Cross-Facility Production and Transportation Planning Problem with Perishable Inventory

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Abstract. This study addresses a production and distribution planning problem in a dynamic, two-stage supply chain. This supply chain consists of a number of facilities and retailers. The model considers that the final product is perishable and therefore has a limited shelf life. We formulate this problem as a network flow problem with a fixed charge cost function which is *NP*-hard. A primal-dual heuristic is developed that provides lower and upper bounds. The models proposed can be used for operational decisions.

1 Introduction

This paper investigates a planning model that integrates production, inventory and transportation decisions in a two-stage supply chain. Production and transportation activities have usually been studied separately by industry and academia, mainly because (*i*) each problem in itself is difficult and therefore the combined problem is not tractable, and (*ii*) different departments in an organization are in charge of each activity. In fact, the two activities can function independently if there is a sufficiently large inventory buffer that completely decouples the two. This, however, would lead to increased holding costs and longer lead times. The pressure of reducing costs in supply chains forces companies to take an integrated view of their production and distribution processes.

The supply chain analyzed in this paper consists of a number of facilities, each with similar production capabilities, and a number of retailers. We assume that retailers' demand for a single perishable product is known deterministically and that there are no production or transportation capacity constraints. Facilities produce the final product and carry inventories to satisfy retailers' demands during the planning period. We assume that there is no transshipment between facilities. This situation is typical in the food and beverage industry, where the retailers are often supermarkets and restaurants that have a very limited storage capacity. Most of the food products are perishable and have a limited lifetime. To account for this, we constraint the number of periods that a product is stored at a facility before being shipped to the retailer. The decisions that need to be made are (*i*) the timing of production; (*ii*) the location and size of inventories; and (*iii*) the timing of shipment.

The proposed model is suitable for tactical and operational planning. We assume that the planning period is a typical one, and it will repeat itself over

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time. This means the model is cyclic in nature. For this reason, we assume a fixed starting and ending period with varying initial and ending inventories. We model the problem as a network flow problem with fixed charge cost functions. This is an *NP*-hard problem since even some of its special cases are known to be *NP*-hard. For example, the single-period problem is a fixed charge network flow problem in bipartite networks (Johnson *et al.* [1]) that is *NP*-hard. The complexity of this problem led us to consider heuristic approaches.

Production/inventory problems for perishable products have been studied. However, very little has been done in the area of integrated production and distribution planning for perishable products. Nahmias [2] presents a review of ordering policies for perishable inventories. Hsu [3] studied the economic lot-sizing problem with perishable products. In this model, the deterioration rate of the inventory and its carrying cost in each period depend on the age of the stock. Myers [4] presents a linear programming model to determine the maximum satisfiable demand for products with limited shelf life.

Previous work of the authors (Ekşioğlu et al. [9]) has been focused on variants of this production and distribution planning problem. In addition to previous work, this paper considers that the final product is perishable and has limited lifetime; and relaxes the assumption of constant initial inventory that is found in most inventory management models.

2 Problem Description and Formulations

This section presents a mixed integer linear programming (MILP) formulation of the cross-facility production and transportation planning problem with perishable inventory. Let F denote the number of facilities that produce and store the final products that are then delivered to R customers. The facilities have identical production capabilities. In other words, the final product can be produced in each facility. However, the setup and transportation costs, as well as the unit production and inventory holding costs, differ from one facility to the other, from one time period to the next. Given the projected demand of each retailer during the T-period planning horizon, the production and transportation planning problem decides how much to produce, transport and hold in inventory at each facility in order to meet demand at minimum cost. We assume that the planning horizon of length T is a typical one and repeats itself over time. All problem data are assumed cyclic with cycle length equal to T ($b_{j,T+1} = b_{j1}, b_{j,T+2} = b_{j2}$, $\ldots$, where b_{jt} is the demand at retailer j in period t). As a result, the inventory pattern at the facilities will be cyclic as well. We model this by letting the initial inventory be equal to the last period inventories.

2.1 Original Problem Formulation

Property 2 of an optimal solution to our problem (Section 2.3) implies that demand in a particular time period is satisfied from exactly one facility. We develop a network flow model based on the single source assignment property. Let

p_{it} denote the unit production cost at facility i in period t; s_{it} is the production setup cost at facility i in period t; h_{it} is the unit inventory cost at facility i in period t; and c_{ijt} is the total transportation cost of shipping b_{jt} from facility i to retailer j in period t. The decision variables are: q_{it} is the amount produced at facility i in period t; I_{it} is the inventory at facility i in the end of period t; x_{ijt} is a binary variable that equals 1 if there is a shipment from facility i to retailer j in period t, and equals 0 otherwise; and y_{it} is a binary variable that equals 1 if production takes place at facility i in period t, and equals 0 otherwise. The following is a MILP formulation of the problem:

$$\text{minimize} \sum_{i=1}^{F}\sum_{j=1}^{R}\sum_{t=1}^{T}\{p_{it}q_{it} + s_{it}y_{it} + h_{it}I_{it} + c_{ijt}x_{ijt}\}$$

subject to (P)

$$q_{it} + I_{i,[T+(t-1)]} - \sum_{j=1}^{R} b_{jt}x_{ijt} - I_{it} = 0 \qquad i = 1,\dots,F; t = 1,\dots,T \quad (1)$$

$$\sum_{i=1}^{F} x_{ijt} = 1 \qquad j = 1,\dots,R; t = 1,\dots,T \quad (2)$$

$$q_{it} - \sum_{\tau=t}^{t+k}\sum_{j=1}^{R} b_{j[\tau]}y_{it} \le 0 \qquad i = 1,\dots,F; t = 1,\dots,T \quad (3)$$

$$I_{it} - \sum_{\tau=t+1}^{t+k}\sum_{j=1}^{R} b_{j[\tau]}x_{ij[\tau]} \le 0 \qquad i = 1,\dots,F; t = 1,\dots,T \quad (4)$$

$$I_{i0} = I_{iT} \qquad i = 1,\dots,F \quad (5)$$

$$q_{it}, I_{it} \ge 0 \qquad i = 1,\dots,F; t = 1,\dots,T \quad (6)$$

$$y_{it}, x_{ijt} \in \{0,1\} \qquad i = 1,\dots,F; j = 1,\dots,R; t = 1,\dots,T. \quad (7)$$

For our convenience, in this formulation we have used the notation $[t]=(t+1)\text{mod } T-1$ i.e., $I_{i[t-1]} = I_{i,t-1}$ for $t = 2,\dots,T$ and $I_{i[0]=I_{iT}}$.

Constraints (1) and (2) are the flow conservation constraints at the production and demand points respectively. Constraints (3) are the setup constraints. Constraints (4) are the perishability constraints, where k ($k \le T-1$) denotes the maximum number of periods that a product can be stored. Constraints (5) model the fact that the initial inventory is equal to the ending inventory and T is a typical sequence of periods that will repeat itself. Setting the initial inventory level equal to the ending inventory means that these inventory levels are not fixed, and the model will determine the ending inventory levels that will prepare the system for future demands. Constraints (6) are the non-negativity constraints, and (7) are the boolean constraints. Standard solvers such as CPLEX can be used to solve small instances of (P). Large problem instances are solved using the primal-dual algorithm.

The transportation cost function $f_{ij}(g_{ijt})$ is considered to be a concave function with respect to the amount shipped, g_{ijt}. Based on the single-source assignment property of an optimal solution to our problem, facility i in period t either will not ship to retailer j or will ship the total demand, b_{jt}. This indicates that the transportation cost function consists of only two points $g_{ijt} = 0$ and $g_{ijt} = b_{jt}$. The LP-relaxation of the transportation cost function passes through the points: $g_{ijt} = 0$ and $g_{ijt} = b_{jt}$. Solving the LP-relaxation of (P) with respect to the transportation cost function gives a solution such that $g_{ijt} = 0$ or $g_{ijt} = b_{jt}$. That means the LP-relaxation gives an exact approximation of the concave transportation cost function. Therefore, $c_{ijt} = f_{ij}(b_{it})$.

In the special case when $F = 1$, retailers' demands are satisfied from the same facility; therefore, there is no decision to be made about which facility will ship the final product. In this case problem (P) reduces to the classical economic lot-sizing problem (Wagner and Whitin [5]).

2.2 Extended Problem Formulation

Linear programming relaxation of formulation (P) that is obtained by replacing the boolean constraints (7) with the nonnegativity constraints is not tight. This is due to the constraints (3). $\sum_{\tau=t}^{t+k}\sum_{j=1}^{R} b_{j[\tau]}$ provides a high upper bound for q_{it}, since the production in a period rarely equals this amount. One way to tighten the formulation is to split the production variables q_{it} by destination into variables $q_{ijt[\tau]}$ ($\tau = t, \ldots, t+k$). The new decision variable $q_{ijt[\tau]}$ presents the amount produced at facility i in period t to satisfy the demand of retailer j in period τ. For these variables, a trivial and tight upper bound is the demand at retailer j in period τ, $b_{j\tau}$.

The following is an equivalent formulation of (P) given with respect to decision variable $q_{ijt[\tau]}$:

$$\text{minimize} \sum_{i=1}^{F}\sum_{t=1}^{T}[\sum_{j=1}^{R}\sum_{\tau=t}^{t+k}\overline{c}_{ijt[\tau]}q_{ijt[\tau]} + s_{it}y_{it}]$$

subject to (Ex-P)

$$\sum_{i=1}^{F}\sum_{t=\tau-k}^{\tau} q_{ij[T+t]\tau} = b_{j\tau} \qquad j = 1,\ldots,R; \tau = 1,\ldots T$$

$$q_{ijt[\tau]} - b_{j[\tau]}y_{it} \le 0 \qquad i = 1,\ldots,F; j = 1,\ldots R; t = 1,\ldots T; t \le \tau \le t+k$$

$$q_{ijt[\tau]} \ge 0 \qquad i = 1,\ldots,F; j = 1,\ldots,R; t = 1,\ldots T; t \le \tau \le t+k$$

$$y_{it} \in \{0,1\} \qquad i = 1,\ldots,F; t = 1,\ldots T,$$

where $\overline{c}_{ijt[\tau]} = p_{it} + c_{ij[\tau]}/b_{j[\tau]} + \sum_{s=t}^{\tau-1} h_{i[s]}$. In the special case when $k = 0$, no inventories are carried from one period to another. In this case the problem decomposes by period. The single-period problem is the facility location problem, which is still a difficult problem to solve.

2.3 Properties of Optimal Solution

Using the network flow interpretation, we establish the required properties of optimal solutions to (P) when the costs are nonnegative.

Theorem 1. *There exists an optimal solution to problem (P) such that the demand at retailer j in period t is satisfied from either production or the inventory of exactly one of the facilities.*

Proof: The uncapacitated, production and transportation planning problem minimizes a concave cost function over a bounded convex set; therefore, its optimal solution corresponds to a vertex of the feasible region (Zangwill [6]). Let (q^*, x^*, I^*) be an optimal solution. In an uncapacitated network flow problem, a vertex is represented by a tree solution. The tree representation of the optimal solution implies that demand in every time period will be satisfied by exactly one of the facilities (in other words, $x^*_{ijt} x^*_{ljt} = 0$, for $i \neq l$ and $t = 1, 2, \ldots, T$). Furthermore, for each facility in each time period, if the inventory level is positive, there will be no production, and vice versa: $q^*_{it} I^*_{i,[t-1]} = 0$, for $i = 1, \ldots, F$, $t = 1, \ldots, T$. □

Theorem 1 implies properties 1 and 2 of the optimal solutions to our problem.

Property 1. The uncapacitated, production and transportation planning problem has an optimal solution that is such that a facility in a time period t either produces or carries inventory from the previous period (or neither), but not both. This property of the optimal solutions is often referred to in the literature as the *Zero Inventory Property* (ZIP). ZIP applies to the classical single-item lot-sizing problem and some of its generalizations (Wagner and Whitin [5]).

Property 2. The optimal solution of the problem is such that the demand in a time period is satisfied from a single facility. This property is equivalent to the single-source assignment property for the uncapacitated facility location problem.

Property 3. Every facility in a given time period t either does not produce or produces the demand for a number of periods in the time interval $t, \ldots, [t+k]$ (the periods do not need to be successive). This property can be easily derived from Theorem 1 and the tree representation of an optimal solution.

3 Solution Procedures

3.1 Exact Solution Approach

Theorem 2. *There exists an algorithm for the uncapacitated, single commodity, integrated production and distribution planning problem with perishable commodities (P) that is polynomial in the number of facilities and exponential in the number of periods and retailers.*

Proof: The following steps describe the algorithm:

1. Consider all the possible assignments of demands b_{jt} ($j = 1, \ldots, R$ and $t = 1, \ldots, T$) to facility i (for $i = 1, \ldots, F$). There is a total of F^{RT} assignments.
2. Given an assignment of demands to facilities, for each facility i ($i = 1, \ldots, F$) we need to solve an uncapacitated, single-commodity lot-sizing problem (ELSP$_i$) that considers the final product to be perishable and is cyclic in nature.
   Without loss of optimality we can assume an optimal solution $\min_{t=1,\ldots,T}$ $I_t = 0$. We can solve problem (ELSP$_i$) for each $t = 1, \ldots, T$, fixing $I_t = 0$ and treating period t as the "last" planning period. The cheapest one among the corresponding solutions is then the optimal solution. One should note that problem (ELSP$_i$) with $I_T = 0$ is in fact the (ELS) problem. This problem can be solved in $O(TlogT)$ (Wagelmans *et al.* [7]), and problem (ELSP$_i$) is solved in $O(T^2 \log T)$.

Therefore, the running time of this algorithm is bound by $O(F^{TR+1}T^2 \log T)$.

3.2 Primal-Dual Heuristic

The dual problem of the LP-relaxation of (Ex-P) has a special structure that allows us to develop a primal-dual based algorithm. The following is the formulation of the dual problem:

$$\text{maximize} \sum_{t=1}^{T} \sum_{j=1}^{R} b_{jt} v_{jt}$$

subject to (D-P)

$$\begin{aligned} \textstyle\sum_{\tau=t}^{t+k} \sum_{j=1}^{R} b_{j[\tau]} w_{ijt[\tau]} &\le s_{it} && i = 1, \ldots, F; t = 1, \ldots, T \\ v_{j[\tau]} - w_{ijt[\tau]} &\le \overline{c}_{ijt[\tau]} && i = 1, \ldots, F; t = 1, \ldots, T; t \le \tau \le t+k \\ w_{ijt[\tau]} &\ge 0 && i = 1, \ldots, F; j = 1, \ldots, R; t = 1, \ldots, T; t \le \tau \le t+k. \end{aligned}$$

In an optimal solution to (D-P), both constraints $w_{ijt[\tau]} \ge 0$ and $w_{ijt[\tau]} \ge v_{j[\tau]} - \overline{c}_{ijt[\tau]}$ should be satisfied. Since $w_{ijt[\tau]}$ is not in the objective function, we can replace it with $w_{ijt[\tau]} = \max(0, v_{j[\tau]} - \overline{c}_{ijt[\tau]})$. This leads to the following condensed dual formulation:

$$\text{maximize} \sum_{t=1}^{T} \sum_{j=1}^{R} b_{jt} v_{jt}$$

subject to (D*-P)

$$\sum_{\tau=t}^{t+k} \sum_{j=1}^{R} b_{j[\tau]} \max(0, v_{j[\tau]} - \overline{c}_{ijt[\tau]}) \le s_{it} \qquad i = 1, \ldots, F; t = 1, \ldots, T.$$

The extended formulation of the multi-facility lot-sizing problem is a special case of the uncapacitated facility location problem. The primal-dual scheme we discuss, in principle, is similar to the primal-dual scheme proposed by Erlenkotter [8] for the facility location problem. However, the implementation of the algorithm is different. Wagelmans *et al.* [7] use a similar primal-dual scheme for the classical lot-sizing problem. They show that this algorithm solves the problem in $O(T \log T)$. The dual variables have the following property: $v_t \geq v_{t+1}$, for $t = 1, \ldots, T$. This property is used to show that the dual ascent algorithm gives the optimal solution to the economic lot-sizing problem. This property does not hold true for (D-P).

Description of the Algorithm. Suppose the linear programming relaxation of (Ex-P) has an optimal solution (q^*, y^*) that is integral. Let (v^*, w^*) denote an optimal dual solution. The complementary slackness conditions for the primal (Ex-P) and dual (D-P) problems are as follow:

$$\begin{array}{ll}
(C_1)\ y^*_{it}[s_{it} - \sum_{j=1}^{R}\sum_{\tau=t}^{t+k} b_{j[\tau]} w^*_{ijt[\tau]}] & = 0 \text{ for } i = 1, \ldots, F; t = 1, \ldots, T \\
(C_2)\ q^*_{ijt[\tau]}[\overline{c}_{ijt[\tau]} - v^*_{j[\tau]} + w^*_{ijt[\tau]}] & = 0 \text{ for } i = 1, \ldots, F; j = 1, \ldots, R; \\
 & \quad t = 1, \ldots, T; t \leq \tau \leq t + k \\
(C_3)\ w^*_{ijt[\tau]}[q^*_{ijt[\tau]} - b_{j[\tau]} y^*_{it}] & = 0 \text{ for } i = 1, \ldots, F; j = 1, \ldots, R; \\
 & \quad t = 1, \ldots, T; t \leq \tau \leq t + k \\
(C_4)\ v^*_{jt}[b_{jt} - \sum_i \sum_{\tau=t-k}^{t} q^*_{ijt[T+\tau]}] & = 0 \text{ for } j = 1, \ldots, R; t = 1, \ldots, T.
\end{array}$$

The simple structure of the dual problem can be exploited to obtain near optimal feasible solutions by inspection. Suppose that the optimal values of the first $f-1$ dual variables of (D*-P) are known. Then, to be feasible, the f-th dual variable $(v_{l\tau})$ must satisfy the following constraints:

$$b_{l\tau} \max(0, v_{l\tau} - \overline{c}_{ilt\tau}) \leq M_{ilt,\tau-1} = s_{it} - \sum_{j=1}^{R}\sum_{s=t}^{\tau-1} b_{j[T+s]} \max(0, v^*_{j[T+s]} - \overline{c}_{ij[T+t][T+s]}) - \sum_{j=1}^{l-1} b_{j\tau} \max(0, v^*_{j\tau} - \overline{c}_{ij[T+t]\tau})$$

for all $i = 1, \ldots, F$ and $t = \tau - k, \ldots, \tau$. In order to maximize the dual problem, we should assign $v_{l\tau}$ the largest value satisfying these constraints. When $b_{l\tau} > 0$, this value is

$$v_{l\tau} = \min_{i=1,\ldots,F;\tau \geq t} \{\overline{c}_{ilt\tau} + \frac{M_{ilt,\tau-1}}{b_{l\tau}}\} \tag{8}$$

Note that if $M_{ilt\tau-1} \geq 0$ implies $v_{l\tau} \geq \overline{c}_{ilt\tau}$.

A dual feasible solution can be obtained simply by calculating the value of the dual variables sequentially (Figure 1). A backward construction algorithm can then be used to generate primal feasible solutions (Figure 2). The primal-dual solutions found using these algorithms may not necessarily satisfy the complementary slackness conditions.

Theorem 3. *The solutions obtained with the primal and dual algorithms are feasible and they always satisfy the complementary slackness conditions* (C_1) *and* (C_2).

Proof: The proof is similar to the proof of Proposition 4.1 in Ekşioğlu *et al.* [9].

Hence, one can determine whether the solution obtained with the primal and dual algorithms is optimal by checking if conditions (C_3) are satisfied or if the

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M_{iτ,τ−1} = s_{iτ} for i = 1,...,F; j = 1,...,R; τ = 1,...,T
for τ = 1 to T do
    for j = 1 to R do
        if b_{jτ} = 0 then v_{jτ} = 0
        else
            v_{jτ} = min_{it}{c̄_{ij[T+t]τ} + M_{i[T+t],τ−1}/b_{jτ}}, τ − k ≤ t ≤ τ
        for t = τ − k to τ do
            for i = 1 to F do
                M_{i[T+t]τ} = max{0, M_{i[T+t],τ−1} − b_{jτ} * max{0, v_{jτ} − c̄_{ij[T+t]τ}}}
            enddo
        enddo
    enddo
enddo
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Fig. 1. Dual algorithm

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 y_{it} = 0, q_{ijtτ} = 0, i = 1,...,F; j=1,...,R; t = 1,...,T; τ ≤ [t + k]
 P = {(j, l)|b_{jl} > 0, for j = 1,...,R; l = 1,...,T}
Start  :  τ = max l ∈ P, t = τ − k
Step 1 : for i = 1 to F do
             for j = 1 to R do
                 repeat t = t + 1
                   until M_{i[T+t]τ} = 0 and c̄_{ij[T+t]τ} − v_{jτ} + max{0, v_{jτ} − c̄_{ij[T+t]τ}} = 0
                   y_{i[T+t]} = 1, and i* = i, t* = t, go to Step 2
              enddo
          enddo
          go to Step 3
Step 2 : for t = t* to t* + k do
             for j = 0 to R do
                 if c̄_{i*jt*[t]} − v_{j[t]} + max{0, v_{j[t]} − c̄_{i*jt*[t]}} = 0
                     then q_{i*jt*[t]} = b_{j[t]}, P = P − (j, [t])
             enddo
          enddo
Step 3 : if P ≠ ∅ then go to Start
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Fig. 2. Primal algorithm

objective function values from the primal and dual algorithms are equal. The running time of this primal-dual algorithm is $O(FRT^2)$.

4 Computational Results

In this section we describe our computational experience in solving the integrated production and transportation planning problem with perishable inventory. Our

Table 1. Summary of results of primal-dual algorithm

Problem	Setup Costs									
	200-300		200-900		600-900		900-1,500		1,200-1,500	
	Error (%)	Cpu (sec)	Error (%)	Cpu (sec)	Error (%)	Cpu (sec)	Error (%)	Cpu (sec)	Error (%)	Cpu (sec)
16	0.14	0.15	0.24	0.15	0.77	0.15	1.37	0.15	1.91	0.15
17	0.19	0.15	0.30	0.20	1.08	0.20	1.91	0.20	2.64	0.20
18	0.26	0.20	0.37	0.25	1.30	0.20	2.28	0.25	3.21	0.25
19	0.14	0.70	0.23	0.55	0.77	0.65	1.40	0.65	1.96	0.06
20	0.19	1.00	0.29	0.85	1.07	0.95	1.85	0.95	2.59	0.95
21	0.24	1.25	0.34	1.25	1.30	1.25	2.26	1.25	3.17	1.25
22	0.14	2.95	0.23	2.90	0.78	2.85	1.40	3.10	1.94	2.95
23	0.19	4.45	0.30	4.45	1.08	4.45	1.89	5.00	2.63	4.95
24	0.24	6.00	0.35	5.85	1.29	6.05	2.23	6.10	3.12	5.80

goal is to provide some indication of both the quality and the computing time of the lower and upper bounds generated using the primal-dual algorithm. The problem instances we use are the same as the ones presented in Ekşioğlu *et al.* [9]. For all problems we set $k = \frac{T}{2}$. The errors presented are calculated as follows:

$$Error(\%) = \frac{\text{Primal Sol.} - \text{Dual Sol.}}{\text{Dual Sol.}} * 100.$$

It has been shown in the literature (Hochbaum and Segev [10], Ekşioğlu *et al.* [9]) that the ratio of setup to variable costs impacts the complexity of the fixed charge network flow problems. The results in Table 1 indicate that an increase in setup costs impacts the quality of the solutions from primal-dual algorithm. However, setup costs do not affect the running time of the algorithm. The results also show that an increase in the number of facilities and time periods impacts the performance of the primal-dual algorithm.

For the same set of problems, formulation (Ex-P) is solved using CPLEX 9 callable libraries. CPLEX gives the optimal solution for problems 16 to 18, but fails to solve the rest of the problems because of the problem size. Table 2 presents the running time of CPLEX for problems 16 to 18.

Table 2. CPLEX running times (in cpu seconds)

Problem	Setup Costs				
	200-300	200-900	600-900	900-1,500	1,200-1,500
16	15.85	15.95	16.05	16.55	16.50
17	26.55	27.05	28.35	28.75	44.40
18	40.25	41.05	42.20	42.10	86.50

5 Conclusions

This paper presents two network flow formulations for an integrated production and transportation planning problem with perishable inventories. The network consists of a number of facilities and retailers. The facilities produce and carry inventory to satisfy retailers' demands during T time periods. Retailers' demands are known deterministically. Unlike the traditional inventory models, the starting and ending inventories are not constant. Section 3 presents an exact solution procedure and a primal-dual algorithm to solve the problem. The exact algorithm is polynomial in the number of facilities and exponential in the number of retailers and time periods. This algorithm runs in $O(F^{TR+1}T^2 \log T)$ times. The primal-dual algorithm is used to generate lower and upper bounds. Its running time is $O(FRT^2)$ times. We tested the performance of the algorithms on a wide range of randomly generated problems. Computational results show high-quality solutions from the primal-dual algorithm. The maximum error gap was 3.12 percent and the maximum running time was 6.10 cpu seconds. Computational results demonstrate that the ratio of fixed to variable costs, the length of time horizon and the number of facilities impacted the running time and the quality of the solutions from the primal-dual algorithm and CPLEX. We identified a number of problems that CPLEX could not solve because it ran out of memory. However, for all problem classes, primal-dual algorithm gave high-quality solutions in a reasonable amount of time.

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