An Integrated Production-Inventory Model for Deteriorating Items with Imperfect Quality and Shortage Backordering Considerations

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Abstract. In this study we present a production-inventory model for deteriorating item with vendor-buyer integration. A periodic delivery policy for a vendor and a production-inventory model with imperfect quality for a buyer are established. Such implicit assumptions (deteriorating items, imperfect quality) are reasonable in view of the fact that poor-quality items do exist during production. Defective items are picked up during the screening process. Shortages are completely backordered. The study shows that our model is a generalization of the models in current literatures. An algorithm and numerical analysis are given to illustrate the proposed solution procedure. Computational results indicate that our model leads to a more realistic result.

1 Introduction

Since the development of the economic order quantity (EOQ) more than four decades ago, a substantial amount of researches have been conducted in the area of inventory lot sizing. However, one of the weaknesses of most researches is the unrealistic assumption of perfect quality items [25]. Cheng [2] proposed an EOQ model with demand-dependent unit production cost and imperfect production process. He proposed a general power function to model the relationship between unit production cost, demand rate and process reliability. Cheng formulated this inventory decision problem as a geometric problem (GP), and applied the theories of GP to derive a closed-form optimal solution. Zhang and Gerchak [30] considered a joint lot sizing and inspection policy, for an EOQ model with a random proportion of defective units. They considered a model where the defective units are replaced by non-defective ones. Rosenblatt and Lee [24] considered the presence of defective products in a small lot size replenishment policy. They assumed that the defective rate from the beginning in-control state until the process goes out of control increased exponentially. The defective items can be reworked instantaneously and kept in stock. Rosenblatt and Lee concluded that the presence of defective products resulted in smaller lot sizes. Schwaller [26] presented a procedure to extend EOQ models by assuming that the defectives of a known proportion were present in the incoming lots,

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and that fixed and variable inspection costs were incurred in finding and removing the items. Porteus [22] incorporated the effect of defective items in the basic EOQ model and invested in process quality improvement. He assumed a probability q would go out of control during production.

Salameh and Jaber [25] presented a modified inventory model for imperfect quality items. They considered poor-quality items are sold as a single batch by the end of the 100% screening process. Rosenblatt and Lee [24] showed that reducing the lot size quantity increased the average percentage of imperfect quality items. The reasonable explanation is that Rosenblatt and Lee [24] assumed defective items were reworked instantaneously and kept in stock. This increases the holding cost that results in lower lot sizes, whereas in this paper, imperfect quality items are withdrawn from stock resulting in lower holding cost and larger lot sizes. Goyal and Gardenas-Barron [10] extended Salameh and Jaber's model and presented a practical approach to determine EPQ for items with imperfect quality. The approach suggested in their study results in nearly a zero penalty as compared to Salameh and Jaber. Later, Goyal *et al.* [11] extended the model of Goyal and Gardenas-Barron [10] to consider vendor-buyer integration. Chung and Hou [3] developed a model to determine an optimal run time for a deteriorating production system with shortages. They assumed the elapsed time is random between the production process shifts.

Recently, Wee and Yu [28] extended the approach by Salameh and Jaber and considered permissible shortage backordering. They found that the traditional EOQ and Salameh and Jaber's modified EPQ/EOQ model are both special cases of the proposed model when the backordering cost is very large. In this paper, the influence of imperfect quality and deterioration is taken into account. Imperfect quality is the result of imperfect machines and processes. Deterioration occurs because many agricultural products, gasoline and medicine do not have constant utility during storage. The distribution of time to deterioration of the item follows the exponential distribution.

Ongoing deteriorating inventory has been studies by several authors in recent decades. Ghare and Schrader [12] were the first authors to consider ongoing deterioration of inventory. They have developed an EOQ model for items with an exponentially decaying inventory. Elsayed and Terasi [5] proposed a deteriorating production-inventory model with Weibull distribution and permissible shortage. Kang and Kim [20] proposed an exponentially deteriorating model considering the price and production level. An exponentially deteriorating production-inventory model with permissible shortage is presented in [23]. Other authors such as Dave [4] and Heng *et al.* [16] assumed either instantaneous or finite production rate with different assumptions on the patterns of deterioration. Yang and Wee [29] developed an integrated economic ordering policy of deteriorating items for a vendor and multiple-buyers.

Collaboration of enterprises, especially in terms of developing strategies, is vital in reducing the overall cost of the enterprise. This is because decision made independently by one player will not result in global optimum. Global optimization will only be realized if the perspectives of all players are considered. One of the advantages of applying joint economic lot size models (JELS) is being able to generate lower total inventory relevant cost for the system so that the net benefit can be shared by both parties. The JELS approach has been studied for years. Goyal [6] was the first to introduce an integrated inventory policy for the single-supplier single-customer problem. He showed that his integrated policy results in minimum joint variable cost for the supplier and the customer. Banerjee [1] developed a joint economic lot size model with lot-for-lot policy for a single-buyer single-vendor system by combining two EOQ models from the buyer and vendor. In his model, he assumed that the vendor makes the production setup as long as the buyer places an order and supplies on a lot for lot basis. He also showed that his JELS model has minimum joint total relevant cost by considering both the buyer and the vendor at the same time. Later Goyal [7] generalized Banerjee's model by relaxing the assumption of the vendor's lot-for-lot policy. He pointed out that the vendor could possibly produce a lot that can supply an integer number of orders from the buyer. Nevertheless the model restricts shipments cannot be triggered before the whole production batch is completed. A review of previous models on buyer-vendor integration until 1990 refers to [8].

Lu [21] developed an algorithm which derived an optimal solution to the single-vendor single-buyer problem, when the delivery quantity to the buyer at each replenishment was identical. Lu's model is synchronous, allowing shipment to take place during production. The model proposed by Halm and Yano [14] is also synchronous, aiming to minimize the manufacturer' and buyer's inventory holding cost, manufacturer's setup cost, as well as the transportation cost. Halm and Yano advocates that for the single-buyer single-item problem, the optimal solution has the property that the production interval is an integer multiple of the delivery interval. Halm and Yano [15] further extend the model to the single-machine multi-component problem. A heuristic procedure was therefore developed to find both the "just-in-time" production runs and the delivery schedule. Goyal [9] relaxes the identical shipment constraint, allowing the quantity of successive shipments to be different in an increasing fashion by a fixed factor of production rate to demand rate. The policy is to deliver whatever that is produced at the replenished time by using the same example as Lu [21]. Goyal shows that a different shipment policy could result in a better solution. Ha and Kim [13] analyze the integration between the buyer and the supplier, and developed a mathematical model using the geometric method.

Though both Goyal's [9] and Hill's [17] models illustrate that delivery in "what is produced" policy is better than delivery in "identical shipment" policy, Viswanathan [27] shows that neither strategy obtains the best solution for all possible problem parameters. More recently, Hill [18] derived a globally optimal batching and shipping policy for the single-vendor single-buyer integrated production-inventory problem. Hoque and Goyal [19] proposed an optimal policy for a single-vendor single-buyer integrated production-inventory system with a limited capacity of transport equipment.

In this study, product deterioration and vendor-buyer integration are considered simultaneously. We propose a production-inventory model for an on-going deterioration item with partial backordering and imperfect quality. Shortages due to imperfect items are completely backordered. This is because not all customers are willing to wait for a new replenishment of stock. Customers encountering shortages will respond differently according to the type of commodities and market environment. In real world, complete backordering is likely only in a monopolistic market. An illustrative example and sensitivity analysis are given to validate the inventory model.

2 Notation and Assumptions

The following notation is used:

$I(t)$ = Inventory level at time t, $0 \le t \le T$;
I_m = Maximum inventory level;
I_s = Inventory level at the end of time t_1;
T_{v1} = The time length of the production stage;
T_{v2} = The time length of the non-production stage;
R = The total production quantity;
Q = Order size;
D = Demand rate for buyer;
X = Screening rate for buyer;
c = Purchasing cost per unit for buyer;
h = Carrying cost per unit per unit time for buyer;
d = Deterioration cost per unit for buyer;
K = Ordering cost per order for buyer;
θ= Deterioration rate;
C_{vh}= Carrying cost per unit per unit time for vendor;
C_{vd}= Deterioration cost per unit for vendor;
p = Defective percentage in demands for vendor;
$f(p)$ = Probability density function of p;
x = Screening cost per unit for buyer;
I_b = Total shortage demand (units/cycle);
b = Backordering cost per unit for buyer;
* = Superscript representing optimal value;

The mathematical models presented in this study have the following assumptions:

(1) A single item with constant deteriorating rate of the on-hand inventory is considered.
(2) Demand rate is a continuous known constant.
(3) Lead-time is a known constant.
(4) Defective items are independent of deterioration.
(5) Replenishment is instantaneous.
(6) Screening process and demand proceeds simultaneously.
(7) Defective percentage, p, has a uniform distribution with [α, β], where $0 \le \alpha < \beta \le 1$.
(8) Shortages are completely backordered.
(9) A single product is considered.

3 Mathematical Model

We derive the cost involved in integrating the lot sizing policies between a vendor and a buyer. The ultimate form of JIT purchasing agreement should be adopted to minimize the total cost by implementing frequent small lots deliveries. Figure 1 depicts the behavior of inventory levels for both the vendor and the buyer. The annual integrated total cost consists of the vendor's annual total cost, and the buyer's annual total cost.

3.1 The Vendor's Total Cost per Unit Time

Figure 1 shows that in periodic delivery, the vendor does not stop producing until all demand is satisfied. For a given R, the values of T_{v1} and T_{v2} can be derived as

$$T_{v1} = R / p \tag{1}$$

and

$$T_{v2} = \frac{1}{\theta} \ln\left(\left(1-\frac{p}{D}\right)\left(\exp\left(\frac{-R\theta}{p}\right)-1\right)+1\right) \tag{2}$$

respectively. The proof is attached in Appendix A.

For $T_{v1} + T_{v2} = T_v$, one has

$$T_{v1} + T_{v2} = \frac{R}{p} + \frac{1}{\theta} \ln\left(\left(1-\frac{p}{D}\right)\left(\exp\left(\frac{-R\theta}{p}\right)-1\right)+1\right) \tag{3}$$

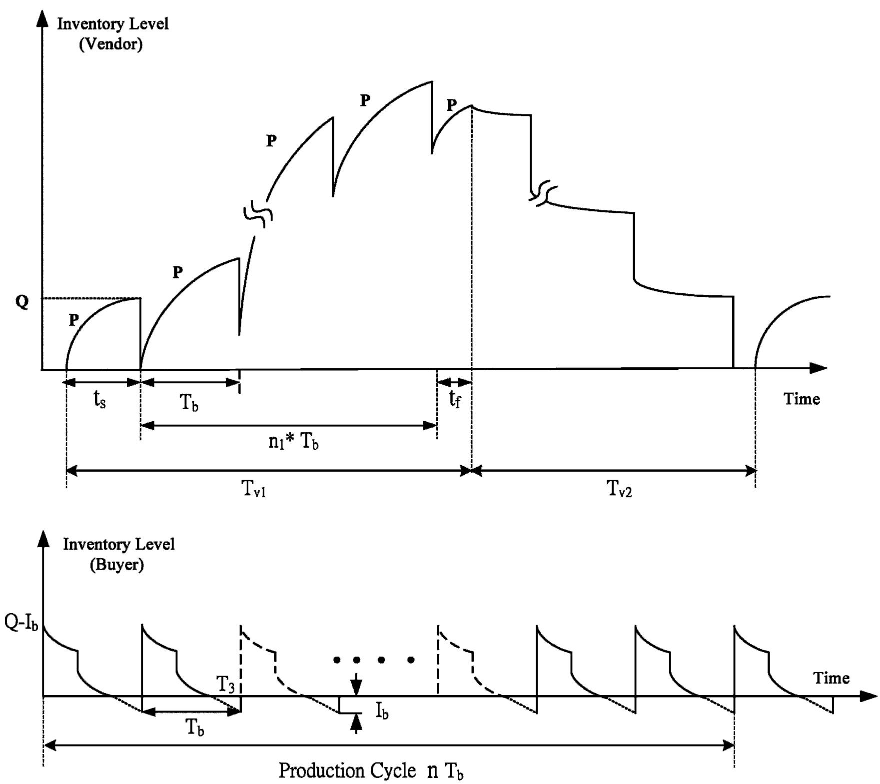

Fig. 1. Inventory level of vendor and buyer with customer demand

The vendor total inventory cost per unit time is depicted by the following formula:

Total cost = setting cost + delivery cost + holding cost + deteriorating cost

A carrying inventory can be derived as follows

$$\text{Carrying inventory } = \frac{\Delta\ I\ \ -D\cdot\Delta t}{\theta} = \frac{pT_{v1}-nQ}{\theta} = \frac{R-nQ}{\theta}$$

Hence, the holding cost per cycle is equal to $C_{vh}\left(\frac{R-nQ}{\theta}\right)$ and the deteriorating cost per cycle is equal to $C_{vd}(R-nQ)$. In addition to the setup cost and the delivery cost, the vendor's annual total cost is given by

$$TC_v = \frac{C_s}{T_v} + \frac{nC_d}{T_v} + \left(\frac{R-nQ}{T_v}\right)\left(\frac{C_{vh}}{\theta} + C_{vd}\right) \tag{4}$$

3.2 The Buyer's Total Cost per Unit Time

Figure 2 shows a lot size of Q units is replenished with an ordering cost of $\$K$ and a purchasing price of $\$c$ per unit. A fraction of each lot received is defective, with a known probability density function f (p). The random variable p has a uniform distribution [α, β], where $0 \le \alpha < \beta \le 1$. A 100% screening process of the item is

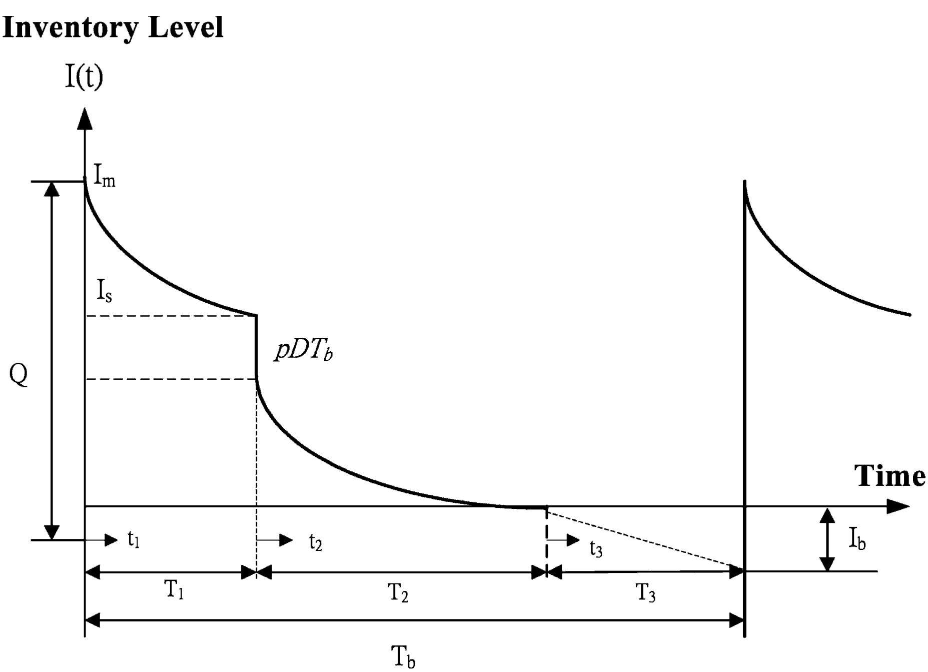

Fig. 2. Buyer's inventory system with backordering

conducted at a rate of X. The defective items are picked up in a single batch during the replenishment period T_1. Shortages of stock are partial backlogged at the beginning of each period. The behaviour of the inventory system is illustrated in Figure 1, where T_b is the cycle length, pDT_b is the maximal number of defectives, and I_b is the total unit backordered.

The buyer's total cost per unit time , TC_b, is depicted as:

Total inventory cost = Ordering cost + Screening Cost + Deteriorating cost + Holding cost + Backordering cost.

For the inventory system depicted in figure 3, the carrying inventory within the time interval between T_1 and T_2 is

$$\frac{\Delta\ I\ \ -D\cdot\Delta t}{\theta}=\frac{1}{\theta}\left(Q-DT_b p-DT_b\right) \tag{5}$$

For $I_b=DT_3=pDT_b$, the backlogged inventory during T_3 is equal to $\frac{1}{2}DT_b^{\ 2}p^2$. Therefore, the total annual inventory cost is

$$TC_b=TC_b(T_b)=\frac{K}{T_b}+\left(c+x+d+\frac{h}{\theta}\right)\frac{Q}{T_b}-\left(d+\frac{h}{\theta}\right)\left(Dp+D\right)+\frac{bDp^{\,2}T_b}{2} \tag{6}$$

The change in the inventory level during an infinitesimal time, dt, is a function of the deterioration rate θ, the demand rate D, and the inventory level $I(t)$. It is formulated as

$$\frac{dI_1(t)}{dt}+\theta\ I_1(t)=-D \quad 0\le t\le T_1 \tag{7}$$

$$\frac{dI_2(t)}{dt}+\theta\ I_2(t)=-D \quad 0\le t\le T_2 \tag{8}$$

$$\frac{dI_3(t)}{dt}=-D \qquad 0\le t\le T_3 \tag{9}$$

$I(t)$ is the inventory level at time t.

From the above differential equations, after adjusting for the constant of integration with various boundary conditions: $I_1(0)=I_m$, $I_2(0)=I_s\text{-}pDT$ and $I_3(0)=0$, the differential equations become:

$$I_1(t_1)=\left[\left(I_s+\frac{D}{\theta}\right)\exp\left((T_1-t_1)\ \theta\right)\right]-\frac{D}{\theta}\ ,\ \ 0\le t_1\le T_1 \tag{10}$$

$$I_2(t_2)=\left[\left(I_s-pDT+\frac{D}{\theta}\right)\exp\left(-\theta t_2\right)\right]-\frac{D}{\theta}\ ,\ \ 0\le t_2\le T_2 \tag{11}$$

and

$$I_3(t_3) = -Dt_3 \quad , \qquad 0 \le t_3 \le T_3 \tag{12}$$

Since the defective items are independent of deterioration, they have a value equal to pDT_b. From figure 2, $I_1(T_1) = I_s = I_2(0) + pDT_b$, one has

$$I_s = pDT_b + \frac{D}{\theta}\left(\exp\left(\theta\ T_2\right) - 1\right) \tag{13}$$

$$I_b = DT_3 = pDT_b \tag{14}$$

and

$$I_1(0) = Q - pDT_b = \left(pDT_b + \frac{D}{\theta}\left(\exp \quad (\theta\ T_2)\ \right)\right) \ \left(\exp \quad (\theta\ T_1)\right) - \frac{D}{\theta} \tag{15}$$

The replenishment, Q, can be derived by substituting $T_1 = DT_b/X$ into Eq. (15). One has

$$Q = Q(T_b) = pDT_b + pDT_b \exp\ \left(\theta\ T_b D / X\right) + \frac{D}{\theta} \exp\ \left(\theta\ T_b(1-p)\right) - \frac{D}{\theta} \tag{16}$$

Expanding the exponential functions and neglecting the third and higher power of $\theta \cdot T$, Eq. (16) becomes:

$$Q = Q(T_b) = (Dp + D)T_b + \left(\frac{p\theta D^2}{X} + \frac{\theta D}{2}(1-p)^2\right)T_b^{\ 2} + \left(\frac{p\theta^2 D^3}{2X^2}\right)T_b^{\ 3} \tag{17}$$

By substituting Eq. (17) into Eq. (6), one has

$$TC_b(T_b) = \frac{K}{T_b} + \left(c + x + d + \frac{h}{\theta}\right)\left(Dp + D + \left(\frac{p\theta D^2}{X} + \frac{\theta\ D}{2}(1-p)^2\right)T_b + \left(\frac{p\theta^2 D^3}{2X^2}\right)T_b^{\ 2}\right)$$
$$- \left(d + \frac{h}{\theta}\right)\left(Dp + D\right) + \frac{bDp^2 T_b}{2} \tag{18}$$

3.3 The Joint Total Cost Per Unit Time

For the vendor, by substituting $T_v = nT_b$ and Eq. (17) into Eq. (4), one has

$$TC_v(T_b) = \left(\frac{R}{nT_b} - (Dp + D) - \left(\frac{p\theta D^2}{X} + \frac{\theta D}{2}(1-p)^2\right)T_b \ - \left(\frac{p\theta^2 D^3}{2X^2}\right)T_b^{\ 2}\right)(\frac{C_{vh}}{\theta} + C_{vd})$$
$$+ \frac{C_s}{nT_b} + \frac{C_d}{T_b} \tag{19}$$

From Eq. (18) and Eq. (19), the total cost per unit time for both the vendor and buyer is: $JTC(T_b,n) = TC_v(T_b,n) + TC_b(T_b)$

For $\alpha \le p \le \beta$, one has

$$E[p] = \frac{\alpha+\beta}{2} = \mu_1$$

$$E[p^2] = \frac{\alpha^2 + \alpha\beta + \beta^2}{3} = \mu_2$$

$$E[(1-p)^2] = \frac{\alpha^2 + \alpha\beta + \beta^2}{3} - (\alpha+\beta) + 1 = \mu_3$$

The expected value of *JTC*, *EJTC*, is

$$\begin{aligned} EJTC\,(T_b) = & \left(\frac{R}{nT_b} - (D\mu_1 + D) - \left(\frac{\mu_1 \theta D^2}{X} + \frac{\theta D}{2}\mu_3 \right) T_b \; - \left(\frac{\mu_1 \theta^2 D^3}{2X^2} \right) T_b{}^2 \right) \left(\frac{C_{vh}}{\theta} + C_{vd} \right) \\ & + \frac{C_s}{nT_b} + \frac{C_d}{T_b} \\ & + \frac{K}{T_b} + \left(c + x + d + \frac{h}{\theta} \right) \left(D\,\mu_1 + D + \left(\frac{\theta\, D^2\mu_1}{X} + \frac{\theta\, D\mu_3}{2} \right) T_b + \left(\frac{\theta^2 D^3 \mu_1}{2X^2} \right) T_b{}^2 \right) \\ & - \left(d + \frac{h}{\theta} \right) (D\mu_1 + D) + \frac{bDT_b\mu_2}{2} \end{aligned} \tag{20}$$

and from Eq. (3),

$$T_b = T_b(n) = \frac{1}{n} \left(\frac{R}{p} + \frac{1}{\theta} \; \ln \left(\left(1 - \frac{p}{D} \right) \left(\exp\left(\frac{-R\theta}{p} \right) - 1 \right) + 1 \right) \right) \tag{21}$$

3.4 Methodology and Solution Search

Our objective is to minimize the expected cost function.

$$\begin{aligned} & \underset{\alpha \le p \le \beta}{Min} \quad EJTC\,(n) \\ & s.t. \qquad Q(T_b) > 0 \\ & \qquad\qquad T_b > 0 \\ & \qquad\qquad n \in N \end{aligned} \tag{22}$$

The problem is to determine the value of *n* that minimizes *EJTC*. Since the number of deliveries per cycle, *n*, is a discrete variable, the value of n can be derived as follows:

Step 1. Input all the system parameters.
Step 2. For a range of *n*-value, equate the first derivative of *EJTC* with respect to T_b to zero. For each *n*, denote the resulting minimum value of T_b by $T_b(n)$.

Step 3. Derive the optimal value of n, denoted by $n^{\#}$. And, the value, n^{*}, is an integer in the vicinity of $n^{\#}$. The optimal value of must satisfy

$$EJTC(n^{*}-1) \geq EJTC(n^{*}) \leq EJTC(n^{*}+1) \tag{23}$$

Step 4. Using (17), the periodic delivery quantity, Q, can be solved.

4 Numerical Example

To illustrate the preceding theory, we compare our analysis with the example from Salameh and Jaber. The following data are assumed: R=150000 unit, P=160000 units/year, C_s=*300*/cycle, C_d=\$25/unit, C_{vh}=\$2/unit/year, C_{vd}=\$30/unit, D=50000 units/year, K=\$100/cycle, h=\$5/unit/year, X=175200 units/year, x=\$0.5/unit, b=\$10/unit/year, c=\$25/unit, d=\$30/unit, s=\$50/unit. The item deteriorates at a constant rate with θ=0.01. The percentage defective random variable, p, can take any value in the range [α, β] where α=0, and β=0.04. It is assumed that p is uniformly distributed with its p.d.f.

$$f(p)=\begin{cases}25, & 0 \leq p \leq 0.04,\\ 0 & otherwise.\end{cases}$$

Since *EJTC* is a very complicated function due to high-power expression of the exponential function, a graphical representation showing the convexity of the *EJTC* is given in Fig. 3. Following the above solution procedure, we compute the optimal value of n that minimizes Eq.(20) as n^{*}=75 ($n^{\#}$=74.8). Substituting n^{*}=75 into Eq.(21), the optimum values of T_b is 0.0396 year. From Eq.(17), the lot size Q^{*} is 2020 units. Therefore, the integrated total cost per year is \$1,194,719.

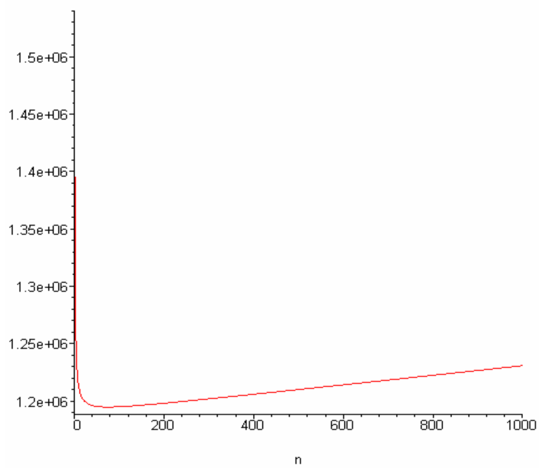

Fig. 3. Graphical representation of a convex *EJTC* (when n^{*}=75)

Based on the numerical example, if the decision is made solely from the buyer's perspective, the optimal value of T_b that minimizes Eq.(18) is T_b^*=0.0272. From Eqs.(17) and (21), the optimal values of Q and n are Q^*= 1388 and n^*= 109. Substituting them into the buyer's expected annual cost and the vendor's expected annual cost, the total cost of the buyer and the vendor is $1,195,172. Therefore, the integrated cost reduction is (1,195,172-1,194,719)= 453. Note that the expected annual integrated total cost has an impressive cost-reduction as compared with an independent decision by the buyer.

5 Conclusions

This study has presented a deteriorating inventory model with unreliable process. The model extends the studies in [25] and considers a single-vendor single-buyer integrated two-echelon supply chain environment. Comparative studies in the example show the benefit of integration. The effect of deterioration should be considered even if it is small. We have shown in this study that the influence of imperfect quality, deterioration and complete backordering are significant. The management of an enterprise can select suppliers based on the defective percentage and the deterioration rate of the products supplied by each supplier.

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Appendix A

From Fig. 2, the vendor's production interval, T_{v1}, and the production interval, T_{v2}, can be denoted by

$$T_{v1} = t_s + n_1 \cdot T_b + t_f \quad \text{and} \quad T_v = T_{v1} + T_{v2} = n \cdot T_b \tag{A1}$$

It is observed that the total inventory level at T_{v1} (the production stage) is the same as total inventory level at T_{v2} (the non-production stage). One has

$$I(T_{v1}) = \left(\frac{p-D}{\theta}\right)\left(1-\exp\left(-T_{v1}\theta\right)\right) = \left(\frac{D}{\theta}\right)\left(\exp\left(T_{v2}\theta\right)-1\right). \tag{A2}$$

Substituting $T_1 = R/P$ into (A2), the values of T_2 can derived as

$$T_{v2} = \frac{1}{\theta}\ \ln\left(\left(1-\frac{p}{D}\right)\left(\exp\left(\frac{-R\theta}{p}\right)-1\right)+1\right) \tag{A3}$$