An Optimal Method for Multiple Observers Sitting on Terrain Based on Improved Simulated Annealing Techniques

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Abstract. The problem of multiple observers sitting on terrain (MOST) is an important part in visibility-based terrain reasoning (VBTR), but it is difficult because of the unacceptable computing time. Recent developments in this field focus on involving spatial optimization techniques, such as a heuristic algorithm. In this paper, a new method is developed based on the Improved Simulated Annealing (ISA) algorithm through the analysis of different terrain characters. A new annealing function and a new state function are designed to make the improved algorithm fit the problem better. Experiment results show that without loss of precision, use of the ISA algorithm reduces time cost 50%~70% when compared with the traditional SA.

1 Introduction

Consider a given terrain, with an observer O at a certain height H. Define the viewshed as the specific terrain visible from O that lies within O's Region Of Interest (ROI), of radius R. The Multiple Observers Sitting on Terrain (MOST) problem consists of finding the fewest possible observers to make the united-viewshed of those observers cover a certain ratio area, given the kind of the observer (person, radar, etc.) and the characteristic of the observer (height, the radius of viewshed, etc.). The MOST problem has many applications, such as locating a telecommunication base station [2, 4, 8], protecting endangered species [3, 6], and locating wind turbines [15].

The current solving method for the MOST problem is to use a greedy algorithm, which compares all possible observer sets, in order to find the best one. The biggest disadvantage of this method is the fact that the computing cost rises exponentially with the increasing complexity of the problem, overrunning the computing capability of the existing computer. In order to solve this problem, W. Franklin et al. provided a toolkit which is based on the visibility-indexes of small cells and the swap algorithm [10, 11]; Rana [19] developed a method which only considers significant features of the terrain such as peaks or ridges; and Y.H. Kim [13] also provided a method based on the terrain features and heuristic algorithms such as a simulated annealing algorithm, a genetic algorithm and a swap algorithm. Y.H. Kim has also claimed that the traditional SA algorithm is the best one among all tested algorithms when considering the balance between time cost and accuracy [14].

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In this paper, an optimal method that is based on an Improved Simulated Annealing (ISA) algorithm is developed. Experiment results show that without loss of precision, use of the ISA algorithm reduces time cost 50%~70% when compared with the traditional SA.

2 Modeling and Time Complexity Analysis

Suppose that the real terrain T is in an XYZ coordinate space. DEM is defined by a set of points $p \equiv (x_p, y_p, z_p)$ where (x_p, y_p) is the coordinate of one point in the X-Y plane, and z_p is the corresponding elevation of (x_p, y_p). Two points p_1 and p_2 are mutually visible (inter-visible) if every point $q \equiv (x, y, z) = p_1 + t(p_2 - p_1)$, $0 < t < 1$, lies above the corresponding point p_q of the terrain, i.e. $z > z_q$ [7, 9].

Suppose on the terrain which has n points, the number of the observers in an observer set be s. Then define the arbitrary observer set O_k which satisfies the condition above as:

$$O_k = \{o_{k,i} \mid o_{k,i} \in T, i = 1,2,\cdots,s\} \tag{1}$$

Define the observer set's united viewshed as:

$$V(O_k) = \bigcup_{i=1}^{s} v(o_{k,i}) \tag{2}$$

where the $v(o_{k,i})$ is the view-shed of the observer o_i in the observer set O_k. The definition of MOST problems is

$$O_{opt} = \{O_k \mid MAX(V(O_k)), k = 1,2,\cdots,C_n^s\} \tag{3}$$

where $MAX(V(O_k))$ is to get the maximum value of the united-viewshed of O_k.

To analyze the time complexity of MOST problems, suppose each observer's radius of viewshed R be n points and use a greedy algorithm. There are two main steps:

A : Compute the viewshed of each point on the terrain using the method described above. The time complexity is $O(n^2)$, according to [5].

B : Select all possible combinations of s observers from the n points, and for each combination, calculate the united-viewshed. The number of selections is

$$C_n^s = \frac{n!}{s!(n-s)!}$$

When n is very large and $n>>s$, this approximates to n^s.Caculating the united-viewshed is $O(n)$ for one combination. So the total time complexity is $n^s * O(n)$, which approximates to $O(n^{s+1})$.Because usually the s>=2 commonly, $O(n^{s+1})$ is by far the larger of the two terms, the overall time complexity is $O(n^{s+1})$.

From analyses above, we can find that the time complexity of MOST problems is rising exponentially with the increase of the terrain area and the number of observers. With a problem of even moderate size, this suggests that the method based on a greedy algorithm will never be computationally tractable. Therefore, a new optimal method must be developed to find a solution of MOST which is both accurate and computationally feasible.

3 An Optimal Method for MOST

From the discussions above, it is clear to see that the key point of the solution for MOST is efficient comparison and selection of the multiple observers set.

Unfortunately, the selection of the best observer set from all possible ones is unfeasible. However, the MOST problem also can be treated as an optimal problem under some certain boundary conditions. Analogues to the solution of other optimal problems, using heuristic algorithms can make the solution feasible and get a nearly best observer set. Two typical heuristic algorithms, Simulated Annealing algorithm (SA) and Genetic Algorithm (GA), have been applied and the former one has been proven to be more suitable for MOST problems [14].

3.1 Principles of Simulated Annealing

Kirkpatrick et al. [16] introduced the conception of annealing in combinatorial optimization. This conception is based on a strong analogy between combinatorial optimization and the physical process of crystallization. This process has inspired Metropolis et al. [18] to propose a numerical optimization procedure known as Metropolis algorithm, which works as follows.

Starting from an initial situation whose ‘energy level’ is *f(0)*, a small perturbation of the state is made in the system. This brings the system into a new state with energy level *f(1)*. If *f(1)* is smaller than *f(0)*, then the state change is accepted. Otherwise, if *f(1)* is greater than *f(0)*, then the change is accepted with a certain probability. The probability of acceptance is given by the Metropolis criterion [1]:

$$P(accept \quad change) = \exp(\frac{f(0) - f(1)}{t_k}) \tag{4}$$

where t_k is a control or freezing parameter. The Simulated Annealing process is ended when the temperature has become a small value [12].

A crucial element of the procedure is the gradual decrease of the freezing parameter t_k.Usually, this is done using a constant multiplication factor:

$$t_k = t_0 \bullet \lambda^k \tag{5}$$

where $0 < \lambda < 1$, k are the annealing iteration times in temperature stage decreasing, and t_0 is the initial temperature stage of the system. This effectively means that jumping to higher energy becomes less and less likely towards the end of the iteration procedure [18].

In the application of MOST, the energy level f(·) is a cost function which correlates with a state at a certain temperature stage. The initial temperature stage t_0, is obtained from the equation

$$t_0 = -|\Delta_{\max}| / \ln p_r \tag{6}$$

where $|\Delta max|$ is the maximum difference of cost corresponding with a group of randomly selected states, and p_r is the initial accept probability, which is usually 0.5. The total number of iterations L per temperature stage is chosen by keeping the temperature stage constant until the cost function has reached a constant value or until it is oscillating around this constant value [20]. The annealing iteration times k are chosen by setting the final temperature stage to a minimal constant value.

3.2 Optimal Method Based on Improved SA

In this study, we developed an optimal method based on Improved Simulated Annealing (ISA) algorithm which consists of three steps as follows.

Step 1: According to the desired number of the observers s, partition the terrain into k average-sized smaller blocks and make each block contain s/k observers. The distribution of observers should be fairly uniform across the terrain.

Step 2: Pick s/k observers randomly and independently in each block. Compute each observer's viewshed and the united-viewshed coverage ratio of the observer set after the individual viewsheds have been combined.

Step 3: Let the result of Step 2 be the initial state, and apply the ISA to get the approximately best observer set. In our optimal method, we select 50 observer sets randomly, and get the original temperature stage of SA algorithm described in 3.1.

Table 1. The statistical information of six samples

	Min	Max	Diff	Mean	SD
Sample 1	693. 1	754. 2	61.1	712. 2	84. 28
Sample 2	939. 3	2531. 5	1592.2	1731. 2	250. 34
Sample 3	1197. 6	2452. 1	1254.5	1636. 8	470. 6
Sample 4	250. 0	461. 3	211.3	364. 4	1272. 6
Sample 5	2153. 5	2570. 1	416.6	2372. 7	2030. 2
Sample 6	930. 4	2481. 7	1551.3	2023. 6	3677. 1

3.3 The Improved Simulated Annealing Algorithm for MOST

For the MOST problem, we have done two analyses: The first analysis is the relationship between the distance separating two observers and the average increase of united-viewshed for these two observers compared to one observer. The second analysis is the relationship between the distance separating two observers and the ratio of the increased view-shed coverage for two observers, compared to one observer. In our analysis and in the following experiments, we assume that the observer height is H=1.6m, which is equal to the height of people's eyes.

3.3.1 The Improved New State Function

In order to get the relationships between the distance separating two observers and the average united-viewshed increase compared to one observer, three steps are included.

1. Select the first observer on the terrain randomly, and compute its viewshed.
2. Select the second observer randomly and let the new observer lie within the first one's neighbor domain whose radius is R. In our analysis, the R is average to 250 sample points (500 m). Compute the united-viewshed of the two observers, and then calculate the difference between the united viewshed and the viewshed of the single observer, as found in step 1. We refer to this resulting term as the "united-viewshed increase".
3. Repeat steps 1 and 2 50,000 times. Finally, get the relationships between the distances separating the two observers and their average united-viewshed increase compared to one observer's viewshed.

Repeat steps1 through 3 for six terrain samples, and get the result (See in Figure 1). The value given to the united-viewshed is the ratio between the number of the visible points in the united-viewshed and the total number of the points on the terrain.

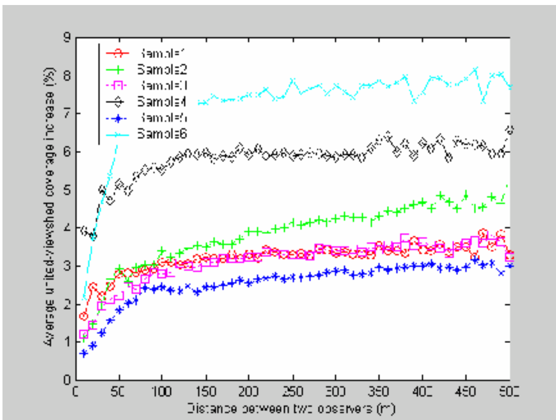

Fig. 1. The relationships between the distance separating two observers and their average united-viewshed increase compared to one observer's viewshed

The Figure 1 shows that the relationship curve between the distance separating two observers and their average united-viewshed increase compared to one observer's viewshed on different terrains is similar. That is, the united-viewshed increases with the distance between two observers. The rate of increase becomes slower and slower.

From the analysis above, we can see that if the observers are near, the average united-viewshed coverage of observer set decreases substantially. Therefore, the re-design of a new state function of SA algorithm should not only consider the average united-viewshed coverage ratio of new observer set but also consider the distance between observers. Accordingly, the new state function of new observer set O_k in an improved SA algorithm $\varphi(o_k)$ contains two parts:

$$\varphi(o_k) = f(o_k) + g(o_k), k = 1,2\cdots n \tag{7}$$

where $f(o_k)=\dfrac{1}{\bigcup\limits_{k=1,2,\cdots n} v(o_k)}$, and $v(o_k)$ is the viewshed of observer k. $g(o_k)$ is a punish function. The degrees of punish increase as the average distance between observers decreases. Therefore, if the average distance is small, the new observer set solution cannot be accepted, and it prevents the occurrence of ‘the assembling of the observers’.

3.3.2 The Improved Annealing Function

In order to show the relationship between the distance separating two observers and the variance ratio about the viewshed coverage of two observers compared to one, we compute the increase ratio at a certain distance based on the result in 3.3.1. The result is seen in Figure 2. The variance ratio is valued as the standard deviation (SD) of the united-viewshed increase.

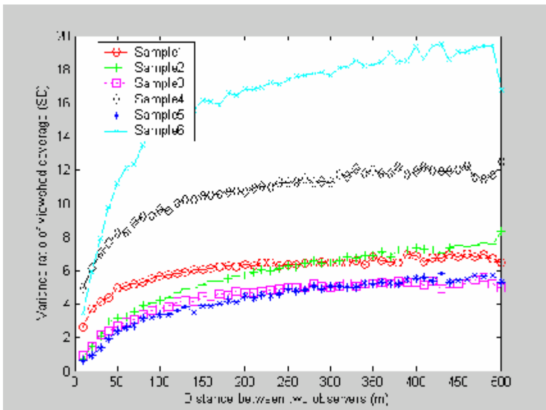

Fig. 2. The relationships between the distance separating two observers and the variance ratio about the viewshed coverage of two observers compared to one

Figure 2 shows that the variance of the united-viewshed mentioned in 3.3.1 increases with the distance between two observers, but the rate of increase slows. It is desirable to increase the annealing iteration times at a high temperature stage in order to help find the global optimal observer set, and to decrease the annealing iteration times at a low temperature stage for time cost savings. Therefore, it is necessary to redesign the annealing function to make it fit the MOST problem better.

Consider the annealing function of the traditional SA algorithm t_k [17], where t_0 is the original temperature stage, λ is the annealing factor, and k is the annealing iteration times. In order to increase the annealing iteration times at a high temperature and decrease the annealing iteration times at a low temperature, we introduce a temperature control function which can accommodate the origin annealing function in the traditional SA algorithm, and get the new annealing function t'_k , that is:

$$t'_k = \begin{cases} c \cdot (t_k^{\alpha})^{1/\gamma} & t_k <= 0.5 \\ \left[1 - c \cdot (1 - t_k)^{\alpha}\right]^{1/\gamma} & t_k > 0.5 \end{cases} \tag{8}$$

where $c = \frac{1}{2} \cdot (0.5)^{\alpha}$.

Suppose that $\alpha/\gamma = \beta$, it can be proved that

$$\lim_{\gamma \to \infty} \{k_{\max} \mid t_{k_{\max}} = t_0\} = \log \frac{1/2^{\beta}}{\lambda^{\beta}} \tag{9}$$

where K_{max} is the maximum step that the annealing temperature stage can keep the same as the initial temperature stage t_0.

In order to control the annealing process, we introduce three boundary conditions:

1. Suppose in the traditional SA algorithm, when annealing temperature stage decreases to 70% of the original temperature stage, the annealing iteration times are k_1. Then in the improved SA, the annealing iteration times of achieving the same instance are $2 \cdot k_1$.
2. Suppose in the traditional SA algorithm, when annealing temperature stage decreases to 10% of the original temperature stage, the annealing iteration times are k_2. Then in the improved SA, the annealing iteration times of achieving the same instance are $k_2/2$.

   According to the two conditions above, we can get $\alpha/\gamma \approx 3.98$ and $k_{\max}$ under different λ (see Table 2).

Table 2. The Max Annealing iteration times $k_{\max}$ Under Different λ

λ	0.5	0.6	0.7	0.8	0.9
$k_{\max}$	1	1.35	1.9	3.1	6.6

3. Suppose in the improved SA algorithm, when annealing temperature stage decreases to 80% of the original temperature stage, the annealing iteration times is $k_{\max}$.

   According to the three conditions above, we can get the α and γ under different λ (see Table 3).

Table 3. Parameter α and γ Under Different λ

λ	0.5	0.6	0.7	0.8	0.9
α	11.43	10.81	9.09	11.43	12.06
γ	2.86	2.70	2.27	2.86	3.03

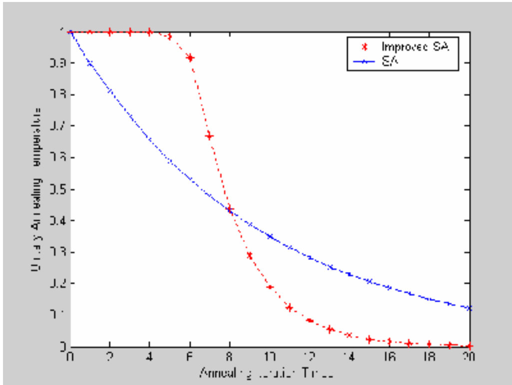

Fig. 3. The annealing function comparing traditional SA and improved SA of λ =0.9

In Figure 3, it is clear to see that the annealing iteration times of improved SA is much longer than the traditional SA in high temperature stages: 6 times, compared to 1 time when the temperature decreases to 90% of the original temperature stage. In contrast, the instance in low temperature stage is opposite: 11 times, compared to 21 times when the temperature decreases to 10% of the initial temperature stage.

3.4 Experiment Result

We use our optimal method to solve the MOST problem for six representative terrains (described in 3.2) in two experiments. Each experiment is repeated 10 times for each terrain. The traditional SA algorithm is used in Step 3 of the optimal method in the first experiment; the Improved SA (ISA) algorithm is used in the second experiment. The two algorithms stop when the temperature decreases to 10% of the original temperature stage. All the experiments are done by using a 2.4 GHz Pentium PC and 1 Gbytes RAM to get the comparison of united-viewshed coverage and time cost between using SA and ISA. The experiment results are presented below:

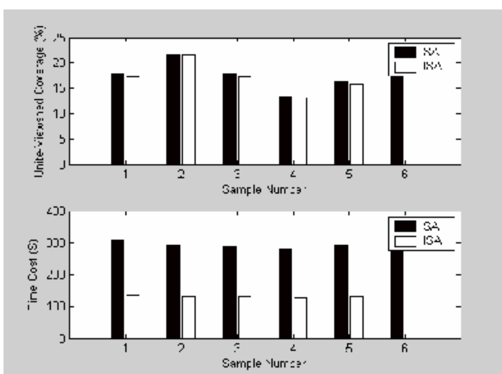

Fig. 4. 4 blocks ,1 observer per block, R=256 sample points, λ =0.9, H=1.6m

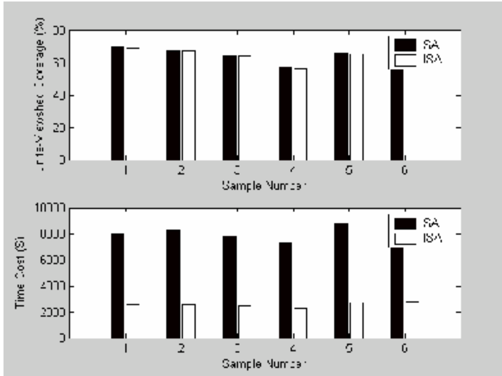

Fig. 5. 16 blocks , 8 observers per block, R=128 sample points, λ =0.9, H=1.6m

The ISA provides great time cost savings compared to the SA because when the annealing temperature decreases to 10% of the original temperature stage, the ISA uses only 11 iterations, while the SA uses 21. While the time cost savings of the ISA is great, the loss of accuracy is prevented by the greater number of annealing iterations at a high temperature (6, compared to 1), and the new state function. Figure 4 and figure 5 clearly show that without loss of precision, use of the ISA algorithm reduces time cost 50%~70% when compared with the traditional SA. The time cost savings become great as the number of observers increases.

4 Conclusions and Future

In visibility-based terrain reasoning, using a heuristic algorithm is an efficient method to solve the multiple observers sitting on terrain problem. However, if we use a general heuristic algorithm without any modification, it usually cannot get the best effect of the balance between precision and efficiency. Therefore, in this paper, an optimal method based on an Improved Simulated Annealing algorithm is developed after a problem-related analysis of six representative terrain samples. This improved algorithm reduces the time redundancy of the traditional SA algorithm.

Using the improved SA algorithm, it is hard to solve the MOST problem in a large terrain area with more observers and high precision in real time. More study is needed on the application of a multi-scale SA algorithm in multi-precision terrain data to solve MOST problems on large terrain. This method has great potential for the solution of MOST in real time.

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