Dependent OWA Operators

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Abstract. Yager [1] introduced several families of ordered weighted averaging (OWA) operators, in which the associated weights depend on the aggregated arguments. In this paper, we develop a new dependent OWA operator, and study some of its desirable properties. The prominent characteristic of this dependent OWA operator is that it can relieve the influence of unfair arguments on the aggregated results. Finally, we give an example to illustrate the developed operator.

1 Introduction

The ordered weighted aggregation (OWA) operator as an aggregation technique has received more and more attention since it's appearance [2]. One important step of the OWA operator is to determine its associated weights. Many authors have focused on this issue, and developed some useful approaches to obtaining the OWA weights. For example, Yager [1] introduced some families of the OWA weights, including the ideal of aggregate dependent weights. Yager [2] introduced an approach to computing the weights of the OWA operator based on Zadeh's [3,4] concept of linguistic quantifiers. O'Hagan [5] established a mathematical programming model maximizing the entropy of the OWA weights for a predefined degree of orness. Xu and Da [6] extended O'Hagan's model to the situations where the weight information is available partially. Filev and Yager [7] developed two procedures to obtain the OWA weights, the first one learns the weights from a collection of samples with their aggregated value, and the second one calculates the weights for a given level of orness. Xu and Da [8] established a linear objective-programming model for obtaining the weights of the OWA operator by utilizing the given arguments under partial weight information. Xu [9] developed a normal distribution based method. We classify all these approaches into the following two categories: argument-independent approaches [1,2,5-7,9-12] and argument-dependent approaches [1,7,8,12-14]. The weights derived by the argument-independent approaches are associated with particular ordered positions of the aggregated arguments, and have no connection with the aggregated arguments, while the argument-dependent approaches determine the weights based on the input arguments. In this paper, we will pay attention on the second category, and develop a new argument-dependent approach to determining the OWA weights.

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2 Dependent OWA Operators

In [2], Yager defined the concept of an ordered weighted averaging (OWA) operator as follows:

An OWA operator of dimension n is a mapping, $OWA: R^n \to R$, that has an associated n vector $w = (w_1, w_2, ..., w_n)^T$ such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Furthermore,

$$OWA(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j a_{\sigma(j)} \tag{1}$$

where $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of $(1,2,..., n)$ such that $a_{\sigma(j-1)} \geq a_{\sigma(j)}$ for all $j = 2, ..., n$.

Clearly, the key point of the OWA operator is to determine its associated weights. Yager [1] introduced the ideal of aggregate dependent weights, which allows the weights to be a function of the aggregated arguments, in this case

$$OWA(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} f_j(a_{\sigma(1)}, a_{\sigma(2)}, ..., a_{\sigma(n)}) a_{\sigma(j)} \tag{2}$$

The first family of the aggregate dependent weights that Yager [1] studied is as follows:

$$w_j = \frac{a_{\sigma(j)}^{\alpha}}{\sum_{j=1}^{n} a_{\sigma(j)}^{\alpha}}, \quad j = 1,2,..., n \tag{3}$$

where $\alpha \in (-\infty, +\infty)$. In this case, it leads to a neat OWA operator:

$$OWA(a_1, a_2, ..., a_n) = \frac{\sum_{j=1}^{n} a_j^{\alpha+1}}{\sum_{j=1}^{n} a_j^{\alpha}} \tag{4}$$

Note: An OWA operator is called neat if the aggregated value is independent of the ordering [1].

Another interesting case of the aggregate dependent weights is

$$w_j = \frac{(1 - a_{\sigma(j)})^{\alpha}}{\sum_{j=1}^{n} (1 - a_{\sigma(j)})^{\alpha}}, \quad j = 1,2,..., n \tag{5}$$

In this case, it follows that

$$OWA(a_1, a_2, ..., a_n) = \frac{\sum_{j=1}^{n} (1 - a_j)^{\alpha} a_j}{\sum_{j=1}^{n} (1 - a_j)^{\alpha}} \tag{6}$$

which is also a neat aggregation.

Yager [2] also considered a case where the aggregation is not neat, that is

$$w_i = \frac{a_{\sigma(n-i+1)}^{\alpha}}{\sum_{j=1}^{n} a_{\sigma(j)}^{\alpha}}, \quad j = 1,2,....,n \tag{7}$$

in this case, it yields

$$OWA(a_1, a_2, ..., a_n) = \frac{\sum_{j=1}^{n} a_{\sigma(n-j+1)}^{\alpha} \, a_{\sigma(j)}}{\sum_{j=1}^{n} a_{\sigma(j)}^{\alpha}} \tag{8}$$

In many actual situations, the arguments $a_1, a_2, ..., a_n$ are usually given by n different individuals. Some individuals may provide unduly high or unduly low preference arguments for their preferred or repugnant objects. In such a case, we shall assign very low weights to these "false" or "biased" opinions, that is to say, the closer a preference argument to the average value, the more the weight [9]. All the above argument-dependent approaches, however, should be unsuitable for dealing with this case. Therefore, it is worth paying attention to this issue, in the following we will develop a novel argument-dependent approach to determining the OWA weights.

Definition 1. Let $a_1, a_2, ..., a_n$ be a collection of arguments, and let μ be the average value of these arguments, i.e., $\mu = \frac{1}{n}\sum_{j=1}^{n} a_j$, $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of $(1,2,...,n)$ such that $a_{\sigma(j-1)} \geq a_{\sigma(j)}$ for all $j = 2,...,n$, then we call

$$s(a_{\sigma(j)}, \mu) = 1 - \frac{|a_{\sigma(j)} - \mu|}{\sum_{j=1}^{n} |a_j - \mu|}, \quad j = 1,2,..., n \tag{9}$$

the similarity degree of the j-th largest argument $a_{\sigma(j)}$ and the average value μ.

Let the $w = (w_1, w_2, ..., w_n)^T$ be the weight vector of the OWA operator, then we define the following:

$$w_j = \frac{s(a_{\sigma(j)}, \mu)}{\sum_{j=1}^{n} s(a_{\sigma(j)}, \mu)}, \quad j = 1,2,..., n \tag{10}$$

where $s(a_{\sigma(j)},\mu)$ is defined by Eq.(9). Clearly, we have $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Since

$$\sum_{j=1}^{n} s(a_{\sigma(j)},\mu) = \sum_{j=1}^{n} s(a_j,\mu) \tag{11}$$

then Eq.(10) can be rewritten as

$$w_j = \frac{s(a_{\sigma(j)},\mu)}{\sum_{j=1}^{n} s(a_j,\mu)}, \quad j = 1,2,\ldots, n \tag{12}$$

In this case, we have

$$OWA(a_1, a_2,\ldots, a_n) = \frac{\sum_{j=1}^{n} s(a_{\sigma(j)},\mu)\, a_{\sigma(j)}}{\sum_{j=1}^{n} s(a_{\sigma(j)},\mu)} = \frac{\sum_{j=1}^{n} s(a_j,\mu)\, a_j}{\sum_{j=1}^{n} s(a_j,\mu)} \tag{13}$$

It is easy to see that this is a neat and dependent OWA operator.

By Eq.(12), we can get the following results easily:

Theorem 1. Let $a_1, a_2,\ldots, a_n$ be a collection of arguments, and μ be the average value of these arguments, $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$ such that $a_{\sigma(j-1)} \geq a_{\sigma(j)}$, for all $j = 2,\ldots,n$, and let $s(a_{\sigma(j)},\mu)$ be the similarity degree of the j-th largest argument $a_{\sigma(j)}$ and the average value μ, if $s(a_{\sigma(i)},\mu) \geq s(a_{\sigma(j)},\mu)$, then $w_i \leq w_j$.

Corollary 1. Let $a_1, a_2,\ldots, a_n$ be a collection of arguments, if $a_i = a_j$, for all i, j, then $w_j = \frac{1}{n}$, for all j.

From Eq.(12) and Theorem 1, we know that a prominent characteristic of this dependent OWA operator are that it can relieve the influence of unfair arguments on the aggregated results by assigning low weights to those "false" or "biased" ones.

Yager [1,2] defined two important measures associated with an OWA operator. The first measure, called the dispersion of the weighting vector w of an OWA operator is defined as

$$disp(w) = -\sum_{j=1}^{n} w_j \ln w_j \tag{14}$$

which measures the degree to which w takes into account the information in the arguments during the aggregation.

The second measure, called orness measure, is defined as

$$orness(w)=\frac{1}{n-1}\sum_{j=1}^{n}(n-j)w_j \tag{15}$$

which lies in the unit interval [0,1], and characterizes the degree to which the aggregation is like an *or* operation. From Eqs.(12), (14) and (15), it follows that

$$disp(w)=-\frac{\sum_{j=1}^{n}s(a_j,\mu)\ln\frac{s(a_j,\mu)}{\sum_{j=1}^{n}s(a_j,\mu)}}{\sum_{j=1}^{n}s(a_j,\mu)} \tag{16}$$

$$orness(w)=\frac{1}{n-1}\frac{\sum_{j=1}^{n}(n-j)s(a_j,\mu)}{\sum_{j=1}^{n}s(a_j,\mu)} \tag{17}$$

Example 1. Suppose that there are seven decision makers $d_j\ (j=1,2,\ldots,7)$, these decision makers provide their individual preferences for a university faculty with respect to the criterion *research.* Assume that the given preference arguments are as follows:

$$a_1=80,\ a_2=75,\ a_3=100,\ a_4=40,\ a_5=90,\ a_6=95,\ a_7=70$$

Therefore, the re-ordered arguments $a_j\ (j=1,2,\ldots,7)$ in descending order are

$$a_{\sigma(1)}=100,\ a_{\sigma(2)}=95,\ a_{\sigma(3)}=90,\ a_{\sigma(4)}=80,\ a_{\sigma(5)}=75$$
$$a_{\sigma(6)}=70,\ a_{\sigma(7)}=40$$

then by Eqs.(9) and (12), we have

$$w_1=0.13145,\ w_2=0.13967,\ w_3=0.14789,\ w_4=0.16432$$
$$w_5=0.16080,\ w_6=0.15258,\ w_7=0.10329$$

which are shown in Fig. 1.

By Eqs. (16) and (17), we have

$$\begin{aligned}disp(w)&=-\sum_{j=1}^{7}w_j\ln w_j\\&=-[0.13145\times\ln(0.13145)+0.13967\times\ln(0.13967)+0.14789\times\ln(0.14789)\\&\quad+0.16432\times\ln(0.16432)+0.16080\times\ln(0.16080)+0.15258\times\ln(0.15258)\\&\quad+0.10329\times\ln(0.10329)]\\&=1.9363\end{aligned}$$

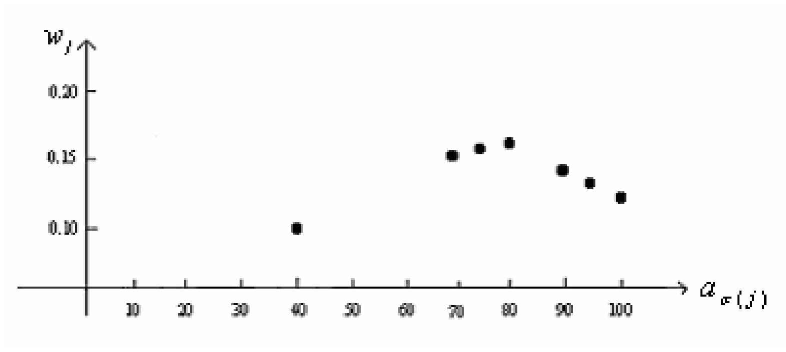

Fig. 1. The weights w_j of $a_{\sigma(j)}$ $(j = 1,2,...,7)$

and

$$orness\,(w) = \frac{1}{7-1}\sum_{j=1}^{7}(7-j)w_j$$

$$= \frac{1}{6}\times(6\times0.13145 + 5\times0.13967 + 4\times0.14789 + 3\times0.16432 + 2\times0.16080$$
$$+1\times0.15258 + 0\times0.10329)$$
$$= 0.5076$$

By Eq.(13), we have

$$OWA(a_1, a_2, ..., a_n) = 0.13145\times100 + 0.13967\times95 + 0.14789\times90$$
$$+0.16432\times80 + 0.16080\times75 + 0.15258\times70 + 0.10329\times40$$
$$= 79.74155$$

hence, the collective preference argument is 79.74155 .

To relieve the influence of unfair arguments on the aggregated results, in the above example, we assign low weights to those "false" or "biased" ones, that is to say, the closer a preference argument to the average value $\mu = 78.57$, the more the weight. For example, we assign the lowest weight $w_7 = 0.10329$ to the lowest preference value $a_4 = 40$, which has the biggest departure from the average value, and assign the second lowest weight $w_1 = 0.13145$ to the maximal preference value $a_3 = 100$, which has the second biggest departure from the average value. We assign the most weight $w_4 = 0.16432$ to the preference value $a_1 = 80$, which is closest to the average value, and assign the second most weight $w_5 = 0.16080$ to the preference value $a_{\sigma(5)} = 75$, which has the second least departure from the average value. We assign the value 0.5076 to the *orness* measure, and give the value 1.9363 to the dispersion measure.

3 Concluding Remarks

In this paper, we have investigated the dependent OWA operators, and developed a new argument-dependent approach to determining the OWA weights, which can relieve the influence of unfair arguments on the aggregated results. We have verified the practicality and effectiveness of the approach with a numerical example.

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