On Search Problems in Complexity Theory and in Logic (Abstract)

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Abstract. A search problem is given by a binary relation $B(x, y)$ in $\mathbf{P}$, such that $\forall x \exists y, |y| \leq poly(|x|) B(x, y)$. The computational task is for given x find such a y. We believe that in general this is not possible in polynomial time and oracles are known for which this is the case.

Many-to-one and Turing reductions between search problems are defined in a natural way. We conjecture that there is no complete search problem.

Our aim is to classify search problems and show relations between the computational complexities of them and the proof complexities of the sentences $\forall x \exists y, |y| \leq poly(|x|) B(x, y)$. A typical example of a class of search problems is the class Polynomial Local Search defined as follows.

A **PLS** problem is given by a $\mathbf{P}$-time relation $R(p, x)$ and a $\mathbf{P}$-time function $F(p, y)$ such that $R(n, n)$ holds for all n. The search problem is for every p and $x \leq p$ to find a $y \leq p$ such that

$$R(p, y) \wedge (\neg F(p, y) < y \vee \neg R(p, F(p, y))).$$

A typical result relating proof complexity and computational complexity of search problems is the following theorem of Buss and Krajíček.

Theorem 1. *A search problem $B(x, y)$ is reducible to a* **PLS** *problem iff*

$$T_2^1 \vdash \forall x \exists y \beta(x, y),$$

for a Σ_0^b formula $\beta(x, y)$ defining the relation $B(x, y)$, (where T_2^1 is a theory that formalizes induction for **NP** *sets.)*

In this lecture we shall present some recent results in this field.

T. Calamoneri, I. Finocchi, G.F. Italiano (Eds.): CIAC 2006, LNCS 3998, p. 5, 2006.