

# Blind Spatial Multiplexing Using Order Statistics for Time-Varying Channels<sup>\*</sup>

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**Abstract.** Spatial multiplexing is currently one of the most promising techniques exploiting the spatial dimension to increase data rates. Most of existing methods are based on coherent detection techniques that imply multichannel estimation. This procedure, especially for time-varying channels, increases the overhead rate due to the periodical training requirement. A suitable approach dealing with this scenario proposes the use of Blind Source Separation (BSS) principles to minimize the mentioned overhead still offering the increased data rate. The authors have developed in previous publications a new BSS technique based on Order Statistics (OS) labeled as ICA-OS with very satisfactory performance in static scenarios. In these studies it was already realized that the amount of data required for convergence was significantly less than other well known methods. Therefore, in this current contribution we present some results showing the capability of our procedure to deal with time-varying channels typical of mobile applications without training requirement.

## 1 Introduction

Modern communications require increased data rates without extra bandwidths in order to provide a suitable service for the incoming applications. Exploitation of the spatial dimension has been traditionally based on standard beamforming or spatial diversity. Additionally, few years ago it was pointed out the possibility of spatial multiplexing to increase data rate without involving other resources but just extra complexity at the RF parts and also more complex reception techniques. [1] and references therein showed that capacity in MIMO (Multiple Input – Multiple Output) systems maybe even be multiplied by the minimum number of antennas at any side. This amazing result has driven the attention of researchers all over the world in the recent years.

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Most of the practical schemes trying to get this promised benefit are based on matrix eigendecompositions (if channel is known at both sides) or in a more attractive approach, BLAST-like implementations just require channel knowledge at the receiver side. Training, especially for time varying channels, reduces significantly the desired data rate due to the periodic interleaving of known data. This overhead is much more significant in MIMO applications because several antennas require specific training reducing the promised data rate gain.

In this contribution we deal with this problem proposing a BSS approach suitable for time-varying channels. The main handicap of existing BSS techniques applied to this scenario is the very large amount of data required to get satisfactory performance, especially those based on Higher Order Statistics (HOS) estimation. This characteristic is acceptable for delay non sensitive applications or static mixtures but seems to be less useful or impractical in MIMO communications.

The authors proposed in several previous publications a new BSS method based on OS. In these proposals the Cumulative Density Function (cdf) estimation using several order statistics was shown to provide suitable means for signals separation. This procedure provided very satisfactory results for both subGaussian and super-Gaussian distributions but specially promising for digital modulations as QAM. More indeed, the amount of data required for appropriate separation was much less than other well known procedures. In those previous publications this remarkable result was just mentioned as a procedure saving computational burden.

In this contribution we are focused on MIMO communications using QAM communications through time-varying channels, showing that ICA-OS is more suitable than other BSS as JADE [10] as one of the most representative schemes. Basically, JADE is a cumulant batch algorithm which optimizes a 4th order measure of independence among the whole set of outputs.

## 2 Our Model for ICA Procedures for Time-Varying Channels

The standard linear mixture can be easily generalized to a time-varying MIMO communication channel.

$$\mathbf{y}[k] = \mathbf{H}[k]\mathbf{x}[k] + \mathbf{n}[k] \quad (1)$$

Where  $\mathbf{y}$  is the received vector whose dimension is the number of receiving antennas  $N_r$  and  $\mathbf{x}$  is the transmitted vector of size  $N_t \times 1$  where  $N_t$  is the number of transmit antennas.  $\mathbf{n}$  is the additive Gaussian noise and  $\mathbf{H}$  is the MIMO matrix sized  $N_r \times N_t$ . We have remarked the time dependence using the variable  $k$ . This model assumes flat fading transmission through a linear mixture with additive noise.

MIMO channel model assumed considers uncorrelated spatial fading. This hypothesis is realistic if the scatterers are located around the antennas and the antennas are enough separated. For terminals, typically this is fulfilled even for less than a lambda separation while for base stations with antennas under the roofs, a few lambdas may be enough. Thus, this model is especially suitable for urban or indoor communications which is the case where more data rate is required and spatial multiplexing is more demanded.

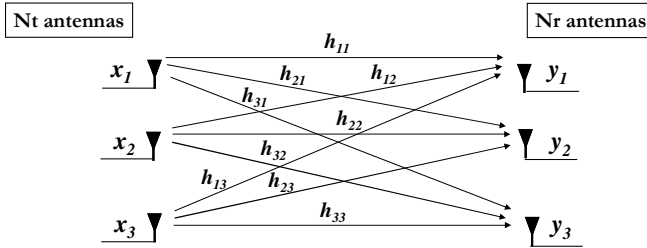


Fig. 1. MIMO channel Mode

Time evolution of the spatial uncorrelated coefficients of matrix  $\mathbf{H}$  is implemented using the procedure described by Hoeher in [2] for the flat fading case. This model is in fact based on an intuitive interpretation of the Bello’s model [3] as an incoherent superposition of  $N$  echoes, where each echo is characterized by a random phase, a random delay and a random Doppler shift. For the flat fading case, each element  $mn$  of matrix  $\mathbf{H}$  collapses to

$$h_{mn} [k] = \lim_{N \rightarrow \infty} \frac{1}{\sqrt{N}} \sum_{l=1}^N e^{j\theta_l} e^{jk 2\pi f_{Dl} l f_s} \tag{2}$$

where  $\theta_l$  are the random phases,  $f_{Dl}$  are the Doppler random variables following the standard Jake’s spectrum characterized by  $f_{Dmax}$ , and  $f_s$  is the sampling frequency. In practical cases,  $N = 25$  seems to provide satisfactory accuracy.

Most of the BSS methods would collect a certain amount of  $\mathbf{y}$  vectors along some time interval in order to estimate the needed statistics from the measured data to perform different algorithms. The degradation of different proposals will depend on the amount of data required for satisfactory separation. This behavior requires static mixtures that can not be guaranteed in time varying scenarios.

### 3 The ICA-OS Method Applied to Communications

Previously, let us remark that the separation scheme in the communication scenery presented at previous section is:

$$\mathbf{w} = \mathbf{B}\mathbf{D}\mathbf{y} \tag{3}$$

where  $\mathbf{w}$  ( $N_r \times n$ ) are the output channels to be updated towards the  $N_r$  estimated sources  $\mathbf{x}$ ,  $\mathbf{B}$  is the ( $N_r \times N_r$ ) unknown orthogonal separation matrix,  $\mathbf{D}$  is the decorrelation matrix obtained through the well known whitening preprocessing scheme [11] and  $\mathbf{y}$  ( $N_r \times n$ ) are the set of  $n$  symbols collected at every  $N_r$  antenna.

ICA-OS has been proposed and explained by the authors in several papers [6-9], therefore let us just expose the main ideas:

ICA-OS is a deflation ICA procedure where at each stage, the separation vector  $\mathbf{b}_i$  is updated by means of the maximization of a certain non-Gaussianity measure; consequently, one non-gaussian original source is obtained at the  $\mathbf{w}_i$  output channel. In

other words, the non-Gaussianity measure  $J(\mathbf{b}_i)$  is used like ICA cost function. In the practical implementation  $J$  is maximized through a gradient rule plus some restriction which forces orthogonality between separation vectors:

$$\begin{aligned} \mathbf{b}_i [t+1] &= \mathbf{b}_i [t] + \mu \nabla J_i |_{\mathbf{b}_i [t]} \\ \mathbf{b}_i &\perp \{\mathbf{b}_1 \mathbf{b}_2 \dots \mathbf{b}_{i-1}\} \end{aligned} \tag{4}$$

In this sense, our main contributions have been a new family of Gaussianity measures based on Order Statistics [7, 8] as well as a new multistage deflation algorithm which decreases the dimension of the problem with each stage [9].

Our recommended Gaussianity distance (among the quoted family) for communication signals is the infinite norm applied to the difference between the implied inverse cdf's -denoted as  $Q$  -:

$$J(w_i(b_i)) = \max_u |Q_{w_i}(u) - Q_g(u)| \tag{5}$$

$Q_{w_i}$  corresponds to the analyzed signal  $w_i$  and  $Q_g$  to the equivalent Gaussian distribution  $g$ .

The practical way to calculate previous distance is based on the estimation of  $Q$ 's through the extreme Order Statistics (OS). It was proved in [6] after some mathematical development that Eq. (5) can be estimated through the following expression:

$$\begin{aligned} \hat{J}(w_i(b_i)) &= |w_{i(k)} - w_{i(l)} + 2Q_g(\frac{l}{n})| \\ \text{where}(k, l) &\cong (n, 1) \end{aligned} \tag{6}$$

Where  $w_{i(k)}$  is the  $k$  order statistic obtained in a simple way ordering a set large enough of  $n$  samples:

$$w_{i(1)} < w_{i(2)} < \dots < w_{i(l)} < \dots < w_{i(k)} < w_{i(n)} \tag{7}$$

and the known value  $Q_g(l/n)$  is the  $l$  order statistic of the Gaussian distribution. The advantages of using previous infinite norm instead of others  $p$ -norms [6, 7] either Gaussian distances are:

- It just needs a couple of order statistics to be estimated in front of the whole set of OS used by other norms.
- The OS are estimated easily just ordering the samples, instead of the complexity involved in HOS [10] and non-linearities used by other Gaussianity measures [11].
- Besides, it works more efficiently and robustly with a few samples (around  $n=100$ ) compared with other ICA methods, especially when the sources are subGaussians which usually are the kind of communication distributions, (see compared performance indexes in [7, 8]). This behavior is the main reason to use this measure like ICA cost Function in slowly variant MIMO channels (see next section).

At this point, it is necessary to expose the gradient expression (for more details see [8, 9]):

$$|\nabla \mathbf{J}|_{\mathbf{b}_i(r)} = S \mathbf{z}(\mathbf{d}_k - \mathbf{d}_l) \tag{8}$$

$$\text{where } S = \text{sign}(w_{i(k)}(\mathbf{b}_l) - w_{i(l)}(\mathbf{b}_l) + 2Q_g \left(\frac{l}{n}\right)_{\mathbf{b}_i(r)})$$

In previous equation  $\mathbf{z}$  is the vector ( $N_r \times n$ ) obtained as  $\mathbf{z} = \mathbf{D}\mathbf{y}$  (from Eq. (3)) after the well known sphering preprocessing [6, 11]. Other wise vector  $\mathbf{d}$  is calculated by means of:

$$\mathbf{d}_r[m] = \begin{cases} 1 & \text{if } w_i[m] = w_{i(r)} \\ 0 & \text{otherwise} \end{cases} \}_{m=1 \dots n} \tag{9}$$

Interested reader may review [6] in order to clarify the whole multistage procedure. We would like to remark that this scheme is also very efficient because in consecutive stages, the dimension of the vector space is consequently reduced.

### 4 Simulations

In order to show the ability of our method to cope with time variant channels we have run a set of simulations for different cases related to the ratio between the Doppler and the sampling frequencies. The maximum Doppler frequency is considered for pedestrian speed, 3 Km/h, and carrier frequency 2.4 GHz. These values are motivated by the fact that the most realistic scenario related to spatial multiplexing is probably the wireless Local Area Networks (WLAN). Assuming isotropic distribution, the Doppler density was derived by Clarke [4] and sometimes dubbed Jakes distribution. The main parameter to control the Doppler rate in terms of the symbol rate is defined in our simulations considering

$$f_s = M f_{D \max} \tag{10}$$

where factor  $M$  controls the relationship between both frequencies. When  $M$  increases, that is the symbol rate is much larger than the fading rate, the channel is nearly constant along the processed data. If  $M$  is not so large, the batch size must be reduced decreasing the expected performance.

In order to get a wide view about the performance of our method we have also implemented one of the most representative BSS methods, JADE [10], and also we have implemented the V-BLAST (see for instance [5] for an exhaustive and detailed review of BLAST and related techniques) as a non blind spatial multiplexing procedure to evaluate the loss of blind methods in front of trained procedures. Comparison with trained schemes includes the effect of some estimation noise modelled as AWGN added to every component of  $\mathbf{H}$  whose power is the effective noise power.

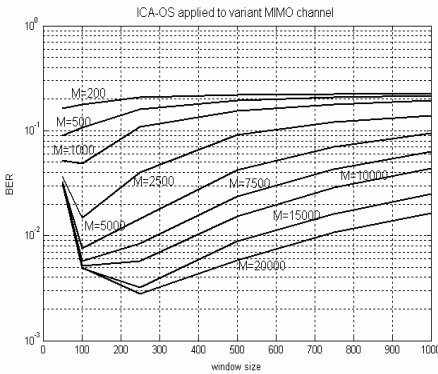
We have to remark that the implemented method requires some minimum training at the beginning of the transmission in order to solve the inherent ambiguity in the order of the recovered sources and also the phase ambiguity (although this point maybe overcome using noncoherent modulations). After ICA-OS a ZF demodulator is used to obtain the set of estimated bits.

Performance will be evaluated in terms of BER for a constellations 4QAM and different values of the parameter  $M$  in Eq. (11).

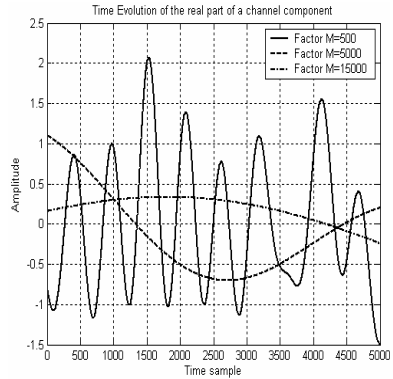
a) Scenario 1. The following simulation shows the degradation of the separation scheme according to the MIMO channel variability increases ( $M$  decreases). Communication sources are 4-QAM with 4 antennas at both ends. Fig. 2 shows the effect of the window size for all the cases under consideration. Noise is added at reception antennas with  $E_b/N_0 = 30$  dBs. It can be observed that there should be a trade off between the window size and the channel variability. Clearly, for static channels, the longer the window size, the better performance. However, for dynamic scenarios there is an optimum window size where the channel remains nearly static and the separation is performed. Some of these values are summarized in the following table:

**Table 1.** Optimal window size

Factor $M$	Windows Size
500	50
5000	100
15000	250



**Fig. 2.** Performance in terms of window and  $M$



**Fig. 3.** Time evolution of the channel

Fig. 3 complements this view with the time evolution of an arbitrary channel component (the real part for simplicity). It can be observed how fast it is the channel evolution depending on the aforementioned factor  $M$ .

b) Scenario 2. Communication sources 4QAM; 4 antennas at both sides and different sampling frequencies where the window size is fixed according to the optimum value provided in Table 1. It can be observed in Fig. 4 that if the channel is highly variant ( $M=500$ ) none of the schemes (ICA-OS and JADE) is able to separate the involved signals. However, as the channel becomes more static, the better performance of the ICA-OS is shown in front of JADE. It has to be remarked that these schemes are able to perform with satisfactory performance for high SNR scenarios, while for values below 10 dB are not suitable. We are currently working towards the

improvement of our approach combining the ICA-OS source separation with more complex MultiUser Detectors (MUDs) in a second stage.

c) Additionally, in a stationary environment we have compared ICA-OS with a second stage using ZF or BLAST criteria in front of trained ZF and BLAST in Fig. 5. Main ideas are the following:

1. ICA-OS method estimates the unknown MIMO channel  $\mathbf{H}$ , afterwards ZF demodulators either is used to recover source bits.
2. On the other side, ICA-OS estimates the unknown channel  $\mathbf{H}$  and afterwards BLAST demodulates the separated symbols using the estimation of  $\mathbf{H}$ .
3. When  $\mathbf{H}$  is known, but with channel estimation errors using instantaneous estimators, BLAST algorithm and ZF schemes obtains the demodulated bit sources

Results shown in Fig. 5 remark that V-BLAST is better than ZF in the trained mode, as expected. Logically blind techniques are worse than the previous ones that know the channel; anyway degradation could be acceptable in many applications. It can be also observed that ICA-OS with blast performs worse than ICA-OS with ZF due probably to the error propagation effect related to BLAST approaches, this last point remarks that BLAST techniques seems to be very sensitive to channel errors associated to estimation procedures. Although these are preliminary results, this approach combining BSS with more complex MUDs seems to be very promising for further work.

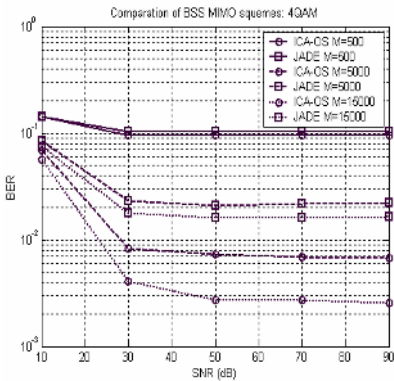


Fig. 4. Comparison of BSS MIMO schemes for 4QAM

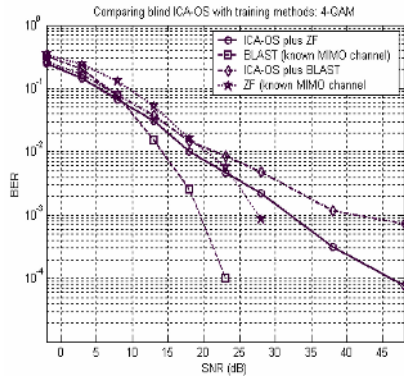


Fig. 5. Comparison of different approaches for blind and trained MIMO processing

## 5 Conclusions

This paper addresses the implementation and assessment of the ICA-OS procedure for spatial multiplexing in MIMO time varying scenarios. This procedure is shown to perform very satisfactorily in front of other well known BSS method where the amount of data available is shortened in order to deal with significant Doppler scenarios. These results may envision the possibility of spatial multiplexing without the overhead related to training the MIMO structure. The loss of performance of trained

ZF and trained BLAST is not very significant. Future work must be done in order to improve present results with the combination of BSS and more complex MUDs as BLAST.

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