Coordinated Inventory Models with Compensation Policy in a Three Level Supply Chain

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Abstract. In this paper, we develop inventory models for the three level supply chain (one supplier, one warehouse, and one retailer) and consider the problem of determining the optimal integer multiple n of time interval, time interval between successive setups and orders in the coordinated inventory model. We consider three types of individual models (independent model, retailer's point of view model, and supplier's point of view model). The focus of this model is minimization of the coordinated total relevant cost, and then we apply the compensation policy for the benefits and losses to our coordinated inventory model. The optimal solution procedure for the developed model is derived and the effects of the compensation policy on the optimal results are studied with the help of numerical examples.

1 Introduction

While SCM is relatively new, the idea of coordinated model is not. The study of multi-echelon inventory/distribution systems began as early as 1960 by Clark and Scarf [5]. Since that time, many researchers have investigated multi-echelon inventory and distribution systems. Many researches have been aimed at coordinated model with two levels, while researchers who studied models with three levels are less. Ergüc *et al.* [7] point out that though a dominant firm in the supply chain usually tends to optimize locally with no regard to its impact on the other members of the chain, there are cases of such firms capable of fostering more cooperative agreements in the chain. An empirical study on buyer-supplier relationship highlighted the importance of strong linkages for efficient JIT operations [3]. They called for replacing the traditional adversarial roles between buyers and sellers with mutual cooperation. Kang *et al.* [13] have reviewed past and present supply chain models and then analyzed those in view of environment factors, operations, solution approaches. Goyal [10] presented an integrated inventory model for a single supplier-single customer problem. Banerjee [1] presented a joint economic-lot-size model where a vendor produces to order for a purchaser on a lot-for-lot basis under deterministic conditions. Goyal [11] further generalized Banerjee [1]'s model by relaxing the assumption of the

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M. Gavrilova et al. (Eds.): ICCSA 2006, LNCS 3982, pp. 600 – 609, 2006.

lot-for-lot policy of the vendor. As a result of using the approach suggested by Goyal [11], significant reduction in inventory cost can be achieved. Several researchers have shown that one partner's gain may exceed the other partners' loss in integrated models. Thus, the net benefit can be shared by both parties in some equitable fashion [12]. Eum *et al.* [8] proposed a new allocation policy considering buyers' demands using the neural network theory. Douglas and Paul [6] defined three categories of operational coordination (buyer-vendor coordination, production-distribution coordination and inventory-distribution coordination).

The value of information sharing among supply chain players has received much attention from researchers [4, 14]. They showed that using the information on the outstanding orders of the products resulted in improvement in system performance in a two-product model. Bourland *et al.* [2] demonstrated the value of obtaining demand information at the retailers. Gavirneni *et al.* [9] captured the value of information flow in a two-echelon capacitated model. Recently, Lee *et al.* [14] addressed the issue of quantifying the benefits of sharing information and identifying the drivers of the magnitude of these drivers.

Most of the works in the literature consider two level supply chain. But our work considers three level supply chain (one supplier, one warehouse and one retailer). While they provided the joint analysis of two level supply chain (the supplier and the buyer), our paper aims to study coordinated analysis among the supplier, the warehouse and the retailer, and strategies to encourage coordination among supply chain partners.

The rest of this paper is organized as follows. We introduce the problem in Section 2. In Section 3 we optimize for individual models. Section 4 establishes the coordinated model and develops the procedure for the minimization of the total cost in a three level supply chain. Section 5 develops compensation policies for benefits and losses using the developed models. We present numerical examples in Section 6 and conclude in Section 7.

2 Problem Definition

We consider a supply chain with three levels. Suppose that a retailer periodically orders some quantity (Q) of an inventory item from a warehouse, while a warehouse periodically orders integer multiple of the retailer's order quantity ($n \cdot Q$) of item from a supplier. Upon receipt of an order, the supplier produces the integer multiple quantity of the item. But the warehouse ships some quantity (Q) to the retailer during the multiple times (n). In addition to the deterministic conditions, we assume that there are no other warehouses and retailers for this item and the supplier is the sole manufacturer. Fig. 1 shows the inventory time plots for n=3. The retailer's time interval between successive orders is T_R, and the supplier and the warehouse's time interval between successive setups and orders are $T_S=3T_R$ and $T_W=3T_R$, respectively. At the end of time interval in the supplier, it delivers the completed lot to the warehouse. At the beginning of time interval in the warehouse, it directly delivers as the retailer's order quantity (Q) to the retailer (similar to cross-docking, a process in which product is exchanged between trucks so that each truck going to a retailer store has products from different suppliers). In the remaining time interval, it delivers two more times with the same quantity to the retailer.

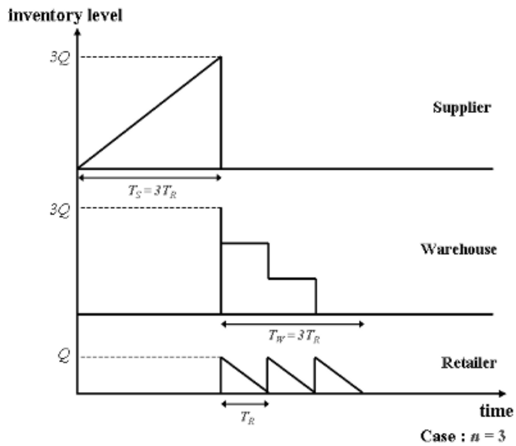

Fig. 1. Graphical representation of the model (n = 3)

The following assumptions are made to develop the models:

(1) The demand is deterministic and constant.
(2) Supplier, warehouse, retailer's lead-time are either zero or replenishment is instantaneous.
(3) The holding cost values are $h_R > h_W$. Because the warehouse takes charge of the storage and distribution professionally, the warehouse's holding cost is less than the retailer.
(4) The supplier's time interval between setups and the warehouse's time interval between orders are integer (n>1) multiple of the retailer's time interval between orders.
(5) Shortages and backlogs are not allowed.
(6) Supplier and retailer's inventory policies can be described by simple EOQ inventory model.

The following notations are used in developing the models:

D : annual demand for the item
S : supplier's setup cost per setup
A_W : warehouse's ordering cost per order
A_R : retailer's ordering cost per order
T_S : time interval between successive setups at supplier
T_W : time interval between successive orders at warehouse
T_R : time interval between successive orders at retailer
h_S : supplier's holding cost per unit per unit time
h_W : warehouse's holding cost per unit per unit time
h_R : retailer's holding cost per unit per unit time
n : positive integer number(n >1)

3 Individual Model

We consider three types of individual models. Firstly, we formulate an independent individual model. In the independent individual model, manufacturing and ordering policies are independent. Secondly, we develop an individual model from retailer's point of view. In this model, the retailer is a decision-maker. Therefore, the other parties follow the retailer's ordering policy. For example, department stores decide the ordering policies regardless of the other parties, because they have a power in the marketplace. Thirdly, we consider an individual model from supplier's point of view. This model is opposite to the retailer's point of view model. Because supplier is a decision-maker, the warehouse and retailer decide ordering policies according to the supplier's decision. For instance, the high-technology products are made by the supplier's decision regardless of the warehouse and retailer's order. Because the supplier and retailer's inventory policies can be described by simple EOQ, we can easily derive the optimal policies. The results of individual optimization are summarized in Table 1.

Table 1. Summary of total costs and individual optimal policies

	Supplier	Warehouse	Retailer
Independent	$TC_S(T_S) = \frac{S}{T_S} + \frac{DT_S}{2}h_S$ $T_S^* = \sqrt{\frac{2S}{Dh_S}}$ $TC_S(T_S^*) = \sqrt{2DSh_S}$	$TC_W(n, T_W) = \frac{A_W}{T_W} + \frac{(n-1)DT_W}{2}h_W$ $n^* = 1$, $T_W^* = T_S^*$ $TC_W(n^*, T_W^*) = \frac{A_W}{T_S^*}$	$TC_R(T_R) = \frac{A_R}{T_R} + \frac{DT_R}{2}h_R$ $T_R^* = \sqrt{\frac{2A_R}{Dh_R}}$ $TC_R(T_R^*) = \sqrt{2DA_R h_R}$
Retailer's point of view	$TC_S(T_S) = \frac{S}{T_S} + \frac{DT_S}{2}h_S$ $T_S^* = T_R^*$ $TC_S(T_S^*) = \frac{S}{T_R^*} + \frac{DT_R^*}{2}h$	$TC_W(n, T_W) = \frac{A_W}{T_W} + \frac{(n-1)DT_W}{2}h_W$ $n^* = 1$, $T_W^* = T_R^*$ $TC_W(n^*, T_W^*) = \frac{A_W}{T_R^*}$	$TC_R(T_R) = \frac{A_R}{T_R} + \frac{DT_R}{2}h_R$ $T_R^* = \sqrt{\frac{2A_R}{Dh_R}}$ $TC_R(T_R^*) = \sqrt{2DA_R h_R}$
Supplier's point of view	$TC_S(T_S) = \frac{S}{T_S} + \frac{DT_S}{2}h_S$ $T_S^* = \sqrt{\frac{2S}{Dh_S}}$ $TC_S(T_S^*) = \sqrt{2DSh_S}$	$TC_W(n, T_W) = \frac{A_W}{T_W} + \frac{(n-1)DT_W}{2}h_W$ $n^* = 1$, $T_W^* = T_S^*$ $TC_W(n^*, T_W^*) = \frac{A_W}{T_S^*}$	$TC_R(T_R) = \frac{A_R}{T_R} + \frac{DT_R}{2}h_R$ $T_R^* = T_S^*$ $TC_R(T_R^*) = \frac{A_R}{T_S^*} + \frac{DT_S^*}{2}h$

The stock in the warehouse is depleted according to the demand and supply. If the warehouse is replenished at a time interval of T_W and the quantity received can satisfy multiple orders, then the total cost per unit time is given by

$$TC_W(n, T_W) = \frac{A_W}{T_W} + \frac{(n-1)DT_W}{2}h_W \tag{3.1}$$

The objective is to find the optimal values of n and T_W which minimize $TC_W(n, T_W)$. Since n is a positive integer and T_W is a real number, we can optimize the total cost per unit time as given below:
For any given n (≥ 1), as the second order derivative of $TC_W(n, T_W)$ is always positive, the necessary condition for the minimum of $TC_W(n, T_W)$ is given by

$$\frac{\partial TC_W}{\partial T_W} = \frac{(n-1)\,D\,h_W}{2} - \frac{A_W}{T_W^2} = 0 \tag{3.2}$$

Solving equation (3.2) we get

$$T_W^* = \sqrt{\frac{2A_W}{(n-1)\,D\,h_W}} \tag{3.3}$$

Substituting T_W from equation (3.3) into equation (3.1), the total cost per unit time can be found for any given n. It is to be observed that there exists a unique optimal solution (n^*, T_W^*) as $TC_W(n, T_W)$ is convex for any given n.

$$TC_W(n) = \sqrt{2(n-1)DA_W h_W} \tag{3.4}$$

Minimizing $TC_W(n)$ is equivalent to minimizing

$$(TC_W(n))^2 = 2(n-1)DA_W h_W \tag{3.5}$$

We define $Y(n)$ which enables our problem to be equivalent to the minimization of $Y(n)$.

$$Y(n) = 2(n-1)DA_W h_W \tag{3.6}$$

However, $Y(n)$ is a linear increasing function which depends on n. Therefore, the optimal minimum value of n is always 1. It means that the supplier directly delivers order quantity to the retailer. The role of the warehouse is similar to the cross-docking (CD) system. Hence, the warehouse is spending only the ordering cost, and the optimal value of T_W is equal to T_S^* and T_R^*.

4 Coordinated Model

The relevant total cost of the coordinated model for the supplier, the warehouse and the retailer can be derived by adding the individual total costs per unit time from the previous section.

$$CTC(n, T_R) = \frac{A_R}{T_R} + \frac{A_W}{nT_R} + \frac{S}{nT_R} + \frac{DT_R h_R}{2} + \frac{(n-1)DT_R h_W}{2} + \frac{nDT_R h_S}{2} \tag{4.1}$$

where $T_W = n \cdot T_R$ and $T_S = n \cdot T_R$

The optimal values of n and T_R can be obtained using the following propositions.

Proposition 1: For any given n (≥ 1), the time interval between successive setups and reorders in the coordinated model can be determined uniquely.

Proof: Differentiating equation (4.1) with respect to T_R, we get

$$\frac{\partial CTC}{\partial T_R} = \frac{D(h_R + nh_W - h_W + nh_S)}{2} - \frac{nA_R + A_W + S}{nT_R^2} \tag{4.2}$$

Differentiating equation (4.2) again with respect to T_R, we get

$$\frac{\partial^2 CTC}{\partial T_R^2} = \frac{2(nA_R + A_W + S)}{nT_R^3} > 0, \ \forall n \geq 1 \tag{4.3}$$

Hence $CTC(n, T_R)$ is convex in T_R when n is given. Therefore, there exists a unique solution of the equation $\partial CTC(n, T_R) / \partial T_R = 0$ which yields

$$T_R^* = \sqrt{\frac{2(nA_R + A_W + S)}{nD\{h_R + (n-1)h_W + nh_S)}} \tag{4.4}$$

Substituting T_R^* into equation (4.1), we obtain the minimum total cost of the coordinated model as follows:

$$CTC(n) = \sqrt{\frac{2D\{h_R - h_W + n(h_W + h_S)\}(nA_R + A_W + S)}{n}} \tag{4.5}$$

We can find the optimal value of n using the following proposition.

Proposition 2: The optimal value of n satisfies the following inequality.

$$n^*(n^* - 1) \leq \frac{(h_R - h_W)(S + A_W)}{A_R(h_S + h_W)} \leq n^*(n^* + 1)$$

Proof: Minimizing $CTC(n)$ is equivalent to minimizing

$$(CTC(n))^2 = 2D\{(h_R - h_W)(A_R + \frac{S + A_W}{n}) + (h_S + h_W)(nA_R + A_W + S)\} \tag{4.6}$$

After ignoring the terms on the right hand side of equation (4.6) which are independent of n, we define $Z(n)$ as follows:

$$Z(n) = \frac{(h_R - h_W)(S + A_W)}{n} + nA_R(h_S + h_W) \tag{4.7}$$

The optimal value of $n = n^*$ is obtained when

$$Z(n^*) \leq Z(n^* - 1) \ \ and \ \ Z(n^*) \leq Z(n^* + 1) \tag{4.8}$$

We get the following inequalities from (4.8)

$$\begin{aligned} &\frac{(h_R - h_W)(S + A_W)}{n^*} + n^* A_R(h_S + h_W) \leq \frac{(h_R - h_W)(S + A_W)}{n^* - 1} + (n^* - 1)A_R(h_S + h_W) \ \ and \\ &\frac{(h_R - h_W)(S + A_W)}{n^*} + n^* A_R(h_S + h_W) \leq \frac{(h_R - h_W)(S + A_W)}{n^* + 1} + (n^* + 1)A_R(h_S + h_W) \end{aligned} \tag{4.9}$$

Accordingly, it follows that

$$A_R(h_S + h_W) \le \frac{1}{n^*(n^*-1)}(h_R - h_W)(S + A_W) \;\; and$$
$$A_R(h_S + h_W) \ge \frac{1}{n^*(n^*+1)}(h_R - h_W)(S + A_W) \tag{4.10}$$

The following condition is obtained from equation (4.10):

$$n^*(n^*-1) \le \frac{(h_R - h_W)(S + A_W)}{A_R(h_S + h_W)} \le n^*(n^*+1) \tag{4.11}$$

< *Procedure for finding* n^* and T_R^* >

Step 1: Determine

$$\frac{(h_R - h_W)(S + A_W)}{A_R(h_S + h_W)}$$

If n is greater than 1 in inequality (4.11), set $n^*=n$. Otherwise, set $n^*=1$. Go to Step 2.

Step 2: Determine the optimal value of T_R using equation (4.4).

5 Compensation Policy

Several researchers have shown that one partner's gain may exceed the other partner's loss in the integrated model [9, 14]. Thus, the net benefit should be shared among parties (the supplier, the warehouse and the retailer) in some equitable fashion. We propose a compensation policy that shares benefits and losses according to the ratio of individual models' total cost per unit time. This method extends Goyal [10]'s method to the three level supply chain.

Applying Goyal's method to our coordinated model, we get

$$Z_S = \frac{TC_S(T_S^*)}{TC_S(T_S^*) + TC_W(n^*, T_W^*) + TC_R(T_R^*)}$$
$$Cost\ of\ supplier = Z_S \cdot CTC(n^*, T_R^*) \tag{5.1}$$

$$Z_W = \frac{TC_W(n^*, T_W^*)}{TC_S(T_S^*) + TC_W(n^*, T_W^*) + TC_R(T_R^*)}$$
$$Cost\ of\ warehouse = Z_W \cdot CTC(n^*, T_R^*) \tag{5.2}$$

$$Z_R = \frac{TC_R(T_R^*)}{TC_S(T_S^*) + TC_W(n^*, T_W^*) + TC_R(T_R^*)}$$
$$Cost\ of\ retailer = Z_R \cdot CTC(n^*, T_R^*) \tag{5.3}$$

Note that Z_S+Z_W+Z_R=1

6 Numerical Examples

For numerical examples, we use the following data:

D = 10,000 *unit/year, S* = \$400*/setup*, A_W = \$200*/order*, A_R = \$50*/order*
h_S = \$3*/unit/year*, h_W = \$3*/unit/year*, h_R = \$3*/unit/year*

The optimal values of n, T_R and total cost for the individual models and the coordinated model are summarized in Table 2.

Table 2. Summary of results

	Individual models			Coordinated model
	Independent	Retailer's point of view	Supplier's point of view	
Supplier's setup interval	0.1633 year	0.0447 year	0.1633 year	0.1581
Warehouse's order interval	0.1633 (n=1)	0.0447 (n=1)	0.1633 (n=1)	0.1581 (n=3)
Retailer's order interval	0.0447	0.0447	0.1633	0.0527
Supplier's annual cost	\$4,898.98	\$9,615.34	\$4,898.98	\$4,901.53
Warehouse's annual cost	\$1,224.75	\$4,472.17	\$1,224.75	\$2,319.00
Retailer's annual cost	\$2,236.07	\$2,236.07	\$4,388.68	\$2,266.30
Total cost	\$8,359.80	\$16,323.58	\$10,512.41	\$9,486.83

Fig. 2 shows that the total cost function $CTC(n, T_R)$ is a convex function in n and T_R and a typical configuration of the surface.

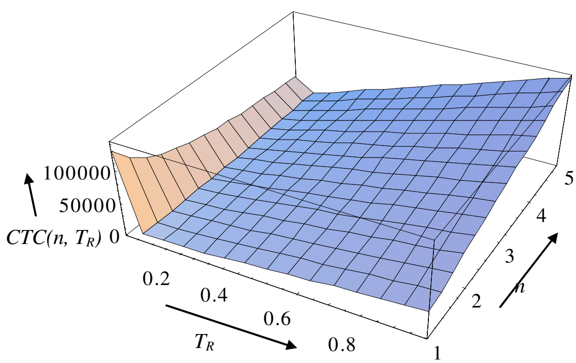

Fig. 2. Graphical Representation of $CTC(n, T_R)$

If we coordinate the three level supply chain, we can reduce \$6,836.75(approximately 42%) of the total cost against retailer's point of view model. Therefore, we need to share the benefits. Applying the compensation policy using equations (5.1), (5.2), and (5.3), we get the following results:

$$Z_S = 0.5890,\ Cost\ of\ \ supplier = \$5{,}587.74$$
$$Z_W = 0.2740,\ Cost\ of\ \ warehouse = \$2{,}599.39$$
$$Z_R = 0.1370,\ Cost\ of\ \ retailer = \$1{,}299.70$$

Fig. 3 summarizes the process of applying the compensation policy and information sharing.

Retailer's point of view models (Total cost=\$16,323.58)		
Supplier	Warehouse	Retailer
Setup interval=0.0447 year	Order interval=0.0447 year	Order interval=0.0447 year
Total cost=\$9,615.34	Total cost=\$4,472.17	Total cost=\$2,236.07

Information sharing **Compensation policy**

Coordinated models (Total cost=\$9,486.83)		
Supplier	Warehouse	Retailer
Setup interval=0.1581 year	Order interval=0.1581 year	Order interval=0.0527 year
Total cost=\$5,587.74	Total cost=\$2,599.39	Total cost=\$1,299.70

Fig. 3. Graphical illustration of the solutions

7 Conclusions

We developed an inventory model for a three level supply chain (one supplier, one warehouse, and one retailer). We proposed a procedure for determining the optimal value of n and T_R for the coordinated model. The compensation policy gives better results than individual models in terms of the total cost per unit time. The total cost per unit time obtained by the coordinated model with compensation policy has been reduced significantly compared to the individual models. We may develop other types of compensation policy (i.e. price quantity discounts policy). In addition, our model can be extended to the case with multiple suppliers, one warehouse, and multiple retailers. . Finally, it must be an interesting extension if one could develop the model by relaxing the assumptions of deterministic demand and lead time.

Acknowledgements

This work was supported by the Regional Research Centers Program(Research Center for Logistics Information Technology), granted by the Korean Ministry of Education & Human Resources Development.

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