Semi-blind Equalization of Wireless MIMO Frequency Selective Communication Channels

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Abstract. The semi-blind equalization for a wireless multiple-input multiple-output (MIMO) system with frequency selective Ricean channels is addressed. A reformulation of the problem accepting higher complexity allows to apply ordinary complex Independent Component Analysis (ICA). An algorithm presented here resolves the increased number of permutations due to the reformulation of the convolutive blind signal separation problem. The remaining ambiguities are as many as in the non-frequency selective case and solved by a short preamble. The efficiency of the proposed method is illustrated by numerical simulations. According to the Bit Error Rate the semi-blind equalization shows a good performance in comparison to training based channel estimation & equalization.

1 Introduction

Multiple-input multiple-output (MIMO) systems provide higher data rates that data-demanding applications require. Training based channel estimation of frequency selective MIMO channels reduces the effective transmission rate. The semi-blind equalization approaches can reduce the amount of training to a minimum. A well-known challenge is to resolve the remaining ambiguities left by blind signal separation.

The blind equalization of frequency selective MIMO channels is also known as convolutive blind signal separation (BSS) or blind separation of convolutive mixtures [1]. Methods for blind separation of convolutive mixtures can be subdivided into direct and indirect approaches. In a direct approach the separated signals are extracted without explicit identification of the mixing matrix, while indirect methods identify the unknown channels before separation and equalization. A variety of two-step algorithms using the indirect approach are proposed. The linear prediction (LP) based approaches [2] and the subspace method [3] are most popular among them. A further approach is proposed in [4].

A different approach [5] uses Orthogonal Frequency Division Multiplexing (OFDM). The convolutive mixture is transformed associating each subcarrier with an instantaneous ICA problem. This special case of performing ICA in the frequency domain requires to solve the frequency dependent permutation problem.

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In this paper the direct approach is considered. A reformulation of the convolutive blind signal separation problem is used to apply ordinary complex ICA. The increased number of ambiguities due to the reformulation of the convolutive blind signal separation problem are resolved by a three step algorithm. In the first step the special structure of the reordered mixing matrix, a generalized Sylvester matrix, is used to reduce the unknown permutations. The remaining ambiguities are as many as in the non-frequency selective case. Considering a communication system with M-QAM modulation, in a second step the phase ambiguity can be reduced up to a multiple of $\pi/2$. Using differential modulation the remaining phase ambiguity can be solved while the permutations remain. As the permutations can not be solved with a differential modulation here a non-differential M-QAM modulation scheme is utilized. To resolve the permutations preamble symbols are transmitted. It will be shown that the required number of preamble symbols is equal to the number of transmit antennas.

The paper is organized as follows. The MIMO system and the frequency selective MIMO channel model is described in Section 2. Section 3 covers the semi-blind equalization of the frequency selective wireless MIMO system. Simulation results are presented in Section 4. As a performance measure the blind equalization method is compared with a training based channel estimation & equalization. A summary and conclusion marks can be found in Section 5.

2 System Description

2.1 Transmitter

We consider a frequency selective MIMO wireless system with n_T transmit and n_R receive antennas (see Fig. 1). The serial data stream is split into n_T parallel substreams, one for every transmit antenna. The substreams are mapped into M-QAM symbols and organized in frames. Each frame of length N_F consists of N_I information symbols and N_P preamble symbols. The preamble sequences are orthogonal to each other. The symbol substreams are subsequently transmitted over the n_T antennas at the same time. The symbol transmitted by antenna m

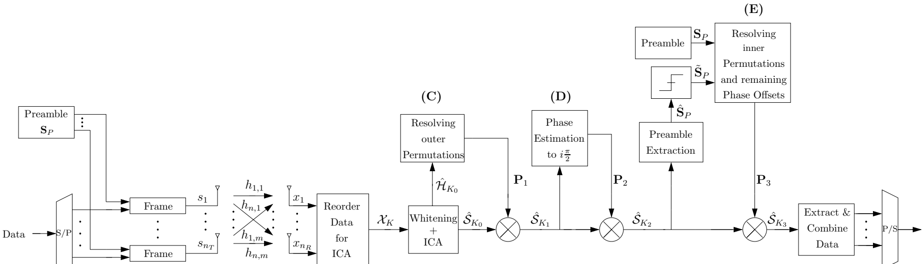


Fig. 1. Wireless MIMO system with semi-blind equalization of frequency selective channels and ambiguity resolution

at time instant k is denoted by $s_m(k)$. The transmitted symbols are arranged in vector $\mathbf{s}(k) = [s_1(k), \ldots, s_{n_T}(k)]^T$ of length n_T, where $(\cdot)^T$ denotes the transpose operation.

2.2 Channel

Between every transmit antenna m and every receive antenna n there is a frequency selective channel impulse response (CIR) of length $L+1$, described by the vector $\mathbf{h}_{n,m} = [h_{n,m}(0), \ldots, h_{n,m}(L)]^T$. Assuming the same channel order L for all channels, the frequency selective MIMO channel can be described by $L+1$ complex channel matrices $\mathbf{H}(k)$, $k = 0, ..., L$, with the dimension $n_R \times n_T$.

$$\mathbf{H}(k) = \begin{bmatrix} h_{1,1}(k) & \cdots & h_{1,n_T}(k) \\ \vdots & \ddots & \vdots \\ h_{n_R,1}(k) & \cdots & h_{n_R,n_T}(k) \end{bmatrix} \quad (1)$$

We suppose that the channels remain constant over the transmission of a frame and vary independently from frame to frame (*block fading channel*).

The elements $h_{n,m}(k), k = 0, \ldots, L$, of the channel impulse responses are complex random variables with a Gaussian distributed real and imaginary part, zero mean and variance $\sigma^2(k)$. The first element $h_{n,m}(0)$ includes also the direct component with the amplitude $p_{n,m}$ and the power $p_{n,m}^2 = c_R \cdot \sigma^2(0)$, where c_R is the Rice factor. The power delay profile describes by the variances $\sigma^2(k), k = 0, \ldots, L$ how the power is distributed over the taps of the channel impulse response. Here the variances decrease exponentially with k.

The channel energy is normalized by the condition

$$\sum_{k=0}^{L} E\left\{|h_{n,m}(k)|^2\right\} = p_{n,m}^2 + \sum_{k=0}^{L} \sigma^2(k) := 1. \quad (2)$$

We assume additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2 per receive antenna.

2.3 Receiver

The symbol received by antenna n at time instant k is denoted by $x_n(k)$. The symbols received by the n_R antennas are arranged in a vector $\mathbf{x}(k) = [x_1(k), \ldots, x_{n_R}(k)]^T$ of length n_R, which can be expressed with (1) and $\mathbf{n}(k)$ as noise vector of length n_R as

$$\mathbf{x}(k) = \sum_{i=0}^{L} \mathbf{H}(i)\, \mathbf{s}(k-i) + \mathbf{n}(k). \quad (3)$$

3 Semi-blind Equalization

3.1 Blind Signal Separation

A. Reorder Data for ICA

The convolutive mixture in (3) is reordered to apply ordinary ICA [1]. Considering the receive vector $\mathbf{x}(k)$ of length n_R at K successive time instances the receive symbols are arranged in a vector $\mathbf{x}_K(k) = [\, \mathbf{x}(k), \ldots, \mathbf{x}(k+K-1)\,]^T$ of length $K \cdot n_R$. The transmitted symbols that are the Independent Components (IC) are arranged in a vector $\mathbf{s}_K(k) = [\, \mathbf{s}(k-L), \ldots, \mathbf{s}(k+K-1)\,]^T$ of length $(K+L)\cdot n_T$. As the channel is assumed to be a block fading channel the received data is processed in frames, which is represented by the receive signal matrix $\mathcal{X}_K = [\mathbf{x}_K(0), \mathbf{x}_K(1), \ldots, \mathbf{x}_K(N_F-K)]$ and the associated transmit signal matrix $\mathcal{S}_K = [\mathbf{s}_K(0), \mathbf{s}_K(1), \ldots, \mathbf{s}_K(N_F-K)]$. With $\mathbf{x}_K(k)$ and $\mathbf{s}_K(k)$ the mixing matrix $\mathcal{H}_K$ becomes a Toeplitz matrix (called generalized Sylvester matrix) of size $K \cdot n_R \times (K+L)\cdot n_T$:

$$\mathcal{H}_K = \begin{bmatrix} \mathbf{H}(L) & \cdots & \mathbf{H}(0) & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{H}(L) & \cdots & \mathbf{H}(0) \end{bmatrix}, \tag{4}$$

containing the channel matrices $\mathbf{H}(k)$, $k = 0, ..., L$. The convolutive mixture in (3) simplifies to a multiplicative one, that is the reformulated ICA problem, given by

$$\mathcal{X}_K = \mathcal{H}_K \cdot \mathcal{S}_K + \mathcal{N}. \tag{5}$$

where $\mathcal{N}$ denotes the noise.

As ICA requires the number of observed mixtures to be at least equal to the number of ICs, the matrix $\mathcal{H}_K$ should have at least as many rows as columns, which is expressed as $K \cdot n_R \geq (K+L)\cdot n_T$. For K the following relation holds

$$K \geq \left\lceil \frac{n_T \cdot L}{n_R - n_T} \right\rceil. \tag{6}$$

As a consequence, applying ICA to frequency selective MIMO systems requires more receive than transmit antennas.

B. Whitening + ICA

Ordinary complex ICA algorithms like complex FastICA [6] with symmetric orthogonalization or JADE [7] are applied to obtain estimates of the separated signals $\hat{\mathcal{S}}_{K_0}$ and an estimate $\hat{\mathcal{H}}_{K_0}$ for the generalized Sylvester matrix. Recalling the restrictions of ICA [1], every estimated IC shows a different unknown phase offset. In addition the estimated ICs are arbitrary permuted. These permutations are divided into outer permutations in $\hat{\mathcal{S}}_{K_0}$, i.e. permutations of the vectors $\mathbf{s}(k)$ in $\mathbf{s}_K(k)$, and inner permutations, i.e. permutations of $s_m(k)$ in $\mathbf{s}(k)$.

3.2 Ambiguity Resolution

C. Resolving Outer Permutations

To resolve the outer permutations, the special structure of the reordered mixing matrix $\mathcal{H}_K$ is used. Due to permutations of the rows in $\hat{\mathcal{S}}_{K_0}$, only the columns of $\hat{\mathcal{H}}_{K_0}$ are permuted. The same applies to the unknown phase offsets.

An algorithm to rearrange the matrix $\hat{\mathcal{H}}_{K_0}$ to a Toeplitz matrix is presented for $n_T = 2$, which can easily be extended to more transmit antennas. The channel matrix $\mathbf{H}(k)$ for $n_T = 2$ is split into subcolumns $\mathbf{H}(k) = [\mathbf{h}_1(k)\, \mathbf{h}_2(k)]$. The algorithm is described in two major steps:

1. The n_T rightmost columns in $\hat{\mathcal{H}}_{K_0}$ can be found by two criteria:
   (a) Assuming an exponential decreasing power delay profile channel, the sum of the absolute values of subcolumn $\mathbf{h}_m(0)$ in (7) is maximal compared to other subcolumns in the same row.
   (b) Considering the part of the column above subcolumn $\mathbf{h}_m(0)$, the sum of the absolutes values should be nearly zero.

   These criteria require $K \geq 2$, elsewise no zero submatrices could be found in $\hat{\mathcal{H}}_{K_0}$. The found columns, in (7) depicted by the boxed columns, will be moved to the right side of the matrix as shown in (8). The columns including $\mathbf{h}_1(0)$ and $\mathbf{h}_2(0)$ can be permuted (inner permutations).

$$\begin{bmatrix} \ddots & \boxed{\begin{matrix} \vdots \\ \mathbf{0}\, e^{j\Delta\varphi_1} \\ \mathbf{h}_1(0)\, e^{j\Delta\varphi_1} \end{matrix}} & \begin{matrix} \\ \cdots \\ \cdots \end{matrix} & \boxed{\begin{matrix} \vdots \\ \mathbf{0}\, e^{j\Delta\varphi_2} \\ \mathbf{h}_2(0)\, e^{j\Delta\varphi_2} \end{matrix}} & \begin{matrix} \cdots \\ \cdots \\ \cdots \end{matrix} \\ \cdots & & & & \\ \cdots & & & & \end{bmatrix} \tag{7}$$

$$\left[\begin{array}{ccc|cc} \ddots & \vdots & & \vdots & \vdots \\ \cdots & \boxed{\mathbf{h}_2(0)\, e^{j\Delta\varphi_3}} & \cdots & \mathbf{0}\, e^{j\Delta\varphi_1} & \mathbf{0}\, e^{j\Delta\varphi_2} \\ \cdots & \mathbf{h}_2(1)\, e^{j\Delta\varphi_3} & \cdots & \mathbf{h}_1(0)\, e^{j\Delta\varphi_1} & \boxed{\mathbf{h}_2(0)\, e^{j\Delta\varphi_2}} \end{array}\right] \tag{8}$$

2. The row one subcolumn above $\mathbf{h}_2(0)$ is searched for a subcolumn having the same absolute values as $\mathbf{h}_2(0)$, depicted by the boxed subcolumns in (8). The matching column is moved left to the already sorted ones as depicted in (9).

$$\left[\begin{array}{c|ccc} \ddots & \vdots & \vdots & \vdots \\ \cdots & \mathbf{h}_2(0)\, e^{j\Delta\varphi_2} & \mathbf{0}\, e^{j\Delta\varphi_1} & \mathbf{0}\, e^{j\Delta\varphi_2} \\ \cdots & \mathbf{h}_2(1)\, e^{j\Delta\varphi_2} & \mathbf{h}_1(0)\, e^{j\Delta\varphi_1} & \mathbf{h}_2(0)\, e^{j\Delta\varphi_2} \end{array}\right] \tag{9}$$

   The boxed submatrices in (8) have the same phase angle but different phase offsets. To relate the phase offsets the differential angle $\Delta\psi$ is determined by

$$\Delta\psi = \angle(\mathbf{h}_2(0)\, e^{j\Delta\varphi_3}) - \angle(\mathbf{h}_2(0)\, e^{j\Delta\varphi_2}) = \Delta\varphi_3 - \Delta\varphi_2. \tag{10}$$

   By multiplying the found column by $e^{-j\Delta\psi}$ we get

$$\mathbf{h}_2(i)\, e^{j\Delta\varphi_3} \cdot e^{-j\Delta\psi} = \mathbf{h}_2(i)\, e^{j\Delta\varphi_2}\ ,\ i = 0, 1 \tag{11}$$

and the same phase offset $\Delta\varphi_2$ is achieved. This processing step is repeated until $\hat{\mathcal{H}}_{K_0}$ is sorted.

The Toeplitz matrix is resorted up to the inner permutations in $\mathbf{H}(k)$. As the rows in $\hat{\mathcal{S}}_{K_0}$ are permuted in the same way as the columns in $\hat{\mathcal{H}}_{K_0}$, during the execution of the algorithm a permutation matrix $\mathbf{P}_1$ is arranged to obtain the corrected signal matrix $\hat{\mathcal{S}}_{K_1} = \mathbf{P}_1 \cdot \hat{\mathcal{S}}_{K_0}$.

D. Phase Estimation to $i\frac{\pi}{2}$
By using M-QAM modulated signals the phase can be estimated by a set of symbols and fourth order cumulants [8] up to a multiple of $\frac{\pi}{2}$. The estimated phase of source m is denoted with φ_{o_m}. With the unitary matrix $\mathbf{P}_2$

$$\mathbf{P}_2 = \mathbf{I}_{K+L} \otimes \mathrm{diag}(e^{-j\hat{\varphi}_{o_1}}, \ldots, e^{-j\hat{\varphi}_{o_{n_T}}}), \tag{12}$$

where $\otimes$ denotes the Kronecker tensor product and $\mathbf{I}$ is the identity matrix, the corrected signal matrix $\hat{\mathcal{S}}_{K_2} = \mathbf{P}_2 \cdot \hat{\mathcal{S}}_{K_1}$ is obtained.

E. Resolving Inner Permutations and the Remaining Phase Offsets
The remaining ambiguities are solved by orthogonal preamble sequences, designed using Hadamard matrices, with a preamble length $N_P \geq n_T$. The symbols of the extracted preamble $\hat{\mathbf{S}}_P$ are hard decided, that is denoted by $\tilde{\mathbf{S}}_P$. The unitary cross correlation matrix $\mathbf{R}_{\tilde{\mathbf{S}}\mathbf{S}}$, given by

$$\mathbf{R}_{\tilde{\mathbf{S}}\mathbf{S}} = \frac{1}{N_P}\tilde{\mathbf{S}}_P\mathbf{S}_P^H, \tag{13}$$

where $(\cdot)^H$ denotes the complex-conjugate (Hermitian) transpose, will be an identity matrix if all phase offsets are zero and no inner permutations exist. As the outer permutations are solved, the matrix

$$\mathbf{P}_3 = \mathbf{I}_{K+L} \otimes \mathbf{R}_{\tilde{\mathbf{S}}\mathbf{S}}^H \tag{14}$$

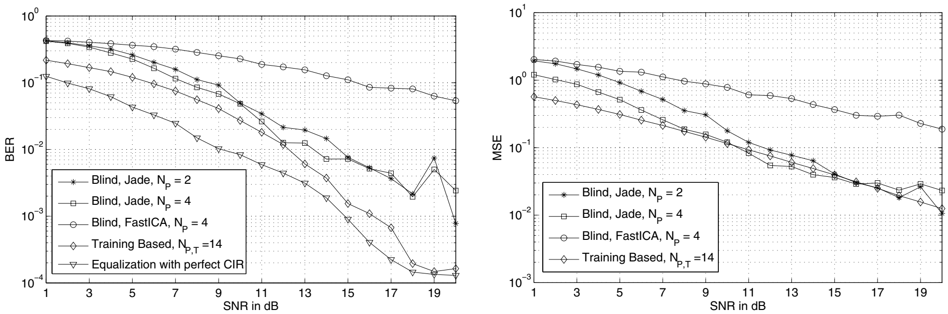
is used to obtain the corrected signal matrix $\hat{\mathcal{S}}_{K_3} = \mathbf{P}_3 \cdot \hat{\mathcal{S}}_{K_2}$.

F. Extract & Combine Data
By solving the ICA problem in (5) the transmitted symbols $s_m(k)$ are estimated $(K+L)$ times. Prior to a demodulation the multiple estimated transmit symbols can be combined to achieve a lower Bit Error Rate.

4 Simulation Results

For a performance comparison a training based channel estimation and equalization is considered. A training sequence is used to estimate the channel matrices by the Minimum Mean Square Error (MMSE) - estimator [9, 10]. We use random training sequences [10] of length $N_{P,T} = 2 \cdot n_T \cdot (L+1) + L$ per transmit antenna, which is slightly above the optimal training sequence length [10] of $N_{P,T} = 1.8 \cdot n_T \cdot (L+1) + L$. The channel estimate is used to equalize the received data. For the training based equalization the successive FS-Blast [11] is used.

In Fig. 2(a) one can see a comparison of semi-blind and training based channel estimation & equalization based on the Bit Error Rate (BER) versus the Signal-to-Noise-Ratio (SNR). Also shown is the BER for the case of perfect knowledge of the channel impulse response (CIR) at the receiver. As expected, the result with training based channel estimation is closer to the result with perfect knowledge than that with semi-blind equalization. The presented semi-blind approach requires a 3 dB increased Signal-to-Noise-Ratio (SNR) to achieve a Bit Error Rate of $4 \cdot 10^{-3}$ compared to training based channel estimation & equalization. Comparing the two ICA algorithms JADE shows a better performance than FastICA. One can also see, that an increased number of preamble symbols N_P does not significantly change the BER. It is sufficient to use the minimal desired number, which is $N_P = 2$ for $n_T = 2$. The mean square error (MSE) versus the



(a) BER versus SNR (b) MSE versus SNR

Fig. 2. Comparison of semi-blind and training based channel estimation & equalization, Simulation with $n_T = 2$ and $n_R = 4$, frequency selective MIMO channel order $L = 2$, $c_R = 1$, 4-QAM modulation, Semi-blind equalization with 500 symbols per frame with combining multiple estimations

SNR for semi-blind and training based channel estimation is shown in Fig. 2(b). The MSE of the channel estimation is calculated as

$$\mathrm{MSE} = E\left\{ \sum_{k=0}^{L} \frac{1}{n_T \cdot n_R} \sum_{m=1}^{n_T} \sum_{n=1}^{n_R} \left| h_{n,m}(k) - \rho_m \hat{h}_{n,m}(k) \right|^2 \right\}, \qquad (15)$$

in which $\hat{h}_{n,m}(k)$ is the estimated channel impulse response and ρ_m is used to compensate the scalar ambiguity associated with the estimation results for the m-th antenna. This calculation of the MSE is equal to the one used in the literature since (2) holds. For high SNR greater than 10 dB the MSE of the training based approach does not differ from the MSE achieved with semi-blind equalization. The BER of the training based approach in contrast is less than the one obtained by semi-blind equalization. It is important to note that the separation of the independent components do not depend on the estimated mixing matrix and their accuracy.

5 Conclusions

We showed that ordinary complex ICA can be used for semi-blind equalization of MIMO frequency selective channels. An algorithm was presented that resolve the increased number of ambiguities due to the reformulation of the convolutive blind signal separation problem. Compared with training based channel estimation & equalization the presented semi-blind approach requires a 3 dB increased Signal-to-Noise-Ratio (SNR) to achieve a Bit Error Rate of $4 \cdot 10^{-3}$.

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