

# Using Constraint Satisfaction Approach to Solve the Capacity Allocation Problem for Photolithography Area

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**Abstract.** This paper addresses the capacity allocation problem for photolithography area (CAPPA) under an advanced technology environment. The CAPPA problem has two characteristics: process window and machine dedication. Process window means that a wafer needs to be processed on machines that can satisfy its process capability (process specification). Machine dedication means that after the first critical layer of a wafer lot is being processed on a certain machine, subsequent critical layers of this lot must be processed on the same machine to ensure good quality of final products. A production plan, constructed without considering the above two characteristics, is difficult to execute and to achieve its production targets. Thus, we model the CAPPA problem as a constraint satisfaction problem (CSP), which uses an efficient search algorithm to obtain a feasible solution. Additionally, we propose an upper bound of load unbalance estimation to reduce the search space of CSP for searching an optimal solution. Experimental results show that the proposed model is useful in solving the CAPPA problem in an efficient way.

## 1 Introduction

Due to its diverse characteristics, such as reentry process, time-constrained operation and different batch sizes for machines, wafer fabrication has received a lot of research attention, especially in photolithography process [13, 14, 7, 10]. The photolithography process uses masks to transfer circuit patterns onto a wafer, and the etching process forms tangible circuit patterns onto the wafer chip. With the required number of processes in the photolithography, integrated circuitry products with preset functions are developed on the wafer.

As wafer fabrication technology advances from micrometer level to nanometer level, more stringent machine selection restrictions, the so-called process window control and machine dedication control, are imposed on the production management of photolithography area for wafer lots.

Process window constraint, also called equipment constraint, is related to the strict limitation to the choice of a machine to process higher-end fabrication technology in the process of a wafer lot to meet increasingly narrower line width, distance between lines, and tolerance limit. In other words, wafer lots could only be processed on

machines that meet certain process capability (process recipe or process specification). On the contrary, wafers that only need a lower-end fabrication technology have less stringent machine selection restriction. Due to the difference in adjustable ability among photolithography machines regarding process recipes, functions of various machines in fact vary to a certain extent even though they are grouped in the same workstation. Hence, the situation is that some machines can handle more process capabilities (simultaneously handle higher- and lower-end fabrication technology) while other machines can handle less process capabilities (only handle lower-end fabrication technology). Some related studies are as follows. Leachman and Carmon [9] and Hung and Cheng [5] use linear programming to obtain a production plan for maximizing the profit. Toktay and Uzsoy [12] transform the capacity allocation problem with machines' capabilities constraint into maximum flow problem. However, only a single product type is considered in the study.

Machine dedication constraint considers layer-by-layer process on wafers, in which the circuit patterns in the layers can be correctly connected in order to provide particular functions. If electrical circuits among the layers cannot be aligned and connected, this will cause defective products. The alignment precision provided by different machines varies to a certain extent, even for machines of the same model, due to some differences, which are referred to as machine difference. It has been stipulated that when the first critical operation of a wafer lot is done on a particular machine, the rest of its subsequent critical processes will need to be processed by the very same machine to avoid the increase in defective rate due to machine difference. A related study was done by Akçali *et al.* [1], in which a study was conducted on the correlation between photolithography process characteristics and production cycle time using a simulation model, and machine dedication policy was set as one of the experiment factors. Experimental results indicate that dedicated assignment policy has a remarkable impact on cycle time.

With advanced fabrication technology, the impact of process window and machine dedication constraints on wafer fabrication is increasingly evident. Capacity requirement planning is difficult because wafer fabrication has special characteristics of reentry and long cycle time, and the number of layers of products, required process window, number and distribution of critical layers are different. As a result, the effectiveness of production planning and scheduling system is seriously impacted if the constraints of process window and machine dedication are not considered.

Up to now, the CAPPA problem has not been tackled except by Chung *et al.* [3], in which a mixed integer-linear programming (MILP) model is devised. However, a practical scale-sized problem may take an exponential time. In this research, we adopt the efficient constraint satisfaction approach, which treats load unbalance among machines as one of the constraints and obtain the optimal solution by constantly narrowing down the upper bound of the load unbalance. Because a relatively large amount of settings and long solving process may still be required, a load unbalance estimation to reduce the search space is also applied.

In section 2, the MILP model, the constraint satisfaction problem, and the load unbalance estimation are introduced. Section 3 demonstrates the effectiveness of proposed model. Section 4 uses a real-world case to show the applicability of proposed model. In the last section, the research results are summarized.

## 2 Model Development

Indices:

- $i$  Index of order number, where  $i = 1, \dots, I$ .
- $j$  Index of layer number, where  $j = 1, \dots, J_i$ .
- $k$  Index of machine number in photolithography area, where  $k = 1, \dots, K$ .
- $l$  Index of process capability number, where  $l = 1, \dots, L$ .
- $t$  Index of planning period, where  $t = 1, \dots, T$ .
- $r$  Index of ranking, where  $r = 1, \dots, R$ .

Parameters:

- $A_{kl}$  = 1 if machine  $k$  has process capability  $l$ ; 0, otherwise.
- $AC_{kt}$  Available capacity of machine  $k$  in planning period  $t$ .
- $AL_t$  Average capacity loading of workstation in planning period  $t$ .
- $CL_{ij}$  = 1 if layer  $j$  of order  $i$  is a critical layer; 0, otherwise.
- $CR_{ijl}$  = 1 if layer  $j$  of order  $i$  has a load on process capability  $l$ ; 0, otherwise.
- $CS_{rt}$  Cumulative available capacity from the first to the  $r$ -th rank process capability.
- $DC_{lt}$  Capacity requirement of process capability  $l$  in planning period  $t$ .
- $DS_{lt}$  Ratio of capacity requirement to available capacity of process capability  $l$  in planning period  $t$ .
- $J_i$  Number of photolithography operations for order  $i$ .
- $LT_{ijt}$  = 1 if layer  $j$  of order  $i$  has a load in planning period  $t$ ; 0, otherwise.
- $ML_t$  The maximum loading level among machines in planning period  $t$ .
- $p_{ij}$  Processing time of layer  $j$  of order  $i$ .
- $SQ(r)$  Function of the processing capacity of the  $r$ -th rank.
- $SC_{lt}$  Available capacity of process capability  $l$  in planning period  $t$ .

Decision Variables:

- $b_{ik}$  = 1 if the first critical layer of order  $i$  is assigned to machine  $k$ ; 0, otherwise.
- $u_{kt}^+$  Positive difference between utilization rate of machine  $k$  and average utilization rate of the entire workstation that machine  $k$  belongs to in planning period  $t$ .
- $u_{kt}^-$  Negative difference between utilization rate of machine  $k$  and average utilization rate of the entire workstation that machine  $k$  belongs to in planning period  $t$ .
- $x_{ijk}$  = 1 if layer  $j$  of order  $i$  is assigned to machine  $k$ ; 0, otherwise.

### 2.1 Mixed Integer-Linear Programming Model

To solve the CAPP problem, we need to know the load occurrence time of photolithography workstation for each layer of each order in the production system. An interview with several semiconductor fabricators found out that X-factor, the ratio of remaining time before delivery to processing time of an order, is used as a reference for controlling the production progress to make sure that the delivery of orders can be accomplished on time (see also [8]). With the information of X-factor, processing time

of an order, production plan and WIP level, the loading occurrence time of each order ( $LT_{ijt}$ ) can be estimated. A MILP model is constructed as follows:

$$\text{Minimize } \sum_{t=1}^T \sum_{k=1}^K (u_{kt}^+ + u_{kt}^-) \tag{1}$$

Subject to

$$\sum_i \sum_k \sum_l \sum_j (x_{ijk} A_{kl} CR_{ijl} LT_{ijt}) = \sum_t \sum_l \sum_j (CR_{ijl} LT_{ijt}) \quad , \text{ for all } i \tag{2}$$

$$\sum_k x_{ijk} = 1 \quad , \text{ for all } i, j \tag{3}$$

$$\sum_t \sum_l \sum_j (x_{ijk} CL_{ij} CR_{ijl} LT_{ijt}) = b_{ik} \times \sum_t \sum_l \sum_j (CL_{ij} CR_{ijl} LT_{ijt}) \quad , \text{ for all } i, k \tag{4}$$

$$u_{kt}^+ - u_{kt}^- = \sum_i \sum_j \sum_l (x_{ijk} p_{ij} A_{kl} CR_{ijl} LT_{ijt}) / AC_{kt} - AL_t \quad , \text{ for all } t, k \tag{5}$$

$$u_{kt}^+ \leq 1 - AL_t \quad , \text{ for all } t, k \tag{6}$$

$$u_{kt}^- \leq AL_t \quad , \text{ for all } t, k \tag{7}$$

$$x_{ijk} \in \{0,1\} \quad , \text{ for all } i, j, k \tag{8}$$

$$b_{ik} \in \{0,1\} \quad , \text{ for all } i, k \tag{9}$$

$$u_{kt}^+ \geq 0 \quad , \text{ for all } t, k \tag{10}$$

$$u_{kt}^- \geq 0 \quad , \text{ for all } t, k \tag{11}$$

The objective function (1) is to balance the capacity utilization rates among machines. Constraint (2) ensures that each layer of an order, including new release orders and WIP orders, must be assigned to a machine k if it has a capacity request in this planning horizon. In the machine assignment, process window constraint must be considered. Constraint (3) is to make sure that each layer of an order can only be assigned to a particular single machine. Constraint (4) states the machine dedication control. If the first critical layer of order i is assigned to machine k for process,  $b_{ik}$  is set to one. Note that the orders in a planning horizon can either be orders planned to release or WIP orders that were released to shop floor in the previous planning horizon. Therefore,  $b_{ik}$  is a decision variable if the order is a planned-to-release order or a WIP order which its first critical layer has not been decided on a particular machine in previous planning horizon, and is a known parameter if the order is a WIP order which its first critical layer has been decided to process on machine k, but unfinished, in the previous planning horizon. Constraint (5) calculates the difference between the utilization rate of machine k and the average utilization rate of the entire workstation in each period of the planning horizon. The detail definition of  $AL_t$  is shown as equation (14) and (15) in section 2.3. Constraint (6) and (7) limit the upper value of  $u_{kt}^+$  and  $u_{kt}^-$ , respectively.

### 2.2 Constraint Satisfaction Problem (CSP)

Constraint satisfaction problem (CSP) searches for a feasible solution which satisfies all constraints under a finite domain of variables. CSP originated from artificial intelligence (AI) in computer science. Through consistency checking techniques, constraint propagation and intelligent search algorithms, CSP has a relatively high solving efficiency in a combinatorial optimization problem, and it has been widely

applied in many research fields, such as vehicle routing related problem, production scheduling, facility layout and resource allocation [2, 11, 4].

Although CSP algorithm primarily aims to derive a feasible solution, it can be adjusted to search for an optimal solution. A feasible solution is generated by CSP first, then the feasible solution is set as the upper bound of the objective function (for a minimization problem), and such a relationship is treated as a constraint to solve the new CSP. With the continuation of lowering the upper bound of the objective function, the optimal solution can finally be obtained as the solution of the previous CSP when the current CSP can no longer be solved [2]. In other words, by deleting the objective function and solving the constraints part, we could convert the problem into a CSP. If the objective function is added into the constraints by setting its upper bound, an optimal solution for the CAPPa problem can be obtained by constantly reducing the upper bound of the objective function ( $E$ ), as shown by equation (12).

$$\sum_t \sum_k (u_{kt}^+ + u_{kt}^-) \leq E \quad (12)$$

Chung *et al.* [3] stated that loading balance is a critical factor for maintaining stability of production cycle time. Thus, we believe that the balance of loading among machines is more suitable than the emphasis of the minimization of the sum of the differences among machine utilization rates in a workstation. As a result, constraint (13) replaces constraint (12) in the solving of the CAPPa by CSP. For a more convenient explanation, we refer a CSP to search for a feasible solution as I-CSP model, and a CSP to search for an optimal solution as O-CSP model.

$$u_{kt}^+ + u_{kt}^- \leq E_t, \quad \text{for each } k, \text{ each } t \quad (13)$$

When CSP is applied to generate an optimal solution for the CAPPa problem (i.e. O-CSP model), the number of iterations for upper bound of load unbalance is difficult to estimate. In consequence, the expectation of a fast-solving and efficient algorithm from CSP may not be attained. Hence, we present a heuristic method to estimate the value of  $E_t$  (upper bound of load unbalance, UBLU). With a good estimation of the upper bound of load unbalance, the search space of O-CSP can be reduced, and the efficiency and quality of solution from CSP to solve the CAPPa problem can be increased.

### 2.3 Upper Bound of Load Unbalance (UBLU) Estimation

Conventionally, the average load level ( $AL_t$ ) of a workstation in a planning period is obtained by dividing total load by the number of machines, and the maximum loading level ( $ML_t$ ) among the machines is assumed equal to the average loading level ( $AL_t$ ). This calculation is based on the assumption that all machines are identical, that is, machines have identical process capability.

Since the types and amount of process capabilities are not exactly the same in the CAPPa problem, the maximum loading level may not equal to the average loading level. Therefore, we propose a two-phase capacity demand-supply assessment, which includes an independent and a dependent assessment, to estimate the maximum loading level among the machines. The results are utilized as the basis for setting the UBLU in equation (13). The concept is described as follows:

Phase I. Independent capacity supply assessment

The independent assessment examines whether capacity requirement is less than capacity supply for each process capability  $l$ . Capacity supply is the sum of the maximum loading level ( $ML_l$ ) of each machine to handle this process capability, and the initial value of  $ML_l$  is set to be the average capacity loading ( $AL_l$ ) in a workstation. If capacity demand is less than supply, the independent assessment is passed, and we can go to the dependent assessment. Otherwise, the maximum loading level of all machines needs to be raised to satisfy the capacity requirement of process capability  $l$ .

Phase II. Dependent capacity supply assessment

Since Phase I evaluates the capacity demand-supply without considering the fact that machines may possess several process capabilities, this phase uses an iterative calculation to assess whether the overall capacity supply is sufficient based on the maximum loading level obtained from Phase I. First, the ratios ( $DS_{lt}$ ) of capacity requirement to capacity supply of each process capability are ranked from large to small, and the sequence is  $SQ(r)$ . Then, whether cumulative capacity requirement is less than cumulative capacity supply is examined according to the ranking of  $DS_{lt}$  (that is,  $SQ(r)$ ). If the answer is affirmative, the capacity supply is sufficient to meet the capacity requirement with the consideration of the types and amount of process capabilities of each machine. Otherwise, further adjustment of the maximum loading level is required to meet the cumulative capacity demand.

The maximum loading level among the machines obtained after the two-phase capacity supply assessment is set as a basis for setting the UBLU. Followings are the detail computation steps:

Capacity requirement of each process capability

Step 1: Calculate capacity requirement ( $DC_{lt}$ ) of process capability in each planning period within the planning horizon.

$$DC_{lt} = \sum_i \sum_j (p_{ij} CR_{ijt} LT_{ijt}) \quad , \text{ for each } l, \text{ each } t \tag{14}$$

Step 2: Calculate average capacity loading ( $AL_l$ ) of machines in photolithography workstation in each planning period.

$$AL_l = \sum_l DC_{lt} / K \quad , \text{ for each } t \tag{15}$$

Phase I. Independent capacity supply assessment

Step 1: Set  $t = 1, l = 1$ .

Step 2: Set  $ML_l = AL_l$ .

Step 3: Verify if independent capacity supply of process capability  $l$  is sufficient in planning period  $t$ . If yes, then go to step 5; else go to step 4.

$$DC_{lt} \leq SC_{lt} \tag{16}$$

where

$$SC_{lt} = ML_l \times \sum_k A_{kl}$$

Step 4: Adjust the maximum loading level ( $ML_t$ ) to satisfy capacity requirement of process capability  $l$ .

$$ML_t = ML_t + (DC_{lt} - SC_{lt}) / \sum_k A_{kl} \tag{17}$$

Step 5: Check if  $l = L$ . If yes, then go to step 6; else let  $l = l + 1$  and go to step 3.

Step 6: Check if  $t = T$ . If yes, then end of Phase I; else let  $t = t + 1$ ,  $l = 1$ , and go to step 2.

**Phase II. Dependent capacity supply assessment**

Step 1: Set  $t = 1$ .

Step 2: Calculate the ratio ( $DS_{lt}$ ) of each process capability.

$$DS_{lt} = DC_{lt} / (ML_t \times \sum_k A_{kl}) \text{ , for each } l \tag{18}$$

Step 3: Rank the values of all  $DS_{lt}$  in planning period  $t$  from large to small. Use  $r$  to represent the rank and  $SQ(r)$  to represent the  $r$ -th process capability.

Step 4: Set  $r = 1$ .

Step 5: Calculate whether dependent capacity supply is sufficient. If equation (19) is satisfied, then go to step 7; else go to step 6.

$$\sum_{r=1}^r DC_{SQ(r),t} \leq CS_{rt} \tag{19}$$

where

$$CS_{rt} = ML_t \times \sum_k \min \left\{ 1, \max \left\{ \sum_{r=1}^r A_{k,SQ(r)}, 0 \right\} \right\}$$

Step 6: Adjust the maximum loading level ( $ML_t$ ) to satisfy cumulative capacity requirement. Then go to step 7.

$$ML_t = ML_t + (\sum_{r=1}^r DC_{SQ(r),t} - CS_{rt}) / CS_{rt} \tag{20}$$

Step 7: Check if  $r = L$ . If yes, then go to step 8; else let  $r = r + 1$  and go to step 5.

Step 8: Check if  $t = T$ . If yes, then end of Phase II; else let  $t = t + 1$ , and go to step 2.

**Setting the upper bound of load unbalance (UBLU)**

After the two-phase assessment above, the upper bound of load unbalance can be set as  $(ML_t - AL_t) / AL_t$ . However, considering the processing time for each layer of each product is not identical and is not divisible. It is revised as follows:

$$E_t = \max \left\{ (ML_t - AL_t) / AL_t \text{ , } \min_{\forall i,j} \{ p_{ij} \} / AL_t \right\} \tag{21}$$

**3 Comparisons Among MILP, I-CSP, and O-CSP Models**

A simple example is presented here to compare the performance between MILP, I-CSP, and O-CSP models. Consider a production environment with three machines, M1, M2 and M3, and each machine possesses different process capabilities as shown in Table 1.

**Table 1.** Process capability of machines

Machine No.	Process capability			
	1	2	3	4
1	1*	1	0	0
2	0	1	1	0
3	0	1	1	1

\* 1 if the machine has this certain process capability; 0, otherwise.

**Table 2.** Processing time, loading occurrence time, process window constraint and critical operation of orders

Order No.	Layer Number						
	1	2	3	4	5	6	7
1	12,1,1,0*	15,1,3,1	19,2,2,0	12,2,3,1	14,3,2,0	-	-
2	11,1,1,0	16,1,3,1	18,2,2,0	11,2,3,1	9,3,2,0	15,3,3,1	17,3,1,0
3	13,1,2,0	15,1,3,0	10,2,4,1	13,2,2,0	20,3,4,1	12,3,3,0	-
4	12,1,2,0	14,2,4,1	14,2,2,0	13,2,4,1	12,3,3,0	15,3,2,0	-
5	13,1,2,0	13,1,3,0	19,1,4,1	12,2,3,0	13,2,4,1	12,3,2,1	16,3,2,0

\* processing time (hr), load occurrence time (week), required process capability, whether a critical operation (1: critical operation; 0: non-critical operation), respectively.

**Table 3.** Performance of different solving models

Model		[k, t]								
		[1, 1]	[2, 1]	[3, 1]	[1, 2]	[2, 2]	[3, 2]	[1, 3]	[2, 3]	[3, 3]
MILP 0.0357 <sup>1</sup> 0.28 <sup>2</sup>	(1) $u_{kr}^+$	0.0099	0.0000	0.0000	0.0019	0.0000	0.0019	0.0000	0.0039	0.0000
	(2) $u_{kr}^-$	0.0000	0.0019	0.0079	0.0000	0.0039	0.0000	0.0019	0.0000	0.0019
	(3) <sup>3</sup>	0.0099	0.0019	0.0079	0.0019	0.0039	0.0019	0.0019	0.0039	0.0019
	(4) <sup>4</sup>		0.0099			0.0039			0.0039	
	(5) <sup>5</sup>		1.6632			0.6552			0.6552	
I-CSP 2.7694 0.00	(1) $u_{kr}^+$	0.4623	0.0000	0.0000	0.3591	0.0000	0.0000	0.5634	0.0000	0.0000
	(2) $u_{kr}^-$	0.0000	0.1865	0.2757	0.0000	0.0634	0.2956	0.0000	0.2817	0.2817
	(3)	0.4623	0.1865	0.2757	0.3591	0.0634	0.2956	0.5634	0.2817	0.2817
	(4)		0.4623			0.3591			0.5634	
	(5)		77.6664			60.3288			94.6512	
O-CSP 0.0357 0.30	(1) $u_{kr}^+$	0.0099	0.0000	0.0000	0.0019	0.0000	0.0019	0.0000	0.0039	0.0000
	(2) $u_{kr}^-$	0.0000	0.0019	0.0079	0.0000	0.0039	0.0000	0.0019	0.0000	0.0019
	(3)	0.0099	0.0019	0.0079	0.0019	0.0039	0.0019	0.0019	0.0039	0.0019
	(4)		0.0099			0.0039			0.0039	
	(5)		1.6632			0.6552			0.6552	

<sup>1</sup> Objective function value. Notice the objective function value of I-CSP and O-CSP is the sum of ( $u_{kr}^+ + u_{kr}^-$ ).

<sup>2</sup> Computational time (sec). Notice the time of O-CSP is the sum of solving time of the 15 iterations in Table 4.

<sup>3,4,5</sup> (3)=(1)+(2). (4) is the maximum value of (3) under t. (5)=(4)×available capacity (168 hours/period).

There are five orders to be released, and the information of processing time (hrs), loading occurrence time (week), required process capability and critical layer process are shown in Table 2. The commercial software ILOG OPL 3.5 [6] is utilized to solve the simplified example by three different models: MILP model, I-CSP model and O-CSP model. The results are shown in Table 3.

Even though I-CSP model uses the least amount of solving time among the three models, its objective function value (2.7694) and workstation utilization rate difference (maximum difference of 0.5634) are the worst. As an extension of I-CSP model,



**Table 4.** Computation process of O-CSP in solving the simple example

# of iterations	UBLU	Objective function value	Solving time	Maximum difference <sup>1</sup>
1	-	2.7694	0.00	0.5634
2	0.5634	1.6782	0.02	0.3432
3	0.3432	1.3051	0.02	0.2599
⋮	⋮	⋮	⋮	⋮
6	0.2480	0.8924	0.03	0.1865
7	0.1865	0.6502	0.02	0.1448
8	0.1448	0.6146	0.02	0.0932
⋮	⋮	⋮	⋮	⋮
14	0.0277	0.1304	0.02	0.0218
15	0.0218	0.0357	0.05	0.0099
16	0.0099	no feasible solution		

<sup>1</sup>The maximum value of  $(u_{kt}^+ + u_{kt}^-)$ .

O-CSP can constantly adjust the UBLU by using equation (13) (see Table 4 for computation process) to eventually derive at an optimal solution (same as the objective function value of MILP model). One drawback of the model is that the required iterations of adjustments are not estimable. With the upper bound of load unbalance estimation introduced in section 2.3, we could set the upper bound of load unbalance to 0.1811. Comparing with the data in Table 4, 0.1811 is the upper bound for the 7th iteration in the O-CSP model. In consequence, the adoption of the setting of UBLU to the O-CSP model can effectively reduce the number of iterations.

### 4 A Real-World Application

To verify the applicability of the proposed model, a real-world case investigated in [3], is examined here. In this wafer fab, there are ten steppers and five different process capabilities. Five types of products, A, B, C, D and E, are manufactured, and each product requires 17, 19, 16, 20 and 19 times of photolithography operations respectively. The total required photolithography operation time for a product is in a range between 597 to 723 minutes. Product A and B require fabrication technology of 0.17 μm, while Product C, D and E adopt 0.14 μm fabrication technology. Production planning and control department sets the planning horizon to be 28 days and planning period to be 7 days. In the planning horizon, there are 474 lots that are expected to be released. Manufacturing execution system (MES) reveals that there are currently 204 lots of WIP on floor.

The CAPP problem is solved by O-CSP using software ILOG OPL 3.5 [6], and the results are shown in Table 5. Through the upper bound of load unbalance estimation, the CAPP problem could have a fairly balanced capacity allocation result (i.e.  $ML_t = AL_t$ ), and the UBLU is set to 0.0014 (=15/10800, where minimum processing time is 15 minutes and available capacity for machines in a planning period is 10,800 minutes.) Table 5 shows that the objective function value derived from the CAPP problem is 0.0205 and the required solving time is 475.22 sec. This result is superior than that generated by Chung *et al.* [3] that the objective function value and required solving time are 0.0291 and 5.3878 hours respectively.

**Table 5.** Objective function value, maximum difference and solving time under different UBLU

UBLU	Objective function value	Maximum difference <sup>1</sup>	Solving time (sec.)
—	9.4434	0.9994	125.94
⋮	⋮	⋮	⋮
0.0014	0.0205	0.0013	475.22
0.0013	0.0167	0.0010	576.53
0.0010	0.0141	0.0008	851.02
0.0008	0.0116	0.0007	903.43
0.0007	0.0107	0.0006	973.98
0.0006	0.0084	0.0005	3477.64
0.0005	no feasible solution		

<sup>1</sup> The maximum value of  $(u_{it}^+ + u_{it}^-)$ .

When the UBLU (0.0014) is used as a basis of O-CSP for solving the CAPP problem, the required number of iterations is only six times. This indicates that the setting of UBLU can effectively reduce the search space of O-CSP. Such a solving process possesses a very good quality and has its application value in real practice.

## 5 Conclusion

In this paper, we consider the capacity allocation problem with the process window and machine dedication constraints that are apparent in wafer fabrication. We model the CAPP problem as a constraint satisfaction problem (CSP), which uses an efficient search algorithm to obtain a feasible solution. A relatively large amount of setting and calculation process is required in CSP because it treats the objective function as one of the constraints for searching an optimal solution while the bound of objective function is narrowed down through an iterative process. Hence, we propose a method for setting the upper bound of load unbalance among machines, and the search space and the number of computations can be decreased effectively in the CAPP problem. The result shows that a very good solution can be obtained in a reasonable time and can be a reference for wafer lot release and dispatching of photolithography machines, and the model thus is valuable in real world application.

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