A New Method for Appraising the Performance of High School Teachers Based on Fuzzy Number Arithmetic Operations

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Abstract. In recent years, some methods have been presented for appraising the performance of high school teachers. In this paper, we present a new method for appraising the performance of high school teachers based on simplified fuzzy number arithmetic operations. The proposed method uses fuzzy numbers to represent fuzzy grades, where the fuzzy weights of criteria are automatically generated from the opinions of evaluators. The proposed method can appraise the performance of high school teachers in a more flexible and more intelligent manner.

1 Introduction

In 1965, Zadeh proposed the theory of fuzzy sets [17]. In recent years, the application of the fuzzy set theory in education has begun [1], [2], [7], [8], [9], [10], [12], [14], [15], [16]. In [1], Biswas presented a fuzzy evaluation method (fem) and a generalized fuzzy evaluation method for students' answerscripts evaluation. In [2], Chang et al. presented a method for fuzzy assessment of junior high school students. In [7], Chen et al. presented new methods for students' evaluation using fuzzy sets. In [8], Cheng and Yang presented a method for the application of the fuzzy set theory in educational grading systems. In [9], Cheng et al. established the criteria for high school teachers appraisal. They used fuzzy linguistic questionnaires to appraise the performance of high school teachers and used fuzzy linguistic integrating operations to calculate teachers' fuzzy grades, and then used Lee-and-Li's method [13] and Chen-and-Cheng's method [3] to rank teachers' fuzzy grades.

In this paper, we present a new method to appraise the performance of high school teachers based on fuzzy number arithmetic operation. The proposed method appraises the performance of high school teachers based on simplified fuzzy number arithmetic. It can appraise high school teachers in a more flexible and more intelligent manner than the method presented in [9].

2 Simplified Fuzzy Arithmetic Operations Between Fuzzy Numbers

In 1965, Zadeh presented the theory of fuzzy sets [17]. A fuzzy number [11], [18] is a fuzzy set in the universe of discourse X that is both convex and normal. A trapezoidal

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fuzzy number $\tilde{A}$ of the universe of discourse X can be characterized by a trapezoidal membership function parameterized by a quadruple (a, b, c, d) as shown in Fig. 1, where a, b, c and d are real values.

In Fig. 1, if b = c, then $\tilde{A}$ becomes a triangular fuzzy number, and it can be parameterized by a triplet (a, b, d). Let $\tilde{A}_1$ and $\tilde{A}_2$ be two triangular fuzzy numbers parameterized by the triplets (a_1, b_1, c_1) and (a_2, b_2, c_2), respectively. The simplified fuzzy number arithmetic operations between the triangular fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ are as follows [11]:

Fuzzy Number Addition $\oplus$:

$$(a_1, b_1, c_1) \oplus (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2). \tag{1}$$

Fuzzy Number Multiplication $\otimes$:

$$(a_1, b_1, c_1) \otimes (a_2, b_2, c_2) = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2). \tag{2}$$

Fuzzy Number Division $\oslash$:

$$(a_1, b_1, c_1) \oslash (a_2, b_2, c_2) = (a_1/c_2, b_1/b_2, c_1/a_2). \tag{3}$$

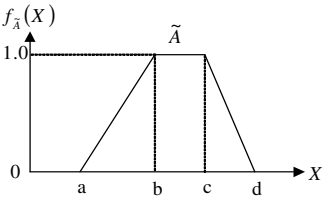

Fig. 1. A trapezoidal fuzzy number

3 A Review of Chen-and-Chen's Method for Ranking Fuzzy Numbers

In [6], Chen and Chen presented a method for ranking fuzzy numbers. The ranking value Rank($\tilde{A}$) of a generalized trapezoidal fuzzy number $\tilde{A}$, $\tilde{A}$ = (a_1, a_2, a_3, a_4), can be calculated as follows:

$$Rank(\tilde{A}) = x^*_{\tilde{A}} + (1 - y^*_{\tilde{A}})^{\hat{S}_{\tilde{A}}}, \tag{4}$$

where

$$x^*_{\tilde{A}} = \frac{y^*_{\tilde{A}}(a_3 + a_2) + (a_4 + a_1)(1 - y^*_{\tilde{A}})}{2}, \tag{5}$$

$$\hat{S}_{\tilde{A}} = \sqrt{\frac{\sum_{i=1}^{4}(a_i - \bar{a})}{3}}, \tag{6}$$

$$y^*_{\tilde{A}} = \begin{cases} \dfrac{1\times\left(\dfrac{a_3 - a_2}{a_4 - a_1} + 2\right)}{6}, & if\ a_1 \neq a_4 \\ \dfrac{1}{2}, & if\ a_1 = a_4 \end{cases} \tag{7}$$

where $\bar{a} = \frac{a_1 + a_2 + a_3 + a_4}{4}$. The larger the value of Rank($\tilde{A}$), the better the ranking of $\tilde{A}$.

4 A New Method for High School Teachers' Appraisal

In this section, we present a new method for high school teachers' appraisal, where the criteria and the fuzzy linguistic questionnaires used for teacher appraisal used in [9] are adopted as shown in Table 1 and Table 2, respectively.

Table 1. The Criteria for Appraising the Performance of High School Teachers [9]

Criteria	Sub-criteria	Contents
X_1 (Teaching)	X_{11}	According to the standard of the course and the needs of the students to achieve the teaching goals
	X_{12}	To make teaching plans and to prepare teaching materials and tools properly
	X_{13}	To present teaching materials clearly and correctly with proper methods
	X_{14}	To bring the motivation of learning and testing and to increase students attention on lessons
	X_{15}	To express clearly in speaking (helpful for comprehension and learning.)
	X_{16}	To express clearly in writing (helpful for comprehension and learning.)
	X_{17}	To make good use of various of teaching methods and teaching media
	X_{18}	To make good use of time while teaching
	X_{19}	Giving proper feedback to students while teaching
X_2 (Class Managing)	X_{21}	Counseling students to obey the rules of life and develop good habits
	X_{22}	To direct students to keep personal and public sanitation
	X_{23}	Management of classroom
	X_{24}	Establishing an appropriate rule of class
	X_{25}	To praise and punish students properly
X_3 (Professional Attitude and Specialty)	X_{31}	To take care of students actively
	X_{32}	Working hard
	X_{33}	Researching and pursuing further education
	X_{34}	Being enthusiastic with working
X_4 (Cooperation in Administration and Relationship with People)	X_{41}	To build various kinds of students' files
	X_{42}	To join activities of school actively.
	X_{43}	To keep good relationship with co-workers in school
	X_{44}	To keep good relationship and interactions with students' parents
	X_{45}	Treating students fairly
	X_{46}	To keep good relationship with students
X_5 (Ability to Use Computers)	X_{51}	To be able to send and receive email
	X_{52}	To be able to design a web page interacting with students
	X_{53}	To be able to use word edit software to edit handouts and test papers
	X_{54}	To be able to search data on the web to be used in teaching

Table 2. A Fuzzy Linguistic Questionnaire for Appraising the Performance of A Teacher [9]

Fuzzy Linguistic Questionnaire				
Satisfaction Levels of the Criterion X_{ij}				
Low				High
←				→
VL	L	M	H	VH

In Table 2, five linguistic levels are used to evaluate teachers performance regarding the sub-criterion X_{ij}, i.e., very low (VL), low (L), middle (M), high (H) and very high (VH), where the triangular fuzzy numbers $\tilde{1}$, $\tilde{2}$, $\tilde{3}$, $\tilde{4}$ and $\tilde{5}$ corresponding to the linguistic satisfaction levels VL, L, M, H and VH are $\tilde{1} = (0, 1, 2)$, $\tilde{2} = (1, 2, 3)$, $\tilde{3} = (2, 3, 4)$, $\tilde{4} = (3, 4, 5)$ and $\tilde{5} = (4, 5, 6)$, respectively. The fuzzy grade $G(\tilde{x}_{ij})$ of the sub-criterion X_{ij} is calculated as follows:

$$G(\tilde{x}_{ij}) = \frac{\sum_{k=1}^{5} \tilde{x}_{ijk} f(\tilde{x}_{ijk})}{\sum_{k=1}^{5} f(\tilde{x}_{ijk})}, \tag{8}$$

where i denotes the index of the criterion, j denotes the index of the sub-criterion, k denotes the index of linguistic levels, $\tilde{x}_{ijk}$ denotes the kth linguistic satisfaction level of the criterion X_{ij}, $1 \le k \le 5$, $\tilde{x}_{ijk} \in \{\tilde{1}, \tilde{2}, \tilde{3}, \tilde{4}, \tilde{5}\}$, $f(\tilde{x}_{ijk})$ denotes the degree of percentage that the teacher satisfies the kth satisfaction level of the criterion X_{ij}, and $\sum_{k=1}^{5} f(\tilde{x}_{ijk}) = 1$. A fuzzy linguistic questionnaire for determining the weights of a criterion is shown in Table 3 [9]. The fuzzy weight $W(\tilde{x}_i)$ of the criterion X_i is calculated as follows:

$$W(\tilde{x}_i) = \frac{\sum_{k=1}^{5} \tilde{x}_{ik} f(\tilde{x}_{ik})}{\sum_{k=1}^{5} f(\tilde{x}_{ik})}, \tag{9}$$

where i denotes the index of the criterion, k denotes the index of linguistic levels, $\tilde{x}_{ik}$ denotes the kth linguistic satisfaction level of the criterion X_i, $1 \le k \le 5$, $\tilde{x}_{ik} \in \{\tilde{1}, \tilde{2}, \tilde{3}, \tilde{4}, \tilde{5}\}$, $f(\tilde{x}_{ik})$ denotes the degree of percentage that the criterion X_i satisfies the kth important level, and $\sum_{k=1}^{5} f(\tilde{x}_{ik}) = 1$.

Table 3. A Fuzzy Linguistic Questionnaire for Determining the Weight of the Criterion X_i [9]

Fuzzy Linguistic Questionnaire				
Importance Levels of the Criterion X_i				
Low ←				→ High
VL	L	M	H	VH

The proposed new method for high school teachers appraisal is now presented as follows:

Step 1: Determine the fuzzy weight of each criterion. Let each evaluator use the fuzzy linguistic questionnaire shown in Table 3 to evaluate the importance of each criterion. Then, use Eq. (9) to calculate the fuzzy weight of each criterion evaluated by each evaluator, where a fuzzy weight represented by a triangular fuzzy number (k, m, n) should satisfy the rules: "if k < 1, then let k = 1; if n > 5, then let n = 5". For example, assume that there are five evaluators E_1, E_2, E_3, E_4, E_5 and nine teachers A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9 being evaluated. After five evaluators fill out the fuzzy

linguistic questionnaires, we get the result shown in Table 4. After applying Eq. (9) to calculate the fuzzy weight of each criterion evaluated by each evaluator, we get the calculation results as shown in Table 5.

Table 4. The Result after Five Evaluators Fill Out the Fuzzy Linguistic Questionnaires

Criteria	Evaluator E_1					Evaluator E_2					Evaluator E_3					Evaluator E_4					Evaluator E_5				
	VL	L	M	H	VH	VL	L	M	H	VH	VL	L	M	H	VH	VL	L	M	H	VH	VL	L	M	H	VH
X_1				80	20			10	80	10			20	60	20				70	30			80	10	10
X_2			10	70	20				40	60			30	60	10			20	70	10			5	15	80
X_3		20	80					20	70	10				70	30			20	70	10			20	80	
X_4			20	60	20			5	85	10			20	70	10			10	75	15			45	55	
X_5			30	50	20			40	40	20			20	70	10			20	30	50			20	70	10

Table 5. The Fuzzy Weight of Each Criterion Evaluated by Each Evaluator

Fuzzy Weights / Evaluators / Criteria	E_1	E_2	E_3	E_4	E_5
X_1	(3.2, 4.2, 5)	(3, 4, 5)	(3, 4, 5)	(3.3, 4.3, 5)	(2.3, 3.3, 4.3)
X_2	(3.1, 4.1, 5)	(3.6, 4.6, 5)	(2.8, 3.8, 4.8)	(2.9, 3.9, 4.9)	(3.75, 4.75, 5)
X_3	(1.8, 2.8, 3.8)	(2.9, 3.9, 4.9)	(3.3, 4.3, 5)	(2.9, 3.9, 4.9)	(2.8, 3.8, 4.8)
X_4	(3, 4, 5)	(3.05, 4.05, 5)	(2.9, 3.9, 4.9)	(3.05, 4.05, 5)	(2.55, 3.55, 4.55)
X_5	(2.9, 3.9, 4.9)	(2.8, 3.8, 4.8)	(2.9, 3.9, 4.9)	(3.3, 4.3, 5)	(2.9, 3.9, 4.9)

Based on Eqs. (4)-(7), we rank the fuzzy weights with respect to each criterion X_i evaluated by the evaluators, where $1 \le i \le 5$, we drop the fuzzy weights with the smallest ranking value and the largest ranking value. Then, we calculate the average of the remaining fuzzy weights using the addition operations and the division operations of fuzzy numbers. Finally, we can get the fuzzy weighted vector $\tilde{W}$, where $\tilde{W} = \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \tilde{w}_3 \\ \tilde{w}_4 \\ \tilde{w}_5 \end{bmatrix}$, $\tilde{w}_i$ denotes the average of the remaining fuzzy weights with respect to the ith criterion, and $1 \le i \le 5$. Therefore, the fuzzy weighted vector $\tilde{W}$ is as follows:

$$\tilde{W} = \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \tilde{w}_3 \\ \tilde{w}_4 \\ \tilde{w}_5 \end{bmatrix} = \begin{bmatrix} (3.07,\ 4.07,\ 5) \\ (3.2,\ 4.2,\ 4.97) \\ (2.87, 3.87, 4.87) \\ (2.98, 3.98, 4.97) \\ (2.9, 3.9, 4.9) \end{bmatrix}.$$

Step 2: Establish the fuzzy grade matrix. Each evaluator uses the fuzzy linguistic questionnaire shown in Table 2 to evaluate the performance of high school teachers.

Then, use Eq. (8) to calculate the fuzzy grade of each sub-criterion of each teacher evaluated by each evaluator, where the fuzzy grade represented by the fuzzy number (k, m, n) should satisfy the rules: "if k < 1, then let k = 1; if n > 5, then let n = 5". For example, assume that the fuzzy grades evaluated by the five evaluators are as shown in Table 6, Table 7, Table 8, Table 9 and Table 10, respectively. Based on Eqs. (4)-(7), we rank the fuzzy grades of a teacher with respect to a sub-criterion evaluated by the evaluators, then drop the fuzzy grades with the smallest ranking value and the largest ranking value, and then calculate the average of the remaining fuzzy grades by using the addition operations and the division operations of fuzzy numbers to get the averaged fuzzy grade of the sub-cirterion. In the same way, we can get the averaged fuzzy grade of each sub-criterion of each teacher. Therefore, we can get the fuzzy grade of each teacher with respect to each sub-criterion to derive the averaged fuzzy grade table shown in Table 11.

Table 6. Fuzzy Grades Evaluated by the Evaluator E_1

Criteria	Sub-Criteria	Teacher A_1	Teacher A_2	Teacher A_3	Teacher A_4	Teacher A_5	Teacher A_6	Teacher A_7	Teacher A_8	Teacher A_9
X_1	X_{11}	(2.5,3.5,4.5)	(2,3,4)	(1.65,2.65,3.65)	(1.5,2.5,3.5)	(2.85,3.85,4.85)	(1.65,2.65,3.65)	(2.9,3.9,4.9)	(2.5,3.5,4.5)	(2.35,3.35,4.35)
	X_{12}	(3.2,4.2,5)	(1.45,2.45,3.45)	(2.9,3.9,4.9)	(2.8,3.8,4.8)	(1.5,2.5,3.5)	(4,5,5)	(1.8,2.8,3.8)	(1.5,2.5,3.5)	(1.8,2.8,3.8)
	X_{13}	(3.75,4.75,5)	(2.9,3.9,4.9)	(3.7,4.7,5)	(3.55,4.55,5)	(1,1.2,2.2)	(3.65,4.65,5)	(3.65,4.65,5)	(2.3,3.3,4.3)	(1.5,2.5,3.5)
	X_{14}	(1.8,2.8,3.8)	(1.25,2.25,3.25)	(2.9,3.9,4.9)	(3.35,4.35,5)	(1.5,2.5,3.5)	(1.65,2.65,3.65)	(3.35,4.35,5)	(1.7,2.7,3.7)	(2.35,3.35,4.35)
	X_{15}	(1,1.6,2.6)	(3.35,4.35,5)	(1,1.2,2.2)	(2.3,3.3,4.3)	(2.5,3.5,4.5)	(1,1.2,2.2)	(1.8,2.8,3.8)	(3.35,4.35,5)	(2.9,3.9,4.9)
	X_{16}	(1.5,2.5,3.5)	(3.7,4.7,5)	(2.45,3.45,4.45)	(1.7,2.7,3.7)	(1.45,2.45,3.45)	(2,3,4)	(3.55,4.55,5)	(1.65,2.65,3.65)	(3.35,4.35,5)
	X_{17}	(1.45,2.45,3.45)	(2.7,3.7,4.7)	(2.85,3.85,4.85)	(1,1.2,2.2)	(3.7,4.7,5)	(1.7,2.7,3.7)	(2.1,3.1,4.1)	(1,1.2,2.2)	(2.1,3.1,4.1)
	X_{18}	(1,1.1,2.1)	(1.6,2.6,3.6)	(2.9,3.9,4.9)	(4,5,5)	(1.25,2.25,3.25)	(1.7,2.7,3.7)	(1.95,2.95,3.95)	(2,3,4)	(1,1.2,2.2)
	X_{19}	(2.5,3.5,4.5)	(3.2,4.2,5)	(2.7,3.7,4.7)	(2.9,3.9,4.9)	(1.45,2.45,3.45)	(1.25,2.25,3.25)	(3.7,4.7,5)	(1.25,2.25,3.25)	(1.7,2.7,3.7)
X_2	X_{21}	(2.15,3.15,4.15)	(2.15,3.15,4.15)	(2.3,3.3,4.3)	(2.6,3.6,4.6)	(2.9,3.9,4.9)	(2.7,3.7,4.7)	(2.6,3.6,4.6)	(1.5,2.5,3.5)	(2.5,3.5,4.5)
	X_{22}	(2.7,3.7,4.7)	(3.2,4.2,5)	(2.6,3.6,4.6)	(1.8,2.8,3.8)	(2.15,3.15,4.15)	(2.15,3.15,4.15)	(1.45,2.45,3.45)	(2.35,3.35,4.35)	(2.9,3.9,4.9)
	X_{23}	(2.6,3.6,4.6)	(1.5,2.5,3.5)	(3.75,4.75,5)	(1.95,2.95,3.95)	(1.6,2.6,3.6)	(3.2,4.2,5)	(1.6,2.6,3.6)	(1.45,2.45,3.45)	(3.35,4.35,5)
	X_{24}	(2.55,3.55,4.55)	(2.6,3.6,4.6)	(2.55,3.55,4.55)	(2.6,3.6,4.6)	(1,1.2,2.2)	(2.6,3.6,4.6)	(1.5,2.5,3.5)	(1.25,2.25,3.25)	(2.9,3.9,4.9)
	X_{25}	(3.2,4.2,5)	(2.5,3.5,4.5)	(1.5,2.5,3.5)	(2.5,3.5,4.5)	(2.9,3.9,4.9)	(2,3,4)	(1.8,2.8,3.8)	(2.7,3.7,4.7)	(2.1,3.1,4.1)
X_3	X_{31}	(2.25,3.25,4.25)	(2.6,3.6,4.6)	(2.15,3.15,4.15)	(2.15,3.15,4.15)	(2.6,3.6,4.6)	(1.65,2.65,3.65)	(1.8,2.8,3.8)	(2.9,3.9,4.9)	(1.7,2.7,3.7)
	X_{32}	(1.7,2.7,3.7)	(1.8,2.8,3.8)	(2.6,3.6,4.6)	(3.75,4.75,5)	(3.75,4.75,5)	(1,1.1,2.1)	(2.5,3.5,4.5)	(2,3,4)	(1.45,2.45,3.45)
	X_{33}	(2.7,3.7,4.7)	(1.25,2.25,3.25)	(2.7,3.7,4.7)	(2.3,3.3,4.3)	(1,1.2,2.2)	(2.5,3.5,4.5)	(3.7,4.7,5)	(1.7,2.7,3.7)	(2.5,3.5,4.5)
	X_{34}	(2.6,3.6,4.6)	(2.15,3.15,4.15)	(1.7,2.7,3.7)	(2.85,3.85,4.85)	(1.45,2.45,3.45)	(3.75,4.75,5)	(3.55,4.55,5)	(1.6,2.6,3.6)	(1.5,2.5,3.5)
X_4	X_{41}	(2.6,3.6,4.6)	(2.75,3.75,4.75)	(2.25,3.25,4.25)	(2.7,3.7,4.7)	(2.35,3.35,4.35)	(3.65,4.65,5)	(2.15,3.15,4.15)	(3.7,4.7,5)	(1.7,2.7,3.7)
	X_{42}	(3.85,4.85,5)	(2.35,3.35,4.35)	(2.25,3.25,4.25)	(1.7,2.7,3.7)	(2.5,3.5,4.5)	(2.6,3.6,4.6)	(1.95,2.95,3.95)	(1.8,2.8,3.8)	(3.7,4.7,5)
	X_{43}	(2.6,3.6,4.6)	(2.55,3.55,4.55)	(1.45,2.45,3.45)	(1.45,2.45,3.45)	(1.8,2.8,3.8)	(1.25,2.25,3.25)	(1,1.2,2.2)	(2.9,3.9,4.9)	(1.6,2.6,3.6)
	X_{44}	(1.7,2.7,3.7)	(2.5,3.5,4.5)	(1.25,2.25,3.25)	(2.7,3.7,4.7)	(1.45,2.45,3.45)	(1.45,2.45,3.45)	(2.6,3.6,4.6)	(1,1.2,2.2)	(3.35,4.35,5)
	X_{45}	(2.6,3.6,4.6)	(2.15,3.15,4.15)	(2.6,3.6,4.6)	(2.35,3.35,4.35)	(2.6,3.6,4.6)	(2.6,3.6,4.6)	(2.9,3.9,4.9)	(2.35,3.35,4.35)	(1,1.2,2.2)
	X_{46}	(1.8,2.8,3.8)	(2.35,3.35,4.35)	(2.85,3.85,4.85)	(2.15,3.15,4.15)	(1.7,2.7,3.7)	(1.7,2.7,3.7)	(1.7,2.7,3.7)	(2.5,3.5,4.5)	(2.9,3.9,4.9)
X_5	X_{51}	(2.25,3.25,4.25)	(2.55,3.55,4.55)	(2.6,3.6,4.6)	(2.9,3.9,4.9)	(2.85,3.85,4.85)	(2.85,3.85,4.85)	(1.6,2.6,3.6)	(3.55,4.55,5)	(3.35,4.35,5)
	X_{52}	(1.45,2.45,3.45)	(2.7,3.7,4.7)	(2.15,3.15,4.15)	(2.35,3.35,4.35)	(2.9,3.9,4.9)	(2.9,3.9,4.9)	(1,1.2,2.2)	(1.5,2.5,3.5)	(1.45,2.45,3.45)
	X_{53}	(2.15,3.15,4.15)	(2.6,3.6,4.6)	(1.5,2.5,3.5)	(1.25,2.25,3.25)	(1.25,2.25,3.25)	(1.25,2.25,3.25)	(1.8,2.8,3.8)	(3.75,4.75,5)	(1,1.2,2.2)
	X_{54}	(2.35,3.35,4.35)	(1.6,2.6,3.6)	(2.85,3.85,4.85)	(1.8,2.8,3.8)	(2.9,3.9,4.9)	(2.9,3.9,4.9)	(1,1.2,2.2)	(2,3,4)	(1.25,2.25,3.25)

Table 7. Fuzzy Grades Evaluated by the Evaluator E_2

Criteria	Sub-criteria	Teacher A_1	Teacher A_2	Teacher A_3	Teacher A_4	Teacher A_5	Teacher A_6	Teacher A_7	Teacher A_8	Teacher A_9
X_1	X_{11}	(2.8,3.8,4.8)	(3.15,4.15,5)	(2.6,3.6,4.6)	(2.85,3.85,4.85)	(1.45,2.45,3.45)	(1.65,2.65,3.65)	(2.75,3.75,4.75)	(2.9,3.9,4.9)	(2.8,3.8,4.8)
	X_{12}	(2.6,3.6,4.6)	(1.8,2.8,3.8)	(3.7,4.7,5)	(2.85,3.85,4.85)	(2.35,3.35,4.35)	(2.6,3.6,4.6)	(2.7,3.7,4.7)	(1.8,2.8,3.8)	(2.35,3.35,4.35)
	X_{13}	(3.15,4.15,5)	(2.9,3.9,4.9)	(1.6,2.6,3.6)	(1.7,2.7,3.7)	(2.6,3.6,4.6)	(1,2,3)	(3.35,4.35,5)	(2.35,3.35,4.35)	(1,2,3)
	X_{14}	(2.5,3.5,4.5)	(2.7,3.7,4.7)	(1.8,2.8,3.8)	(3.65,4.65,5)	(3.75,4.75,5)	(1.8,2.8,3.8)	(2.75,3.75,4.75)	(3.15,4.15,5)	(2.75,3.75,4.75)
	X_{15}	(3.7,4.7,5)	(2.5,3.5,4.5)	(2.75,3.75,4.75)	(1.25,2.25,3.25)	(1.7,2.7,3.7)	(2.3,3.3,4.3)	(1.25,2.25,3.25)	(2.7,3.7,4.7)	(2.1,3.1,4.1)
	X_{16}	(2.6,3.6,4.6)	(2.85,3.85,4.85)	(2.5,3.5,4.5)	(2.6,3.6,4.6)	(1,1.2,2.2)	(3.15,4.15,5)	(2.1,3.1,4.1)	(1.95,2.95,3.95)	(2.9,3.9,4.9)
	X_{17}	(3.15,4.15,5)	(1.25,2.25,3.25)	(1.8,2.8,3.8)	(2,3,4)	(1.25,2.25,3.25)	(1.45,2.45,3.45)	(2.3,3.3,4.3)	(1.25,2.25,3.25)	(2.35,3.35,4.35)
	X_{18}	(1.25,2.25,3.25)	(3.15,4.15,5)	(1.5,2.5,3.5)	(2.7,3.7,4.7)	(3.7,4.7,5)	(2.6,3.6,4.6)	(1,1.1,2.1)	(2.1,3.1,4.1)	(1.45,2.45,3.45)
	X_{19}	(1.25,2.25,3.25)	(2.15,3.15,4.15)	(2.6,3.6,4.6)	(2.9,3.9,4.9)	(3.2,4.2,5)	(3.7,4.7,5)	(1.45,2.45,3.45)	(2.35,3.35,4.35)	(3.7,4.7,5)
X_2	X_{21}	(2.6,3.6,4.6)	(2.7,3.7,4.7)	(3.15,4.15,5)	(2.6,3.6,4.6)	(2.35,3.35,4.35)	(2.6,3.6,4.6)	(2.1,3.1,4.1)	(2.6,3.6,4.6)	(1.5,2.5,3.5)
	X_{22}	(2.7,3.7,4.7)	(2.6,3.6,4.6)	(1.5,2.5,3.5)	(1.25,2.25,3.25)	(3.7,4.7,5)	(2,3,4)	(3.7,4.7,5)	(3.35,4.35,5)	(2.75,3.75,4.75)
	X_{23}	(3.7,4.7,5)	(2.3,3.3,4.3)	(2.6,3.6,4.6)	(2.75,3.75,4.75)	(2.85,3.85,4.85)	(1,1.2,2.2)	(1.7,2.7,3.7)	(4,5,5)	(3.35,4.35,5)
	X_{24}	(1.25,2.25,3.25)	(3.15,4.15,5)	(1.8,2.8,3.8)	(2.1,3.1,4.1)	(1.5,2.5,3.5)	(3.55,4.55,5)	(1,1.2,2.2)	(2.35,3.35,4.35)	(2.9,3.9,4.9)
	X_{25}	(3.15,4.15,5)	(2.75,3.75,4.75)	(3.7,4.7,5)	(3.7,4.7,5)	(2.35,3.35,4.35)	(3.25,4.25,5)	(2.3,3.3,4.3)	(2.75,3.75,4.75)	(1.7,2.7,3.7)
X_3	X_{31}	(3.7,4.7,5)	(2.6,3.6,4.6)	(3.7,4.7,5)	(2.85,3.85,4.85)	(2.6,3.6,4.6)	(2.35,3.35,4.35)	(2.8,3.8,4.8)	(2.7,3.7,4.7)	(2.5,3.5,4.5)
	X_{32}	(2.1,3.1,4.1)	(2.8,3.8,4.8)	(3.15,4.15,5)	(2.35,3.35,4.35)	(1.25,2.25,3.25)	(2.3,3.3,4.3)	(2.7,3.7,4.7)	(2.9,3.9,4.9)	(3.35,4.35,5)
	X_{33}	(3.7,4.7,5)	(3.7,4.7,5)	(2.5,3.5,4.5)	(2.6,3.6,4.6)	(2.1,3.1,4.1)	(2.35,3.35,4.35)	(2.5,3.5,4.5)	(2.35,3.35,4.35)	(1.65,2.65,3.65)
	X_{34}	(2.3,3.3,4.3)	(2.5,3.5,4.5)	(2.85,3.85,4.85)	(1.8,2.8,3.8)	(3.15,4.15,5)	(3.65,4.65,5)	(3.15,4.15,5)	(1.5,2.5,3.5)	(2.5,3.5,4.5)
X_4	X_{41}	(2.5,3.5,4.5)	(1.5,2.5,3.5)	(2.6,3.6,4.6)	(2.7,3.7,4.7)	(1.8,2.8,3.8)	(2.9,3.9,4.9)	(1.8,2.8,3.8)	(2.9,3.9,4.9)	(1.45,2.45,3.45)
	X_{42}	(2.6,3.6,4.6)	(2.7,3.7,4.7)	(1.8,2.8,3.8)	(2.75,3.75,4.75)	(1.5,2.5,3.5)	(2.35,3.35,4.35)	(1.7,2.7,3.7)	(1,1.2,2.2)	(2.9,3.9,4.9)
	X_{43}	(3.7,4.7,5)	(1.25,2.25,3.25)	(2.75,3.75,4.75)	(2.5,3.5,4.5)	(3.75,4.75,5)	(1,1.2,2.2)	(2.5,3.5,4.5)	(2.5,3.5,4.5)	(3.35,4.35,5)
	X_{44}	(1.5,2.5,3.5)	(2.3,3.3,4.3)	(3.7,4.7,5)	(1.8,2.8,3.8)	(3.2,4.2,5)	(1.7,2.7,3.7)	(2.1,3.1,4.1)	(1.65,2.65,3.65)	(3.2,4.2,5)
	X_{45}	(2.7,3.7,4.7)	(1.5,2.5,3.5)	(3.2,4.2,5)	(2.35,3.35,4.35)	(2.9,3.9,4.9)	(1.25,2.25,3.25)	(1.45,2.45,3.45)	(3.7,4.7,5)	(1,1.1,2.1)
	X_{46}	(2.35,3.35,4.35)	(2.7,3.7,4.7)	(2.9,3.9,4.9)	(1.8,2.8,3.8)	(2.5,3.5,4.5)	(1.45,2.45,3.45)	(2.5,3.5,4.5)	(2.1,3.1,4.1)	(1.7,2.7,3.7)
X_5	X_{51}	(2.85,3.85,4.85)	(2.1,3.1,4.1)	(2.75,3.75,4.75)	(2.5,3.5,4.5)	(2.85,3.85,4.85)	(2,3,4)	(1,1.2,2.2)	(1.95,2.95,3.95)	(1.7,2.7,3.7)
	X_{52}	(1.8,2.8,3.8)	(1.25,2.25,3.25)	(2.9,3.9,4.9)	(1.45,2.45,3.45)	(2,3,4)	(1,1.2,2.2)	(1.8,2.8,3.8)	(2,3,4)	(3.35,4.35,5)
	X_{53}	(2.1,3.1,4.1)	(3.15,4.15,5)	(1.25,2.25,3.25)	(1.8,2.8,3.8)	(2.9,3.9,4.9)	(2.1,3.1,4.1)	(1.25,2.25,3.25)	(3.2,4.2,5)	(2,3,4)
	X_{54}	(2.8,3.8,4.8)	(2.5,3.5,4.5)	(2.15,3.15,4.15)	(2.85,3.85,4.85)	(2.5,3.5,4.5)	(1.7,2.7,3.7)	(1.5,2.5,3.5)	(1.45,2.45,3.45)	(1.25,2.25,3.25)

Table 8. Fuzzy Grades Evaluated by the Evaluator E_3

Criteria	Sub-criteria	Teacher A_1	Teacher A_2	Teacher A_3	Teacher A_4	Teacher A_5	Teacher A_6	Teacher A_7	Teacher A_8	Teacher A_9
X_1	X_{11}	(3.7,4.7,5)	(3.1,4.1,5)	(2.25,3.25,4.25)	(3.1,4.1,5)	(3.1,4.1,5)	(2.7,3.7,4.7)	(2.8,3.8,4.8)	(2.75,3.75,4.75)	(2.9,3.9,4.9)
	X_{12}	(2.6,3.6,4.6)	(2.15,3.15,4.15)	(3.15,4.15,5)	(3,55,4.55,5)	(2.35,3.35,4.35)	(2.95,3.95,4.95)	(2.1,3.1,4.1)	(3,35,4,35,5)	(3.65,4.65,5)
	X_{13}	(2.9,3.9,4.9)	(2.5,3.5,4.5)	(2.75,3.75,4.75)	(2.5,3.5,4.5)	(2.75,3.75,4.75)	(1.45,2.45,3.45)	(2.6,3.6,4.6)	(1.5,2.5,3.5)	(2.75,3.75,4.75)
	X_{14}	(1.45,2.45,3.45)	(1.8,2.8,3.8)	(2.8,3.8,4.8)	(1,2,3)	(3.7,4.7,5)	(1.25,2.25,3.25)	(0.2,1.2,2.2)	(2.3,3.3,4.3)	(2.1,3.1,4.1)
	X_{15}	(2.15,3.15,4.15)	(2.6,3.6,4.6)	(2.95,3.95,4.95)	(2.9,3.9,4.9)	(2.25,3.25,4.25)	(2.6,3.6,4.6)	(2.75,3.75,4.75)	(2.35,3.35,4.35)	(1.7,2.7,3.7)
	X_{16}	(2,3,3,3,4.3)	(3.65,4.65,5)	(3.65,4.65,5)	(3.1,4.1,5)	(2.6,3.6,4.6)	(3.7,4.7,5)	(2.7,3.7,4.7)	(2.95,3.95,4.95)	(1.95,2.95,3.95)
	X_{17}	(3.1,4.1,5)	(2.9,3.9,4.9)	(1.45,2.45,3.45)	(3.15,4.15,5)	(2.5,3.5,4.5)	(2.7,3.7,4.7)	(1.25,2.25,3.25)	(1.5,2.5,3.5)	(1,45,2.45,3.45)
	X_{18}	(2.85,3.85,4.85)	(1.8,2.8,3.8)	(3.7,4.7,5)	(2.95,3.95,4.95)	(3.1,4.1,5)	(1.7,2.7,3.7)	(2.35,3.35,4.35)	(0.2,1.2,2.2)	(2.9,3.9,4.9)
	X_{19}	(3.15,4.15,5)	(3.15,4.15,5)	(2.85,3.85,4.85)	(3.15,4.15,5)	(1,45,2.45,3.45)	(2,3,4)	(2.75,3.75,4.75)	(1,45,2.45,3.45)	(1.5,2.5,3.5)
X_2	X_{21}	(3.1,4.1,5)	(2.95,3.95,4.95)	(2.6,3.6,4.6)	(3.7,4.7,5)	(2.9,3.9,4.9)	(1.65,2.65,3.65)	(2.3,3.3,4.3)	(3.65,4.65,5)	(1.45,2.45,3.45)
	X_{22}	(1,2,3)	(2.75,3.75,4.75)	(2.5,3.5,4.5)	(1.8,2.8,3.8)	(2.6,3.6,4.6)	(2.7,3.7,4.7)	(1.8,2.8,3.8)	(1.25,2.25,3.25)	(2.1,3.1,4.1)
	X_{23}	(2.6,3.6,4.6)	(2.15,3.15,4.15)	(3.1,4.1,5)	(2.8,3.8,4.8)	(2,3,4)	(1.45,2.45,3.45)	(2.9,3.9,4.9)	(3.65,4.65,5)	(2.8,3.8,4.8)
	X_{24}	(2.25,3.25,4.25)	(3.15,4.15,5)	(2.85,3.85,4.85)	(2.6,3.6,4.6)	(3.2,4.2,5)	(2,3,4)	(3.75,4.75,5)	(1.45,2.45,3.45)	(3.65,4.65,5)
	X_{25}	(2.9,3.9,4.9)	(2.8,3.8,4.8)	(3.1,4.1,5)	(3.1,4.1,5)	(2.75,3.75,4.75)	(3.7,4.7,5)	(2.7,3.7,4.7)	(2.95,3.95,4.95)	(1.45,2.45,3.45)
X_3	X_{31}	(3.65,4.65,5)	(2.15,3.15,4.15)	(1.8,2.8,3.8)	(3.15,4.15,5)	(2.6,3.6,4.6)	(2.1,3.1,4.1)	(1,45,2.45,3.45)	(1.8,2.8,3.8)	(2.25,3.25,4.25)
	X_{32}	(1.25,2.25,3.25)	(3.65,4.65,5)	(2.95,3.95,4.95)	(1,2,3)	(0.1,1.1,2.1)	(2.5,3.5,4.5)	(2.8,3.8,4.8)	(1,2,3)	(1.7,2.7,3.7)
	X_{33}	(2.75,3.75,4.75)	(3.2,4.2,5)	(2.7,3.7,4.7)	(2.5,3.5,4.5)	(2.3,3.3,4.3)	(2.75,3.75,4.75)	(1.25,2.25,3.25)	(1.5,2.5,3.5)	(2.9,3.9,4.9)
	X_{34}	(2.95,3.95,4.95)	(2.6,3.6,4.6)	(2.1,3.1,4.1)	(3.1,4.1,5)	(2.35,3.35,4.35)	(1,45,2.45,3.45)	(1.8,2.8,3.8)	(2.75,3.75,4.75)	(1.25,2.25,3.25)
X_4	X_{41}	(2.85,3.85,4.85)	(2.95,3.95,4.95)	(2.7,3.7,4.7)	(1.8,2.8,3.8)	(3.1,4.1,5)	(2.9,3.9,4.9)	(0.2,1.2,2.2)	(2.95,3.95,4.95)	(2.85,3.85,4.85)
	X_{42}	(2.75,3.75,4.75)	(2.75,3.75,4.75)	(1.8,2.8,3.8)	(3.15,4.15,5)	(1.8,2.8,3.8)	(2.95,3.95,4.95)	(1,45,2.45,3.45)	(1.7,2.7,3.7)	(2.25,3.25,4.25)
	X_{43}	(3.65,4.65,5)	(1.7,2.7,3.7)	(2.75,3.75,4.75)	(3,55,4.55,5)	(3.2,4.2,5)	(3.7,4.7,5)	(2.85,3.85,4.85)	(1.7,2.7,3.7)	(0.1,1.1,2.1)
	X_{44}	(2.75,3.75,4.75)	(1,2,3)	(1.5,2.5,3.5)	(2,3,3,3,4.3)	(1.65,2.65,3.65)	(2.75,3.75,4.75)	(2.5,3.5,4.5)	(1.45,2.45,3.45)	(3.35,4.35,5)
	X_{45}	(3.65,4.65,5)	(1.8,2.8,3.8)	(1.5,2.5,3.5)	(3.2,4.2,5)	(2.25,3.25,4.25)	(2.8,3.8,4.8)	(2.5,3.5,4.5)	(3.75,4.75,5)	(1.7,2.7,3.7)
	X_{46}	(2.95,3.95,4.95)	(1.7,2.7,3.7)	(2.8,3.8,4.8)	(1.8,2.8,3.8)	(2.7,3.7,4.7)	(1.45,2.45,3.45)	(1.7,2.7,3.7)	(3.15,4.15,5)	(2.9,3.9,4.9)
X_5	X_{51}	(2.25,3.25,4.25)	(2,3,3,3,4.3)	(2.85,3.85,4.85)	(2,3,3,4.3)	(3.65,4.65,5)	(2.85,3.85,4.85)	(2.9,3.9,4.9)	(1,45,2.45,3.45)	(1.5,2.5,3.5)
	X_{52}	(1.8,2.8,3.8)	(2.25,3.25,4.25)	(1,1.2,2.2)	(3.1,4.1,5)	(1.8,2.8,3.8)	(1.7,2.7,3.7)	(1.5,2.5,3.5)	(1.5,2.5,3.5)	(0.2,1.2,2.2)
	X_{53}	(3.15,4.15,5)	(3.7,4.7,5)	(2,3,4)	(1.45,2.45,3.45)	(2.1,3.1,4.1)	(3.7,4.7,5)	(2,3,4)	(1.5,2.5,3.5)	(1.5,2.5,3.5)
	X_{54}	(2.9,3.9,4.9)	(1,45,2.45,3.45)	(2,3,4)	(1.7,2.7,3.7)	(3.15,4.15,5)	(1.5,2.5,3.5)	(2.1,3.1,4.1)	(0.2,1.2,2.2)	(3.75,4.75,5)

Table 9. Fuzzy Grades Evaluated by the Evaluator E_4

Criteria	Sub-criteria	Teacher A_1	Teacher A_2	Teacher A_3	Teacher A_4	Teacher A_5	Teacher A_6	Teacher A_7	Teacher A_8	Teacher A_9
X_1	X_{11}	(2.7,3.7,4.7)	(2.95,3.95,4.95)	(2,3,3,3,4.3)	(3.1,4.1,5)	(3.1,4.1,5)	(1,45,2.45,3.45)	(2.1,3.1,4.1)	(1,45,2.45,3.45)	(1.8,2.8,3.8)
	X_{12}	(2,3,3,3,4.3)	(2.7,3.7,4.7)	(2.35,3.35,4.35)	(2.85,3.85,4.85)	(1.45,2.45,3.45)	(2.75,3.75,4.75)	(2,3,3,3,4.3)	(1.95,2.95,3.95)	(2.8,3.8,4.8)
	X_{13}	(2.8,3.8,4.8)	(2.75,3.75,4.75)	(2.6,3.6,4.6)	(2.25,3.25,4.25)	(3.15,4.15,5)	(2.25,3.25,4.25)	(1,1.2,2.2)	(1.8,2.8,3.8)	(1.7,2.7,3.7)
	X_{14}	(2.7,3.7,4.7)	(2,3,3,3,4.3)	(2.1,3.1,4.1)	(2.7,3.7,4.7)	(2.75,3.75,4.75)	(1.8,2.8,3.8)	(2.25,3.25,4.25)	(2.75,3.75,4.75)	(1.7,2.7,3.7)
	X_{15}	(2.35,3.35,4.35)	(2.25,3.25,4.25)	(1,1.2,2.2)	(3.65,4.65,5)	(1.5,2.5,3.5)	(2,3,3,3,4.3)	(2.75,3.75,4.75)	(1,45,2.45,3.45)	(1.25,2.25,3.25)
	X_{16}	(3.65,4.65,5)	(3.2,4.2,5)	(2.75,3.75,4.75)	(1,1.2,2.2)	(2.5,3.5,4.5)	(1.45,2.45,3.45)	(3.15,4.15,5)	(1.8,2.8,3.8)	(2.85,3.85,4.85)
	X_{17}	(3.15,4.15,5)	(3.7,4.7,5)	(2.6,3.6,4.6)	(4,5,6)	(1.45,2.45,3.45)	(2,3,3,3,4.3)	(2.5,3.5,4.5)	(3.35,4.35,5)	(2.25,3.25,4.25)
	X_{18}	(3.7,4.7,5)	(2.75,3.75,4.75)	(3.1,4.1,5)	(2.25,3.25,4.25)	(1.8,2.8,3.8)	(1.65,2.65,3.65)	(2.35,3.35,4.35)	(1.45,2.45,3.45)	(3.35,4.35,5)
	X_{19}	(3.2,4.2,5)	(2.8,3.8,4.8)	(2.6,3.6,4.6)	(2.95,3.95,4.95)	(2.75,3.75,4.75)	(1.8,2.8,3.8)	(2.75,3.75,4.75)	(2.35,3.35,4.35)	(2.95,3.95,4.95)
X_2	X_{21}	(3.65,4.65,5)	(3.15,4.15,5.15)	(2.25,3.25,4.25)	(2.75,3.75,4.75)	(2,3,4)	(1,45,2.45,3.45)	(1.8,2.8,3.8)	(1,2,3)	(1.8,2.8,3.8)
	X_{22}	(2.6,3.6,4.6)	(2.6,3.6,4.6)	(2,3,3,3,4.3)	(1,2,3)	(1.8,2.8,3.8)	(2.25,3.25,4.25)	(2,3,3,3,4.3)	(1.8,2.8,3.8)	(2.25,3.25,4.25)
	X_{23}	(1.45,2.45,3.45)	(1.45,2.45,3.45)	(2.7,3.7,4.7)	(3.1,4.1,5)	(3.75,4.75,5)	(2.35,3.35,4.35)	(2,3,3,3,4.3)	(1.25,2.25,3.25)	(2.75,3.75,4.75)
	X_{24}	(2.75,3.75,4.75)	(1.8,2.8,3.8)	(3.2,4.2,5)	(3,55,4.55,5)	(1.8,2.8,3.8)	(2.25,3.25,4.25)	(2.25,3.25,4.25)	(2.35,3.35,4.35)	(1,45,2.45,3.45)
	X_{25}	(2,3,3,3,4.3)	(2.75,3.75,4.75)	(2.1,3.1,4.1)	(2.8,3.8,4.8)	(2.6,3.6,4.6)	(2.7,3.7,4.7)	(1.45,2.45,3.45)	(2.95,3.95,4.95)	(2,3,3,3,4.3)
X_3	X_{31}	(2.7,3.7,4.7)	(2.85,3.85,4.85)	(2.85,3.85,4.85)	(2.7,3.7,4.7)	(2,3,3,3,4.3)	(2.75,3.75,4.75)	(1.8,2.8,3.8)	(2.75,3.75,4.75)	(2.35,3.35,4.35)
	X_{32}	(3.7,4.7,5)	(2.25,3.25,4.25)	(2.75,3.75,4.75)	(1,1.2,2.2)	(2.75,3.75,4.75)	(1.7,2.7,3.7)	(2.25,3.25,4.25)	(1.8,2.8,3.8)	(1.65,2.65,3.65)
	X_{33}	(2.6,3.6,4.6)	(2.35,3.35,4.35)	(2.25,3.25,4.25)	(2.75,3.75,4.75)	(2.8,3.8,4.8)	(1.5,2.5,3.5)	(2.75,3.75,4.75)	(1,45,2.45,3.45)	(2.8,3.8,4.8)
	X_{34}	(1,45,2.45,3.45)	(1.8,2.8,3.8)	(2.6,3.6,4.6)	(2.7,3.7,4.7)	(1.25,2.25,3.25)	(1.25,2.25,3.25)	(1.5,2.5,3.5)	(2,3,4)	(2.85,3.85,4.85)
X_4	X_{41}	(2.25,3.25,4.25)	(3.15,4.15,5)	(2.25,3.25,4.25)	(2.7,3.7,4.7)	(1.8,2.8,3.8)	(3.1,4.1,5)	(2.9,3.9,4.9)	(2.85,3.85,4.85)	(1,45,2.45,3.45)
	X_{42}	(2,3,3,3,4.3)	(2.7,3.7,4.7)	(1.8,2.8,3.8)	(2.75,3.75,4.75)	(3.15,4.15,5)	(2.75,3.75,4.75)	(3.15,4.15,5)	(2.25,3.25,4.25)	(1,45,2.45,3.45)
	X_{43}	(2.25,3.25,4.25)	(2.75,3.75,4.75)	(3.15,4.15,5)	(3.1,4.1,5)	(1.7,2.7,3.7)	(1.1,1.2,1)	(2.5,3.5,4.5)	(1.65,2.65,3.65)	(3.35,4.35,5)
	X_{44}	(2.75,3.75,4.75)	(2.5,3.5,4.5)	(2.75,3.75,4.75)	(2.25,3.25,4.25)	(2.25,3.25,4.25)	(3.65,4.65,5)	(3.15,4.15,5)	(1,45,2.45,3.45)	(1.8,2.8,3.8)
	X_{45}	(2.5,3.5,4.5)	(2.1,3.1,4.1)	(1.45,2.45,3.45)	(3.1,4.1,5)	(3.7,4.7,5)	(3.15,4.15,5)	(1.7,2.7,3.7)	(2,3,4)	(2.1,3.1,4.1)
	X_{46}	(2,3,3,3,4.3)	(2.75,3.75,4.75)	(2,3,3,3,4.3)	(2.25,3.25,4.25)	(2.75,3.75,4.75)	(1.8,2.8,3.8)	(1.45,2.45,3.45)	(2.25,3.25,4.25)	(1.45,2.45,3.45)
X_5	X_{51}	(1.8,2.8,3.8)	(2,3,3,3,4.3)	(1,1.2,2.2)	(1.8,2.8,3.8)	(3.15,4.15,5)	(1,2,3)	(1,1.2,2.2)	(1.8,2.8,3.8)	(1.7,2.7,3.7)
	X_{52}	(2.85,3.85,4.85)	(2.75,3.75,4.75)	(1.5,2.5,3.5)	(2.75,3.75,4.75)	(2.75,3.75,4.75)	(1.8,2.8,3.8)	(1.45,2.45,3.45)	(2.9,3.9,4.9)	(1.5,2.5,3.5)
	X_{53}	(2.35,3.35,4.35)	(2.8,3.8,4.8)	(2.85,3.85,4.85)	(3.2,4.2,5)	(1.5,2.5,3.5)	(1.45,2.45,3.45)	(1.45,2.45,3.45)	(1.45,2.45,3.45)	(2,3,4)
	X_{54}	(3.65,4.65,5)	(2.25,3.25,4.25)	(2.75,3.75,4.75)	(2.25,3.25,4.25)	(2.1,3.1,4.1)	(1,1.2,2.2)	(3.15,4.15,5.15)	(2,3,4)	(3.35,4.35,5)

Table 10. Fuzzy Grades Evaluated by the Evaluator E_5

Criteria	Sub-criteria	Teacher A_1	Teacher A_2	Teacher A_3	Teacher A_4	Teacher A_5	Teacher A_6	Teacher A_7	Teacher A_8	Teacher A_9
X_1	X_{11}	(2,5,3.5,4.5)	(2.25,3.25,4.25)	(1.65,2.65,3.65)	(3.2,4.2,5)	(2.95,3.95,4.95)	(3.2,4.2,5)	(2.75,3.75,4.75)	(1.65,2.65,3.65)	(2.95,3.95,4.95)
	X_{12}	(3.7,4.7,5)	(1.45,2.45,3.45)	(3.15,4.15,5)	(3.65,4.65,5)	(2.75,3.75,4.75)	(1.45,2.45,3.45)	(1.65,2.65,3.65)	(3.9,4.9,5)	(2.25,3.25,4.25)
	X_{13}	(2.85,3.85,4.85)	(2.75,3.75,4.75)	(2.75,3.75,4.75)	(2.75,3.75,4.75)	(3.1,4.1,5)	(2.75,3.75,4.75)	(1.45,2.45,3.45)	(3.15,4.15,5)	(2.95,3.95,4.95)
	X_{14}	(3.7,4.7,5)	(2.25,3.25,4.25)	(1,45,2.45,3.45)	(2.25,3.25,4.25)	(2,3,4)	(1.5,2.5,3.5)	(2.5,3.5,4.5)	(1,45,2.45,3.45)	(1.65,2.65,3.65)
	X_{15}	(3.65,4.65,5)	(2.75,3.75,4.75)	(2.9,3.9,4.9)	(1.8,2.8,3.8)	(1.45,2.45,3.45)	(1.95,2.95,3.95)	(2.95,3.95,4.95)	(1.5,2.5,3.5)	(2.8,3.8,4.8)
	X_{16}	(2.25,3.25,4.25)	(2.95,3.95,4.95)	(2.75,3.75,4.75)	(3.2,4.2,5)	(2,3,4)	(3.3,4.3,5)	(2.85,3.85,4.85)	(1.95,2.95,3.95)	(2.95,3.95,4.95)
	X_{17}	(2.25,3.25,4.25)	(2.9,3.9,4.9)	(2.5,3.5,4.5)	(2.75,3.75,4.75)	(1.65,2.65,3.65)	(2.95,3.95,4.95)	(1,1.2,2.2)	(1,1.2,2.2)	(1,1.2,2.2)
	X_{18}	(1.45,2.45,3.45)	(1.7,2.7,3.7)	(3.1,4.1,5)	(1.95,2.95,3.95)	(2.75,3.75,4.75)	(3.2,4.2,5)	(1.8,2.8,3.8)	(1.45,2.45,3.45)	(3.35,4.35,5)
	X_{19}	(3.15,4.15,5)	(2.85,3.85,4.85)	(3.4,4.4,5)	(1.8,2.8,3.8)	(2.85,3.85,4.85)	(2.75,3.75,4.75)	(2.1,3.1,4.1)	(2.95,3.95,4.95)	(2.85,3.85,4.85)
X_2	X_{21}	(1.8,2.8,3.8)	(2.75,3.75,4.75)	(3.7,4.7,5)	(2.75,3.75,4.75)	(2.25,3.25,4.25)	(0.2,1.2,2.2)	(2.75,3.75,4.75)	(1.65,2.65,3.65)	(1.45,2.45,3.45)
	X_{22}	(1.5,2.5,3.5)	(1.65,2.65,3.65)	(3.15,4.15,5)	(3.6,4.6,5)	(2.5,3.5,4.5)	(1.45,2.45,3.45)	(1.8,2.8,3.8)	(2.8,3.8,4.8)	(1.45,2.45,3.45)
	X_{23}	(1.45,2.45,3.45)	(2.8,3.8,4.8)	(3.65,4.65,5)	(3,35,4,35,5)	(2.75,3.75,4.75)	(2.25,3.25,4.25)	(1.5,2.5,3.5)	(2.1,3.1,4.1)	(1.1,2.2,2)
	X_{24}	(1.8,2.8,3.8)	(1.45,2.45,3.45)	(3.2,4.2,5)	(2.8,3.8,4.8)	(2.75,3.75,4.75)	(2.8,3.8,4.8)	(1.25,2.25,3.25)	(1.5,2.5,3.5)	(1.7,2.7,3.7)
	X_{25}	(1.45,2.45,3.45)	(2.75,3.75,4.75)	(2.35,3.35,4.35)	(2.85,3.85,4.85)	(3.7,4.7,5)	(1.25,2.25,3.25)	(1.8,2.8,3.8)	(2.75,3.75,4.75)	(2.1,3.1,4.1)
X_3	X_{31}	(2.75,3.75,4.75)	(2.25,3.25,4.25)	(2.75,3.75,4.75)	(2.95,3.95,4.95)	(1.45,2.45,3.45)	(3.1,4.1,5)	(1.8,2.8,3.8)	(2.25,3.25,4.25)	(1.65,2.65,3.65)
	X_{32}	(1.45,2.45,3.45)	(1.45,2.45,3.45)	(1.45,2.45,3.45)	(2,3,4)	(4,5,5)	(3.15,4.15,5)	(2.8,3.8,4.8)	(1.7,2.7,3.7)	(2.25,3.25,4.25)
	X_{33}	(2.75,3.75,4.75)	(2.8,3.8,4.8)	(1.8,2.8,3.8)	(1.65,2.65,3.65)	(3.7,4.7,5)	(1.95,2.95,3.95)	(3.8,4.8,5)	(2.5,3.5,4.5)	(1.25,2.25,3.25)
	X_{34}	(1.8,2.8,3.8)	(2.75,3.75,4.75)	(3.7,4.7,5)	(2.75,3.75,4.75)	(3.35,4.35,5)	(1.8,2.8,3.8)	(3.7,4.7,5)	(2.85,3.85,4.85)	(2.1,3.1,4.1)
X_4	X_{41}	(2.75,3.75,4.75)	(3.15,4.15,5)	(1.45,2.45,3.45)	(3.15,4.15,5)	(1.5,2.5,3.5)	(1,45,2.45,3.45)	(1.45,2.45,3.45)	(1.45,2.45,3.45)	(2.8,3.8,4.8)
	X_{42}	(3.75,4.75,5)	(1.45,2.45,3.45)	(2.9,3.9,4.9)	(1.45,2.45,3.45)	(2.25,3.25,4.25)	(3.35,4. 35,5)	(2.25,3.25,4.25)	(2.25,3.25,4.25)	(1.7,2.7,3.7)
	X_{43}	(3.65,4.65,5)	(3,4,5)	(2.75,3.75,4.75)	(2.75,3.75,4.75)	(3.2,4.2,5)	(1.65,2.65,3.65)	(3.35,4.35,5)	(1.45,2.45,3.45)	(2.8,3.8,4.8)
	X_{44}	(1.45,2.45,3.45)	(3.1,4.1,5)	(2.5,3.5,4.5)	(2.95,3.95,4.95)	(1.8,2.8,3.8)	(2.85,3.85,4.85)	(2.7,3.7,4.7)	(3.2,4.2,5)	(1.65,2.65,3.65)
	X_{45}	(1.45,2.45,3.45)	(2.75,3.75,4.75)	(1.65,2.65,3.65)	(1.95,2.95,3.95)	(3.7,4.7,5)	(1.45,2.45,3.45)	(3.6,4.6,5)	(2.8,3.8,4.8)	(2.1,3.1,4.1)
	X_{46}	(2.1,3.1,4.1)	(1.8,2.8,3.8)	(2.75,3.75,4.75)	(2.1,3.1,4.1)	(2.8,3.8,4.8)	(3.1,4.1,5)	(3.55,4.55,5)	(3.7,4.7,5)	(3.25,4.25,5)
X_5	X_{51}	(1.5,2.5,3.5)	(2.75,3.75,4.75)	(3.7,4.7,5)	(3.7,4.7,5)	(3.35,4.35,5)	(3.1,4.1,5)	(1.1,1.2,1)	(1.8,2.8,3.8)	(2.9,3.9,4.9)
	X_{52}	(2.8,3.8,4.8)	(3.15,4.15,5)	(1.45,2.45,3.45)	(2.5,3.5,4.5)	(3.95,4.95,5)	(1.65,2.65,3.65)	(2,3,4)	(1.95,2.95,3.95)	(1,1.2,2.2)
	X_{53}	(1.95,2.95,3.95)	(3.7,4.7,5)	(1.45,2.45,3.45)	(2.8,3.8,4.8)	(3.7,4.7,5)	(1,1.2,2.2)	(1,1.2,2.2)	(3.35,4.35,5)	(1.7,2.7,3.7)
	X_{54}	(1.45,2.45,3.45)	(1.65,2.65,3.65)	(2.1,3.1,4.1)	(1,1.1,2.1)	(3.35,4.35,5)	(2.7,3.7,4.7)	(2,3,4)	(1.5,2.5,3.5)	(1,1.2,2.2)

Table 11. The Averaged Fuzzy Grade Table

Criteria	Sub-criteria	Teacher A_1	Teacher A_2	Teacher A_3	Teacher A_4	Teacher A_5	Teacher A_6	Teacher A_7	Teacher A_8	Teacher A_9
X_1	X_{11}	(2.67,3.67,4.67)	(2.76,3.76,4.73)	(2.07,3.07,4.07)	(3.02,4.02,4.95)	(2.97,3.97,4.93)	(2,3,4)	(2.77,3.77,4.77)	(2,3,3.3,4.3)	(2.68,3.68,4.68)
	X_{12}	(2.8,3.8,4.73)	(1.8,2.8,3.8)	(3.07,4.07,4.97)	(3.07,4.07,4.9)	(2.07,3.07,4.07)	(2.77,3.77,4.77)	(2.07,3.07,4.07)	(2.37,3.37,4.25)	(2.47,3.47,4.47)
	X_{13}	(2.97,3.97,4.92)	(2.8,3.8,4.8)	(2.7,3.7,4.7)	(2.5,3.5,4.5)	(2.82,3.82,4.78)	(2.15,3.15,4.15)	(2.47,3.47,4.35)	(2.15,3.15,4.15)	(1.98,2.98,3.98)
	X_{14}	(2.33,3.33,4.33)	(2.11,3.11,4.11)	(2.23,3.23,4.23)	(2.77,3.77,4.65)	(2.82,3.82,4.67)	(1.65,2.65,3.65)	(2.5,3.5,4.5)	(2.25,3.25,4.25)	(2.05,3.05,4.05)
	X_{15}	(2.72,3.72,4.5)	(2.61,3.61,4.61)	(2.22,2.95,3.95)	(2.33,3.33,4.33)	(1.82,2.82,3.82)	(2.18,3.18,4.18)	(2.43,3.43,4.43)	(2.18,3.18,4.18)	(2.2,3.2,4.2)
	X_{16}	(2.38,3.38,4.38)	(3.26,4.26,4.98)	(2.78,3.78,4.78)	(2.47,3.47,4.43)	(1.98,2.98,3.98)	(2.82,3.82,4.67)	(2.9,3.9,4.85)	(1.9,2.9,3.9)	(2.9,3.9,4.9)
	X_{17}	(2.83,3.83,4.75)	(2.83,3.83,4.83)	(2.3,3.3,4.3)	(2.63,3.63,4.5)	(1.87,2.87,3.87)	(2.23,3.23,4.23)	(1.88,2.88,3.88)	(1.25,1.98,2.98)	(1.94,2.94,3.94)
	X_{18}	(1.85,2.85,3.85)	(2.08,3.08,4.08)	(3.03,4.03,4.97)	(2.63,3.63,4.63)	(2.55,3.55,4.52)	(2,3,4)	(2.03,3.03,4.03)	(1.63,2.63,3.63)	(2.57,3.57,4.45)
	X_{19}	(2.93,3.93,4.83)	(2.93,3.93,4.93)	(2.71,3.71,4.71)	(2.92,3.92,4.92)	(2.35,3.35,4.35)	(2.18,3.18,4.18)	(2.53,3.53,4.53)	(2.05,3.05,4.05)	(2.5,3.5,4.5)
X_2	X_{21}	(2.61,3.61,4.58)	(2.8,3.8,4.8)	(2.68,3.68,4.63)	(2.7,3.7,4.7)	(2.5,3.5,4.5)	(1.9,2.9,3.9)	(2.33,3.33,4.33)	(1.92,2.92,3.92)	(1.58,2.58,3.58)
	X_{22}	(2.26,3.26,4.26)	(2.65,3.65,4.65)	(2.46,3.46,4.46)	(1.62,2.62,3.62)	(2.42,3.42,4.33)	(2.13,3.13,4.13)	(1.97,2.97,3.97)	(2.32,3.32,4.32)	(2.37,3.37,4.37)
	X_{23}	(2.21,3.21,4.21)	(1.98,2.98,3.98)	(3.15,4.15,4.9)	(2.88,3.88,4.85)	(2.53,3.53,4.53)	(2.02,3.02,4.02)	(1.87,2.87,3.87)	(2.38,3.38,4.18)	(2.97,3.97,4.85)
	X_{24}	(2.2,3.2,4.2)	(2.51,3.51,4.46)	(3.06,4.06,4.8)	(2.67,3.67,4.67)	(2.02,3.02,4.02)	(2.55,3.55,4.55)	(1.67,2.67,3.67)	(1.77,2.77,3.77)	(2.5,3.5,4.5)
	X_{25}	(2.78,3.78,4.73)	(2.75,3.75,4.75)	(2.51,3.51,4.48)	(2.92,3.92,4.88)	(2.75,3.75,4.75)	(2.65,3.65,4.57)	(1.97,2.97,3.97)	(2.82,3.82,4.82)	(1.97,2.97,3.97)
X_3	X_{31}	(3.03,4.03,4.82)	(2.48,3.48,4.48)	(2.58,3.58,4.58)	(2.83,3.83,4.83)	(2.5,3.5,4.5)	(2.4,3.4,4.4)	(1.8,2.8,3.8)	(2.57,3.57,4.57)	(2.1,3.1,4.1)
	X_{32}	(1.75,2.75,3.75)	(2.28,3.28,4.28)	(2.76,3.76,4.76)	(1.78,2.78,3.78)	(3.75,4.75,5.75)	(2.17,3.17,4.17)	(2.67,3.67,4.67)	(1.83,2.83,3.83)	(1.87,2.87,3.87)
	X_{33}	(2.73,3.73,4.73)	(2.78,3.78,4.72)	(2.48,3.48,4.48)	(2.47,3.47,4.47)	(2.58,3.58,4.58)	(2.27,3.27,4.27)	(2.98,3.98,4.75)	(1.85,2.85,3.85)	(2.32,3.32,4.32)
	X_{34}	(2.23,3.23,4.23)	(2.41,3.41,4.41)	(2.51,3.51,4.51)	(2.77,3.77,4.77)	(2.32,3.32,4.27)	(2.3,3.3,4.08)	(2.83,3.83,4.6)	(2.12,3.12,4.12)	(2.03,3.03,4.03)
X_4	X_{41}	(2.61,3.61,4.61)	(2.95,3.95,4.9)	(2.36,3.36,4.36)	(2.7,3.7,4.7)	(1.98,2.98,3.98)	(2.97,3.97,4.93)	(1.8,2.8,3.8)	(2.9,3.9,4.9)	(1.98,2.98,3.98)
	X_{42}	(3.03,4.03,4.78)	(2.58,3.58,4.58)	(1.95,2.95,3.95)	(2.4,3.4,4.4)	(2.18,3.18,4.18)	(2.77,3.77,4.77)	(1.97,2.97,3.97)	(1.92,2.92,3.92)	(2.28,3.28,4.28)
	X_{43}	(3.3,4.3,4.87)	(2.33,3.33,4.33)	(2.75,3.75,4.75)	(2.78,3.78,4.75)	(2.73,3.73,4.6)	(1.3,2.03,3.03)	(2.62,3.62,4.62)	(1.95,2.95,3.95)	(2.58,3.58,4.47)
	X_{44}	(1.98,2.98,3.98)	(2.43,3.43,4.42)	(2.8,3.8,4.8)	(2.42,3.42,4.42)	(1.9,2.9,3.9)	(2.43,3.43,4.43)	(2.6,3.6,4.6)	(1.52,2.52,3.52)	(2.78,3.78,4.6)
	X_{45}	(2.6,3.6,4.6)	(2.01,3.01,4.01)	(1.95,2.95,3.95)	(2.6,3.6,4.57)	(3.07,4.07,4.83)	(2.18,3.18,4.18)	(2.37,3.37,4.37)	(2.95,3.95,4.72)	(1.6,2.33,3.33)
	X_{46}	(2.25,3.25,4.25)	(2.38,3.38,4.38)	(2.1,3.1,4.1)	(2.02,3.02,4.02)	(2.65,3.65,4.65)	(1.57,2.57,3.57)	(1.97,2.97,3.97)	(2.63,3.63,4.58)	(2.5,3.5,4.5)
X_5	X_{51}	(2.1,3.1,4.1)	(2.36,3.36,4.36)	(2.73,3.73,4.73)	(2.57,3.57,4.57)	(3.12,4.12,4.95)	(2.45,3.45,4.45)	(1.23,1.67,2.67)	(1.85,2.85,3.85)	(2.1,3.1,4.1)
	X_{52}	(2.13,3.13,4.13)	(2.56,3.56,4.56)	(1,7,2.7,3.7)	(2.47,3.47,4.47)	(2.55,3.55,4.55)	(1.72,2.72,3.72)	(1.58,2.58,3.58)	(1.82,2.82,3.82)	(1.32,2,3)
	X_{53}	(2.2,3.2,4.2)	(3.21,4.21,4.93)	(1.65,2.65,3.65)	(2.02,3.02,4.02)	(2.17,3.17,4.17)	(1.72,2.72,3.72)	(1.5,2.5,3.5)	(2.68,3.68,4.5)	(1.73,2.73,3.73)
	X_{54}	(2.68,3.68,4.68)	(1.81,2.81,3.81)	(2.33,3.33,4.33)	(1.92,2.92,3.92)	(2.85,3.85,4.88)	(1.73,2.73,3.73)	(1.87,2.87,3.87)	(1.65,2.65,3.65)	(1.95,2.95,3.83)

Then, we apply the simplified fuzzy numbers addition operations and division operations to the averaged fuzzy grade table shown in Table 11 to get the fuzzy grade of each criterion of each teacher with respect to each criterion. Finally, we can get the fuzzy grade matrix $\widetilde{G}$ defined as follows:

$$\widetilde{G} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & \cdots & X_k \end{array} \\ \begin{bmatrix} \widetilde{g}_{11} & \widetilde{g}_{12} & \cdots & \widetilde{g}_{1k} \\ \widetilde{g}_{21} & \widetilde{g}_{22} & \cdots & \widetilde{g}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{g}_{n1} & \widetilde{g}_{n2} & \cdots & \widetilde{g}_{nk} \end{bmatrix} \end{array},$$

where $\widetilde{g}_{ij}$ denotes the fuzzy grade of the ith teacher A_i with respect to the jth criterion X_j, $1 \le i \le n$, $1 \le j \le k$, n denotes the number of teachers, and k denotes the number of criteria. Therefore, we can get the fuzzy grade matrix $\widetilde{G}$, shown as follows:

$$\widetilde{G} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \end{array} \begin{array}{c} \begin{array}{ccccc} X_1 & X_2 & X_3 & X_4 & X_5 \end{array} \\ \begin{bmatrix} (2.61,3.61,4.55) & (2.41,3.41,4.40) & (2.44,3.44,4.38) & (2.63,3.63,4.52) & (2.28,3.28,4.28) \\ (2.58,3.58,4.54) & (2.54,3.54,4.53) & (2.49,3.49,4.47) & (2.45,3.45,4.45) & (2.49,3.49,4.42) \\ (2.57,3.54,4.52) & (2.77,3.77,4.65) & (2.58,3.58,4.58) & (2.32,3.32,4.32) & (2.10,3.10,4.10) \\ (2.70,3.70,4.65) & (2.56,3.56,4.54) & (2.46,3.46,4.46) & (2.49,3.49,4.48) & (2.25,3.25,4.25) \\ (2.36,3.36,4.33) & (2.44,3.44,4.43) & (2.79,3.79,4.78) & (2.42,3.42,4.36) & (2.67,3.67,4.64) \\ (2.33,3.33,4.20) & (2.25,3.25,4.23) & (2.29,3.29,4.23) & (2.20,3.20,4.15) & (1.91,2.91,3.91) \\ (2.40,3.40,4.38) & (1.96,2.96,3.96) & (2.57,3.57,4.46) & (2.22,3.22,4.22) & (1.55,2.41,3.41) \\ (2.01,2.98,3.97) & (2.24,3.24,4.24) & (2.09,3.09,4.09) & (2.31,3.31,4.27) & (2,3,3.96) \\ (2.37,3.37,4.35) & (2.28,3.28,4.25) & (2.08,3.08,4.08) & (2.29,3.24,4.19) & (1.78,2.70,3.67) \end{bmatrix} \end{array}.$$

Step 3: Calculate the total fuzzy grade vector $\widetilde{R}$, where

$$\widetilde{R} = \widetilde{G} \otimes \widetilde{W} = \begin{bmatrix} \widetilde{g}_{11} & \widetilde{g}_{12} \cdots \widetilde{g}_{1k} \\ \widetilde{g}_{21} & \widetilde{g}_{22} \cdots \widetilde{g}_{2k} \\ \vdots & \vdots \ddots \vdots \\ \widetilde{g}_{n1} & \widetilde{g}_{n2} \cdots \widetilde{g}_{nk} \end{bmatrix} \otimes \begin{bmatrix} \widetilde{w}_1 \\ \widetilde{w}_2 \\ \vdots \\ \widetilde{w}_k \end{bmatrix} = \begin{bmatrix} \widetilde{g}_{11} \otimes \widetilde{w}_1 \oplus \widetilde{g}_{12} \otimes \widetilde{w}_2 \oplus \cdots \oplus \widetilde{g}_{1k} \otimes \widetilde{w}_k \\ \widetilde{g}_{21} \otimes \widetilde{w}_1 \oplus \widetilde{g}_{22} \otimes \widetilde{w}_2 \oplus \cdots \oplus \widetilde{g}_{2k} \otimes \widetilde{w}_k \\ \vdots \\ \widetilde{g}_{n1} \otimes \widetilde{w}_1 \oplus \widetilde{g}_{n2} \otimes \widetilde{w}_2 \oplus \cdots \oplus \widetilde{g}_{nk} \otimes \widetilde{w}_k \end{bmatrix} = \begin{bmatrix} \widetilde{R}_1 \\ \widetilde{R}_2 \\ \vdots \\ \widetilde{R}_k \end{bmatrix}, \quad (10)$$

where $\tilde{R}_i$ denotes the total fuzzy grade of the ith teacher A_i and $1 \le i \le n$. Therefore, we can get the total fuzzy grade vector $\tilde{R}$, shown as follows:

$$\tilde{R} = \begin{bmatrix} \tilde{R}_1 \\ \tilde{R}_2 \\ \tilde{R}_3 \\ \tilde{R}_4 \\ \tilde{R}_5 \\ \tilde{R}_6 \\ \tilde{R}_7 \\ \tilde{R}_8 \\ \tilde{R}_9 \end{bmatrix} = \begin{bmatrix} (37.18, 69.57, 109.39) \\ (37.72, 70.29, 110.76) \\ (37.16, 69.4, 109.58) \\ (37.49, 69.57, 110.62) \\ (38.02, 70.72, 111.35) \\ (33.02, 64.02, 102.41) \\ (32.13, 62.30, 100.98) \\ (32.02, 62.57, 101.47) \\ (32.58, 62.84, 101.55) \end{bmatrix}.$$

Step 4: Use Eqs. (4)-(7) to rank the total fuzzy grades of the teachers. The larger the value of Rank($\tilde{R}_i$), the better the ranking of $\tilde{R}_i$ and the better the choice of teacher A_i, where $1 \le i \le n$ and n denotes the number of teachers. For example, after applying Eqs. (4)–(7) to rank the total fuzzy grades $\tilde{R}_1$, $\tilde{R}_2$, $\tilde{R}_3$, $\tilde{R}_4$, $\tilde{R}_5$, $\tilde{R}_6$, $\tilde{R}_7$, $\tilde{R}_8$ and $\tilde{R}_9$ obtained in Step 3, we can get the ranking results as shown in Table 12. From Table 12, we can see that Rank($\tilde{R}_5$) > Rank($\tilde{R}_2$) > Rank($\tilde{R}_4$) > Rank($\tilde{R}_1$) > Rank($\tilde{R}_3$) > Rank($\tilde{R}_6$) > Rank($\tilde{R}_9$) > Rank($\tilde{R}_8$) > Rank($\tilde{R}_7$). Because $\tilde{R}_5$ has the largest ranking value, teacher A_5 is the best choice.

Table 12. Ranking Values of the Total Fuzzy Grades

Total Fuzzy Grades $\tilde{R}_i$	X*	Y*	$\hat{S}$	Rank($\tilde{R}_i$)
$\tilde{R}_1$	72.0467	0.3333	29.5575	72.04670624
$\tilde{R}_2$	72.9233	0.3333	29.9055	72.9233
$\tilde{R}_3$	72.0467	0.3333	29.6541	72.046706
$\tilde{R}_4$	72.6933	0.3333	29.9482	72.6933
$\tilde{R}_5$	73.3633	0.3333	30.0242	73.3633
$\tilde{R}_6$	66.4833	0.3333	28.4086	66.4833
$\tilde{R}_7$	65.1367	0.3333	28.2150	65.1367
$\tilde{R}_8$	65.3533	0.3333	28.4551	65.3533
$\tilde{R}_9$	65.6567	0.3333	28.2623	65.6567

5 Conclusions

In this paper, we have presented a new method for appraising the performance of high school teachers based on simplified fuzzy number arithmetic operations. The proposed method uses fuzzy numbers to represent fuzzy grades, where fuzzy weights of criteria are automatically generated from the opinions of evaluators. The proposed method can appraise the performance of high school teachers in a more flexible and more intelligent manner.

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