde{0.8}) \times [(1-0.6) \times L(VG) + 0.6 \times T(VG)] + DEF(\tilde{0.7}) \times [(1-0.6) \times L(G) + 0.6 \times T(G)] + DEF(\tilde{0.5}) \times [(1-0.6) \times L(MG) + 0.6 \times T(MG)]}{DEF(\tilde{0.8}) + DEF(\tilde{0.7}) + DEF(\tilde{0.5})},$$

$$= \frac{0.8 \times [(1-0.6) \times L(VG) + 0.6 \times T(VG)] + 0.7 \times [(1-0.6) \times L(G) + 0.6 \times T(G)] + 0.5 \times [(1-0.6) \times L(MG) + 0.6 \times T(MG)]}{0.8 + 0.7 + 0.5},$$

$$= \frac{0.8 \times (0.4 \times 0.81 + 0.6 \times 0.90) + 0.7 \times (0.4 \times 0.71 + 0.6 \times 0.80) + 0.5 \times (0.4 \times 0.61 + 0.6 \times 0.70)}{0.8 + 0.7 + 0.5},$$

$$= 0.7790,$$

$$D(Q.4) = \frac{DEF(\tilde{0.5}) \times [(1-0.6) \times L(B) + 0.6 \times T(B)] + DEF(\tilde{0.9}) \times [(1-0.6) \times L(VB) + 0.6 \times T(VB)] + DEF(\tilde{0.2}) \times [(1-0.6) \times L(VVB) + 0.6 \times T(VVB)]}{DEF(\tilde{0.5}) + DEF(\tilde{0.9}) + DEF(\tilde{0.2})},$$

$$= \frac{0.5 \times [(1-0.6) \times L(B) + 0.6 \times T(B)] + 0.9 \times [(1-0.6) \times L(VB) + 0.6 \times T(VB)] + 0.2 \times [(1-0.6) \times L(VVB) + 0.6 \times T(VVB)]}{0.5 + 0.9 + 0.2},$$

$$= \frac{0.5 \times (0.4 \times 0.25 + 0.6 \times 0.40) + 0.9 \times (0.4 \times 0.10 + 0.6 \times 0.24) + 0.2 \times (0.4 \times 0.01 + 0.6 \times 0.09)}{0.5 + 0.9 + 0.2},$$

$$= 0.2170.$$

Table 5. Defuzzified Values of Fuzzy Numbers of the Extended Fuzzy Grade Sheet Shown in Table 4

Question No.	Satisfaction Levels											Degree of Satisfaction
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
Q.1	0.8	0.9	0	0	0	0	0	0	0	0	0	
Q.2	0	0	0	0.6	0.9	0.5	0	0	0	0	0	
Q.3	0	0	0.8	0.7	0.5	0	0	0	0	0	0	
Q.4	0	0	0	0	0	0	0	0.5	0.9	0.2	0	
												Total Mark =

Step 3: By applying formula (7), the total mark of the student can be evaluated as follows:

$$\begin{aligned}
& 20 \times D(Q.1) + 30 \times D(Q.2) + 25 \times D(Q.3) + 25 \times D(Q.4) \\
&= 20 \times 0.9778 + 30 \times 0.6690 + 25 \times 0.7790 + 25 \times 0.2170 \\
&= 19.556 + 20.07 + 19.475 + 5.425 \\
&= 64.526 \\
&\cong 65 \text{ (assuming that no half mark is given in the total mark).}
\end{aligned}$$

4 A Generalized Fuzzy Evaluation Method Using Fuzzy Numbers

In this section, we present a generalized fuzzy evaluation method for students' answerscripts evaluation using fuzzy numbers. Consider a candidate's answerscript to a paper of 100 marks. Assume that there are n questions to be answered:

TOTAL MARKS = 100,
Q.1 carries s_1 marks,
Q.2 carries s_2 marks,
$\vdots$
Q.n carries s_n marks,

where $\sum_{i=1}^{n} s_i = 100$, $0 \le s_i \le 100$, and $1 \le i \le n$. Assume that the degree of optimism of the evaluator is λ, where $\lambda \in [0, 1]$. If $0 \le \lambda < 0.5$, then the evaluator is a pessimistic evaluator. If $\lambda = 0.5$, then the evaluator is a normal evaluator. If $0.5 < \lambda \le 1.0$, then the evaluator is an optimistic evaluator. Assume that an evaluator evaluates the questions of students' answerscripts using the following four criteria [1]:

C_1: Accuracy of information,
C_2: Adequate coverage,
C_3: Conciseness,
C_4: Clear expression,

and assume that the weights of the criteria C_1, C_2, C_3 and C_4 are w_1, w_2, w_3 and w_4, respectively, where $0 \le w_i \le 1$ and $1 \le i \le 4$. Furthermore, assume that the evaluator can evaluate each question of students' answerscripts using the above four criteria based on the method described in Section 3. In this case, an evaluator can evaluate students' answerscripts using a generalized extended fuzzy grade sheet as shown in Table 6, where the evaluating values in Table 6 are represented by fuzzy numbers and the degrees of satisfaction of the question Q.i of a student's answerscript regarding to the criteria C_1, C_2, C_3 and C_4 evaluated by the proposed method presented in Section 3 are $D(C_{i1})$, $D(C_{i2})$, $D(C_{i3})$, and $D(C_{i4})$, respectively, where $0 \le D(C_{i1}) \le 1$, $0 \le D(C_{i2}) \le 1$, $0 \le D(C_{i3}) \le 1$, $0 \le D(C_{i4}) \le 1$, and $1 \le i \le n$.

The degree of satisfaction P(Q.i) of the question Q.i of the student's answerscript can be evaluated as follows:

$$P(Q.i) = \frac{w_1 \times D(C_{i1}) + w_2 \times D(C_{i2}) + w_3 \times D(C_{i3}) + w_4 \times D(C_{i4})}{w_1 + w_2 + w_3 + w_4}, \tag{8}$$

where $0 \le P(Q.i) \le 1$ and $1 \le i \le n$. The total mark of the student can be evaluated and is equal to

$$s_1 \times P(Q.1) + s_2 \times P(Q.2) + \cdots + s_n \times P(Q.n). \tag{9}$$

Put this total score in the appropriate box at the bottom of the extended fuzzy grade sheet.

Table 6. A Generalized Extended Fuzzy Grade Sheet [5]

Question No.	Criteria	Satisfaction Levels											Degree of Satisfaction for Criteria	Degree of Satisfaction for Questions
		EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB		
Q.1	C_1												$D(C_{11})$	P(Q.1)
	C_2												$D(C_{12})$	
	C_3												$D(C_{13})$	
	C_4												$D(C_{14})$	
Q.2	C_1												$D(C_{21})$	P(Q.2)
	C_2												$D(C_{22})$	
	C_3												$D(C_{23})$	
	C_4												$D(C_{24})$	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Q.*n*	C_1												$D(C_{n1})$	P(Q.*n*)
	C_2												$D(C_{n2})$	
	C_3												$D(C_{n3})$	
	C_4												$D(C_{n4})$	
								Total Mark = $s_1 \times P(Q.1) + s_2 \times P(Q.2) + \cdots + s_n \times P(Q.n)$						

5 Conclusions

In this paper, we have presented new methods for evaluating students' answerscripts, where the evaluating values are represented by fuzzy numbers, and an optimism index λ determined by the evaluator is used to indicate the degree of optimism of the evaluator for evaluating students' answerscripts, where $\lambda \in [0, 1]$. The universe of discourse is formed by a set of satisfaction levels. The fuzzy mark awarded to the answer of each question of a student's answerscript is represented by a type-2 fuzzy set. The proposed methods can overcome the drawbacks of the methods presented in [1] and [5]. It can evaluate students' answerscripts in a more flexible and more intelligent manner.

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