Wavelet Chaotic Neural Networks and Their Application to Optimization Problems

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Abstract. In this paper, we first review Chen's chaotic neural network model and then propose a novel wavelet chaotic neural network. Second, we apply them to search global minima of a continuous function, respectively. Meanwhile, the time evolution figures of the corresponding most positive Lyapunov exponent are given. Third, 10-city traveling salesman problem (TSP) is given to make a comparison between them. Finally we conclude that the novel wavelet chaotic neural network is more valid.

1 Introduction

Many combinatorial optimization problems arising from science and technology are often difficult to solve entirely. Hopfield and Tank first applied the continuous-time, continuous-output Hopfield neural network (HNN) to solve TSP [1], thereby initiating a new approach to optimization problems [2, 3]. However, using the HNN to solve continuous-time nonlinear searching optimization and TSP suffers from several shortcomings. First, the network is often trapped at a local minimum in the complex energy terrain because of its gradient descent property. Second, HNN may converge to an infeasible solution. At last, sometimes, HNN does not converge at all within prescribed iteration.

Chaotic neural networks have been proved to be powerful tools for escaping from local minima. From then on, there have been some researches on chaotic neural networks in the field. Chen and Aihara proposed chaotic simulated annealing (CSA) to illustrate the features and effectiveness of a transiently chaotic neural network (TCNN) in solving optimization problems [4]; and Wang proposed a stochastic noisy chaotic simulated annealing method (SCSA) [5] by combining stochastic simulated annealing (SSA) and chaotic simulated annealing (CSA). All above researches are based on simulated annealing methods; distinctly, now we do research on the activation function.

In this paper, we first review the Chen's chaotic neural network model. Second, we propose a novel chaotic neural network model. Third, we apply both of them to search global minima of a continuous nonlinear function and then the time evolution figures of

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their most positive Lyapunov exponents are given. At last, we apply both of them to 10-city traveling salesman problem (TSP) in order to make a comparison. Finally we conclude the novel chaotic neural network we proposed is more valid.

2 Chaotic Neural Network Models

In this section, two chaotic neural network models are given. And the first is proposed by Chen, the second is proposed by us.

2.1 Chaotic Simulated Annealing with Decaying Self-coupling

Chen and Aihara's transiently chaotic neural network [4] is described as follows:

$$x_i(t) = f(y_i(t)) = \frac{1}{1 + e^{-y_i(t)/\varepsilon}} \tag{1}$$

$$y_i(t+1) = ky_i(t) + \alpha\left[\sum_j W_{ij}x_j + I_i\right] - z_i(t)(x_i(t) - I_0) \tag{2}$$

$$z_i(t+1) = (1-\beta)z_i(t) \tag{3}$$

where $x_i(t)$ is output of neuron i ; $y_i(t)$ denotes internal state of neuron i ; W_{ij} describes connection weight from neuron j to neuron i , $W_{ij} = W_{ji}$; I_i is input bias of neuron i , a a positive scaling parameter for neural inputs, k damping factor of nerve membrane, $0 \le k \le 1$, $z_i(t)$ self-feedback connection weight (refractory strength) ≥ 0, β damping factor of $z_i(t)$, $0 < \beta < 1$, I_0 a positive parameter, ε steepness parameter of the output function, $\varepsilon > 0$.

2.2 Morlet-Sigmoid Chaotic Neural Network (M-SCNN)

Morlet-Sigmoid chaotic neural network is a novel model proposed by us, described as follows:

$$y_i(t+1) = ky_i(t) + \alpha\left[\sum_j W_{ij}x_j + I_i\right] - z_i(t)(x_i(t) - I_0) \tag{4}$$

$$x_i(t) = f(y_i(t)) \tag{5}$$

$$z_i(t+1) = (1-\beta)z_i(t) \tag{6}$$

$$\eta_i(t+1) = \frac{\eta_i(t)}{\ln(e + \lambda(1 - \eta_i(t)))} \tag{7}$$

$$f(y_i(t)) = \gamma e^{-\frac{(u_1 y_i(t)(1+\eta_i(t)))^2}{2}} \cos(5u_1 y_i(t)(1+\eta_i(t))) + \frac{1}{1 + e^{-u_0 y_i(t)(1+\eta_i(t))}} \tag{8}$$

where $x_i(t)$, $y_i(t)$, W_{ij} , α , k , I_i , β , $z_i(t)$, I_0 are the same with the above. And $\eta_i(t)$ is the other simulated annealing factor, $\eta_i(t) > 0$; λ is a positive parameter, which controls the speed of this annealing process; u_0 and u_1 are important parameters of activation function which should be varied with kinds of special optimization problems.

3 Application to Search Optimization of Continuous Nonlinear Function

In this section, we apply the two chaotic neural networks to search the minimum points of a famous Six-Hump Camel-Back Function [6] which can be described as follows:

$$f(x_1, x_2) = 4x_1^2 - 2.1x_1^4 + x_1^6 / 3 + x_1 x_2 - 4x_2^2 + 4x_2^4 \quad \left|x_i\right| \leq 5 \qquad (9)$$

Its minimum point is (-0.08983,0.7126) or (0.08983,-0.7126), and its corresponding minimum value is -1.0316285.

Moreover, the time evolution figures of the corresponding most positive Lyapunov exponent are given.

3.1 Chen's Chaotic Neural Network

The parameters are set as follows:

α=0.5, k=1, β=0.004,I_0=0.8,ε=1 ,z (0)=[17.5,17.5],y(0)=[0,0].

The time evolution figure of the corresponding most positive Lyapunov exponent is shown as Fig.1.

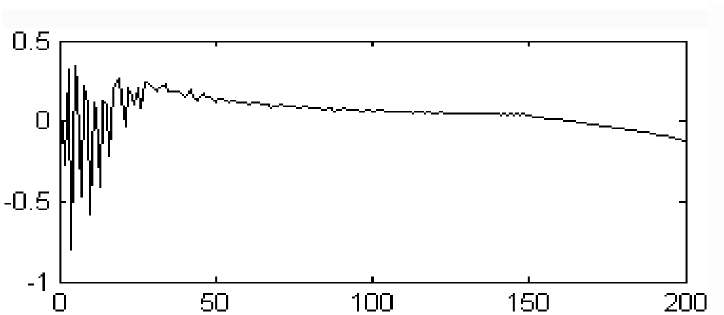

Fig. 1. Lyapunov exponent time evolution figure

We find out that when these parameters α、k、β、z (0) and y(0) are invariable, however we change the parameters I_0 and ε, the minimum point computed by Chen's model can not reach (-0.08983,0.7126) or (0.08983,-0.7126), and nor does the minimum energy -1.0316285. When these parameters are set as above, its minimum point is (1.3705e-131, 0.70717), and its corresponding minimum value is -1 within 2000 iterations.

Now, these parameters α、k、β、z (0) and y(0) are fixed so as to make a comparison between Chen's and our model.

3.2 Morlet-Sigmoid Chaotic Neural Network (M-SCNN)

The parameters are set as follows:

α=0.5, k =1, β =0.004, I_0=0.5,μ_0=0.05,μ_1=20,λ=0.002, z (0)=[17.5,17.5], y(0)=[0,0],η(0)=[0.05,0.05].

The time evolution figure of the corresponding most positive Lyapunov exponent is shown as Fig.5.

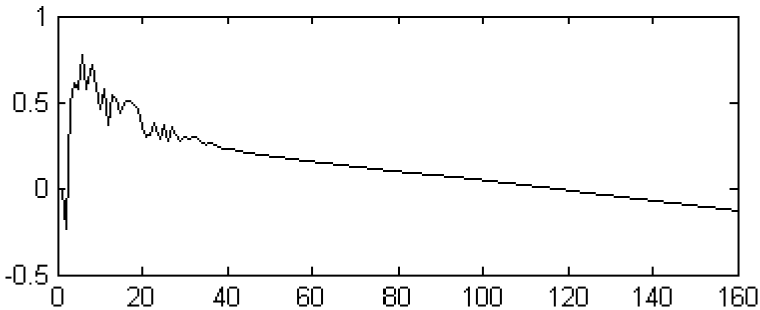

Fig. 2. Lyapunov exponent time evolution figure

Under these parameters, its minimum point is (-0.088431, 0.71251), and its corresponding minimum value is -1.0316 within 2000 iterations. Seen from the above analysis, the result is more accurate than Chen's, and the velocity of convergence is much faster than that of Chen's.

In order to verify the availability of our novel model, we apply it to the traveling salesman problem (TSP).

4 Application to Traveling Salesman Problem

The coordinates of 10-city is as follows:

(0.4, 0.4439)、(0.2439, 0.1463)、(0.1707, 0.2293)、(0.2293, 0.716)、(0.5171,0.9414)、(0.8732, 0.6536)、(0.6878, 0.5219)、(0.8488, 0.3609)、(0.6683, 0.2536)、(0.6195, 0.2634). The shortest distance of the 10-city is 2.6776.

Here are the results of the test about Chen's and M-SCNN.

The objective function we adopt is that provided in the reference [7].The parameters of the objective function are set as follows: A=2.5, D=1.

The parameters of Chen's are set as follows :

α=0.5, k =1, I_0=0.5, ε =1/20 ,z (0)=[0.5,0.5].

The parameters of M-SCNN are set as follows :

α=0.5, k =1,u_0=10,u_1=0.8,I_0=0.5,z(0)=[0.5,0.5],λ=0.001,η(0)=[0.8,0.8].

We make the test for 200 iterations in different β , as is shown in table 1. (VN= valid number; GN= global number; VP= valid percent; GP=global percent.)

Table 1. Test result of two chaotic neural network

β	Reference	VN	GN	VP	GP
0.04	M-SCNN	191	188	95.5%	94%
	Chen's	180	177	90%	88.5%
0.01	M-SCNN	191	188	95.5%	94%
	Chen's	180	177	90%	88.5%
0.008	M-SCNN	195	191	97.5%	95.5%
	Chen's	183	182	91.5%	91%

The time evolution figures of the energy function of M-SCNN and Chen's in solving TSP are respectively given in Fig.3 and Fig.4 when β =0.008.

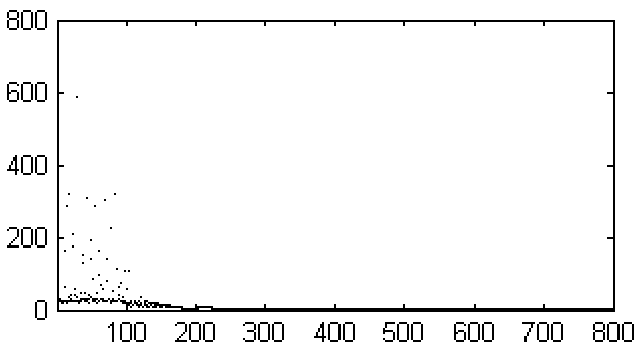

Fig. 3. Energy time evolution figure of M-SCNN

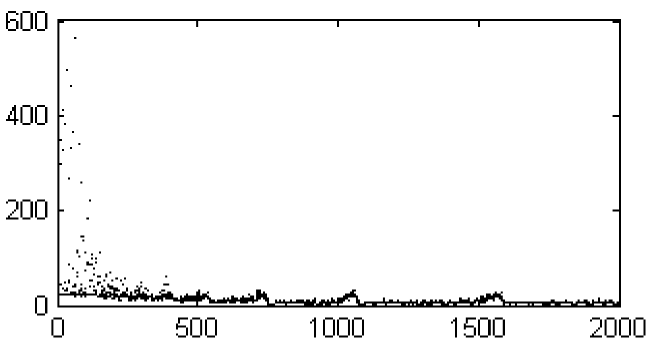

Fig. 4. Energy time evolution figure of Chen's

By comparison, it is concluded that M-SCNN is superior to Chen's model. From the Fig.3, Fig.4, one can see that the velocity of convergence of M-SCNN is much faster than that of Chen's in solving TSP.

The superiority of M-SCNN contributes to several factors: First, because of the quality of Morlet wavelet function, the activation function of M-SCNN has a further performance in solving combinatorial optimization problems than Chen's. Second, it is easier to produce chaotic phenomenon [8] in that the activation function is non-monotonic. Third, $\eta_i(t)$ is varied with time, which denotes steepness parameter of M-SCNN.

5 Conclusions

We have introduced two models of chaotic neural networks. To verify the availability of them, we have made comparison with Chen's model in optimization problems. By comparison, one can conclude that M-SCNN is superior to Chen's in searching global minima of continuous nonlinear function.

Different from Chen's model, the activation function of M-SCNN is composed by Morlet wavelet and Sigmoid. So, besides it has the nature of sigmoid activation, the activation function of M-SCNN has a higher nonlinear nature than Sigmoid, which is easier to produce chaotic phenomenon [8] because of its non-monotonic. Due to these factors, M-SCNN is superior to Chen's in solving TSP.

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