# Separability of Convolutive Mixtures: Application to the Separation of Combustion Noise and Piston-Slap in Diesel Engine

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**Abstract.** We focus on convolutive mixtures, expressed in time-domain. Separation is known to be obtained by testing the independence between delayed outputs. This criterion can be much simplified and we prove in this paper that testing the independence between the contributions of all sources on the same sensor at same time index also leads to separability. We recover the contribution by using Wiener filtering (or Minimal Distorsion Principal) which is included in the separation filters. The independence is tested here with the mutual information. It is minimized only for non-delayed outputs of the Wiener filters. The test is easier and shows good results on simulation and experimental signals for the separation of piston slap and combustion in diesel engine.

## 1 Introduction

Blind source separation (BSS) is a method for recovering a set of unknown source signals from the observation of their mixtures. Among open issues, recovering the sources from their linear convolutive mixtures remains a challenging problem. Many solutions have been addressed in the frequency-domain, particularly for the separation of non-stationary audio signals. In the BSS of stationary signals, two problems remain open in time domain. It has been proved [1] that convolutive mixtures are separable, that is, the independence of the outputs insures the separation of the sources, up to a few indeterminacies. However, the meaning of the independence is not the same in convolutive and instantaneous contexts. In the convolutive context, the outputs have to be independent in the sense of stochastic processes [2] which requires the independence of the random variables $y_i(n)$ and $y_j(n-m)$ for all discrete times $n$ and $m$. The independence criteria are therefore more complicated and computationally expensive. Several ideas are given in [3,4] to test the independence in function of time delays $m$, using the mutual information criterion. The second problem is coming from the inherent indeterminacy of the definition of a source in the BSS model. Indeed, any linear transform of a source can also be considered as a source and there is an infinity of separators that can extract sources. Some constraints can be added either on the source signals (they are usually supposed to be normalized) or on the separator system (Minimal Distorsion Principal [5]). In [5], one proposition is to choose the separator

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which minimizes the quadratic error between sensors and outputs, also known as Wiener filter. In this paper, we deal with convolutive mixtures and express the model in time-domain. The aim is to quantify the proportions of mechanical noise coming from piston slap or thermal noise and received on accelerometers, placed on one cylinder of a diesel engine. We are only interested in the contribution of these two sources recorded on each sensor. These signals are uniquely defined, which removes the filter indeterminacy. It can also help to simplify the independence criterion and we prove in this paper that testing the independence between the contributions of all sources on the same sensor at same time index *n* also leads to separability. We recover these contributions $z_i(n)$ by using Wiener filters which are included in the separation filters. The independence criterion is therefore less complicated as it requires only the independence between the outputs $z_i(n)$ and $z_j(n)$ (and no more $y_i(n)$ and $y_j(n-m)$). The mutual information is used here and shows good results on simulation and experimental signals for the separation of piston slap and combustion noise in diesel engine.

## 2 Modelization of the Observations

Let us consider the standard convolutive mixing model with M inputs and M outputs. Each sensor $x_j(n)$ (*j=1, .., M*) receives a linear convolution (noted *) of each source $s_i(n)$ (*i=1,...,M*) at discrete time *n*:

$$x_j(n) = \sum_{i=1}^{M} h_{ji} * s_i(n) \tag{1}$$

where $h_{ij}$ represents the impulse response from source *i* to sensor *j*. The inverse of mixing filters are not necessarily causal, so the aim of BSS is to recover non-causal filters with impulse responses $g_{ij}$ between sensor *i* and output *j*, such that the output vector $\boldsymbol{y}(n)$ estimates the sources, up to a linear filter :

$$y_j(n) = \sum_{i=1}^{M} \sum_{k=-L}^{L} g_{ji}(k) x_i(n-k) \tag{2}$$

Any linear transform of a source can also be considered as a source and there is an infinity of separators $g_{ij}$ that can extract sources. We focus here on the estimation of the signals $h_{ij} * s_i(n)$, coming from source *i* on sensor *j*. These signals are uniquely defined, which removes the filter indeterminacy. Let be a 2 sources 2 sensors scheme. For sake of simplicity, we call here sources the two contributions on the first sensor. So, $x_1(n)$ is equal to : $x_1(n) = s_1(n) + s_2(n)$ . Let be $y_1(n)$ and $y_2(n)$, two outputs :

$$y_j(n) = \sum_{i=1}^{2} \sum_{k=-L}^{L} g_{ji}(k) x_i(n-k) \tag{3}$$

If $y_j(n)$ is any linear filtering of one source, than the contribution of this source on the first sensor is calculated by an (eventually non causal) Wiener filter $W_j(z)$ such that the quadratic error between $x_1(n)$ and $y_j(n)$ is minimized. The two contributions on the first sensor are so given by:

$$z_j(n) = \sum_{i=1}^{2} \sum_{k=-L}^{L} w_j(k) y_i(n-k) \tag{4}$$

where the discrete Fourier Transforms (DFT) of the Wiener filters $w_j(k)$ are computed in function of the cross-spectra of $x_1(n)$ and $y_j(n)$ :

$$W_1(f) = \frac{\gamma_{Y1X1}(f)}{\gamma_{Y1}(f)} \quad ; \quad W_2(f) = \frac{\gamma_{Y2X1}(f)}{\gamma_{Y2}(f)} \tag{5}$$

## 3 Separability of the Source Contributions on One Sensor

In specific cases, testing the independence between $y_1(n)$ and $y_2(n)$ is sufficient [6] to ensure the separation. For example, for i.i.d. normalized sources, the sum of fourth-order cumulants of the outputs is a contrast function [7] under a condition on separating filters [6]. For linear filtering of i.i.d. signals, the same result is obtained after a first step of whitening of the data. However, in a general case, delays must be introduced in the contrast function and the separability of convolutive mixtures is obtained only when the components of the output vector $\mathbf{y}(n)$ are independent in the sense of stochastic variables : $y_1(n)$ and $y_2(n-m)$ have to be independent for all discrete time delays $m$. For example, a solution is to minimize the criterion $J$ :

$$J = \sum_m I\left(y_1(n), y_2(n-m)\right) \tag{6}$$

where $I$ represents the mutual information (7). $I$ is nonnegative and equal to zero if and only if the components are statistically independent.

$$I(y) = \int_R p_y(y) \ln \left( \frac{p_y(y)}{\prod_{i=1}^{M} p_{yi}(y_i)} \right) dy \tag{7}$$

The delays $m$ can be taken in an *a priori* set [-K, .., K], which depends on the degree of the filters corresponding to the whole mixing-separating system. The criterion (6) is computationally expensive. In [3], a gradient-based algorithm minimizes (6): at each time iteration, a random value of delay $m$ is chosen and $I(y_1(n), y_2(n-m))$ is used as the current separation criterion.

We propose to study here the separability of $z_1(n)$ and $z_2(n)$ (4) versus $y_1(n)$ and $y_2(n)$. We show that it is simpler and that no time delay $(n-m)$ is needed. Suppose now any outputs $y_1(n)$ and $y_2(n)$. To ensure the separation, it is necessary (but not sufficient) that the mutual information $I(y_1(n), y_2(n))$ is zero. Two cases can happen. If each output $y_j(n)$ only depends on one source, the outputs are also independent in the sense of stochastic processes (the separation has been effected) and it will be also verified for $z_1(n)$ and $z_2(n)$. So $I(z_1(n), z_2(n))=0$. In the second case, the outputs $y_j(n)$ can be independent ($I(y_1(n), y_2(n))=0$ at time delay 0) but remain mixtures of sources. For example, in the case of i.i.d sources, the two following outputs $y_j(n)$ are independent (8):

$$y_1(n) = s_1(n) + s_2(n)$$
$$y_2(n) = s_1(n-1) + s_2(n-1) \tag{8}$$

It occurs (typically for i.i.d. sources) when one source is common in the two outputs but with two different time index *(n-n0)* and *(n-n1)*. In that case, $y_j(n)$ are independent but surely not the components of $z_j(n)=W_i(z)\, y_j(n)$, as common time index can appear after linear filtering. It can be seen intuitively, since Wiener filtering aims at the maximization of the correlation between $z_1(n)$ and $x_1(n)$ (respectively $z_2(n)$ and $x_1(n)$). We will prove theoretically that indeed $I(z_1(n), z_2(n))$ is not equal to zero. As a consequence, testing the cancellation of $I(y_1(n), y_2(n))$ and $I(z_1(n), z_2(n))$ will ensure the separability.

Suppose that $y_1(n)$ and $y_2(n)$ are mixtures of the sources (even if $I(y_1(n), y_2(n))=0$). So are $z_1(n)$ and $z_2(n)$ after Wiener filtering. Let be $Z_1(f)$ and $Z_2(f)$, their DFT's. They are of the form (10). The transfer functions $W_1(f)$ and $W_2(f)$ (5) of the Wiener filters are expressed in function of the DFT of filters $g_{ij}(k)$, $G_{ji}(f)$, and the source spectra:

$$W_1(f) = \frac{\overline{G}_{11}(f)\gamma_{S1}(f) + \overline{G}_{12}(f)\gamma_{S2}(f)}{\gamma_{Y1}(f)} \quad ; \quad W_2(f) = \frac{\overline{G}_{21}(f)\gamma_{S1}(f) + \overline{G}_{22}(f)\gamma_{S2}(f)}{\gamma_{Y2}(f)} \tag{9}$$

$$\begin{aligned} Z_1(f) &= \frac{|G_{11}(f)|^2 \gamma_{S1}(f) + G_{11}(f)\overline{G}_{12}(f)\gamma_{S2}(f)}{\gamma_{Y1}(f)} S_1(f) + \frac{\overline{G}_{11}(f)G_{12}(f)\gamma_{S1}(f) + |G_{12}(f)|^2 \gamma_{S2}(f)}{\gamma_{Y1}(f)} S_2(f) \\ Z_2(f) &= \frac{|G_{21}(f)|^2 \gamma_{S1}(f) + G_{21}(f)\overline{G}_{22}(f)\gamma_{S2}(f)}{\gamma_{Y2}(f)} S_1(f) + \frac{\overline{G}_{21}(f)G_{22}(f)\gamma_{S1}(f) + |G_{22}(f)|^2 \gamma_{S2}(f)}{\gamma_{Y2}(f)} S_2(f) \end{aligned} \tag{10}$$

$z_j(n)$ are linear filtering of $s_1(n)$ and $s_2(n)$ as $y_1(n)$ and $y_2(n)$. Call $u_{ij}(k)$, the new mixing filters: $u_{ij}(k)=[w_j * g_{ij}](k)$ where * stands for the linear convolution. $z_j(n)$ are expressed as :

$$z_j(n) = \sum_{i=1}^{2} \sum_{k=-L}^{L} u_{ij}(k) s_i(n-k) \tag{11}$$

The two signals $z_1(n)$ and $z_2(n)$ cannot be independent ($I(z_1(n), z_2(n))$ is not zero) if some coefficients $u_{11}(k)$ and $u_{12}(k)$ are non zero for common time delays $k$. And, at least, we prove that one coefficient, $u_{ij}(k)(0)$, is non zero. Suppose that the DFT is computed on N time samples :

$$u_{11}(0) = \sum_{f=0}^{N-1} \frac{|G_{11}(f)|^2 \gamma_{S1}(f) + G_{11}(f)\overline{G}_{12}(f)\gamma_{S2}(f)}{\gamma_{Y1}(f)} \tag{12}$$

$$\begin{aligned} |u_{11}(0)|^2 &= \left( \sum_f |G_{11}(f)|^2 \frac{\gamma_{S1}(f)}{\gamma_{Y1}(f)} \right)^2 + \left| \sum_f G_{11}(f)\overline{G}_{12}(f) \frac{\gamma_{S2}(f)}{\gamma_{Y1}(f)} \right|^2 \\ &+ \sum_f |G_{11}(f)|^2 \frac{\gamma_{S1}(f)}{\gamma_{Y1}(f)} \left( \sum_f \frac{\gamma_{S2}(f)}{\gamma_{Y1}(f)} (G_{11}(f)\overline{G}_{12}(f) + \overline{G}_{11}(f)G_{12}(f)) \right) \end{aligned} \tag{13}$$

If the third term of the sum is positive or null, then $u_{11}(k)(0)$ cannot be null. If it is negative, $(u_{11}(k)(0))^2$ is always superior to a strictly positive value (14). Similar computations can be done with $u_{12}(k)(0)$, $u_{21}(k)(0)$ and $u_{22}(k)(0)$.

$$\begin{aligned}
\left|u_{11}(0)\right|^2 &> \left(\sum_f \left|G_{11}(f)\right|^2 \frac{\gamma_{S1}(f)}{\gamma_{Y1}(f)}\right)^2 + \left|\sum_f G_{11}(f)\overline{G}_{12}(f)\frac{\gamma_{S2}(f)}{\gamma_{Y1}(f)}\right|^2 \\
&-\sum_f \left|G_{11}(f)\right|^2 \frac{\gamma_{S1}(f)}{\gamma_{Y1}(f)}\left(\left|\sum_f \frac{\gamma_{S2}(f)}{\gamma_{Y1}(f)}(G_{11}(f)\overline{G}_{12}(f)\right| + \left|\sum_f \frac{\gamma_{S2}(f)}{\gamma_{Y1}(f)}\overline{G}_{11}(f)G_{12}(f))\right|\right) \\
&\geq \left|\sum_f \left|G_{11}(f)\right|^2 \frac{\gamma_{S1}(f)}{\gamma_{Y1}(f)} - \left|\frac{\gamma_{S2}(f)}{\gamma_{Y1}(f)}(G_{11}(f)\overline{G}_{12}(f)\right|\right|^2
\end{aligned} \tag{14}$$

So, for any outputs $y_j(n)$ which verify $I(y_1(n), y_2(n))=0$, then after Wiener filtering projected on the same sensor (here the first one) $I(z_1(n), z_2(n))$ is non zero. The only exception concerns the outputs $y_j(n)$ which depend on one source and it means that the separation has been achieved. Same results can also be obtained with M sources.

As a consequence, testing $I(y_1(n), y_2(n))=0$ and $I(z_1(n), z_2(n))=0$, ensures the separability. The criterion is much more easier to test than the mutual information of delayed outputs as it can be verified in an iterative way. Moreover the outputs are directly the contribution of the sources on the processed sensor.

## 4 Separating Algorithm and Simulations

The final separating algorithm for convolutive mixtures is based here on the minization of the mutual information as in [3] but the previous proof of separability could be exploited with another independence test.

Initialization : $\mathbf{y}(n)= \mathbf{x}(n)$
Repeat until convergence :

- Estimate the score function difference between $y_1(n)$ and $y_2(n)$: $\beta(y_1(n),y_2(n))$
- Update : $\mathbf{y}(n) \leftarrow \mathbf{y}(n)- \mu\, \beta( y_1(n),y_2(n))$
- Compute the Wiener filters $W_i(z)$, and the contributions : $z_j(n)=W_i(z)\, y_j(n)$
- Replace : $\mathbf{y}(n) \leftarrow z(n)$

The performances are shown in figures 1 and 2 with simulations results. Each source (of 1500 samples) is constituted of the sum of a uniform random signal and a sinusoid. They are mixed with filters :

$$H(z) = \begin{bmatrix} 1+0.2z^{-1}+0.1z^{-2} & 0.5+0.3z^{-1}+0.1z^{-2} \\ 0.5+0.3z^{-1}+0.1z^{-2} & 1+0.2z^{-1}+0.1z^{-2} \end{bmatrix} \tag{15}$$

The mutual information (between $z_1(n)$ and $z_2(n)$) and the quadratic error between $z_1(n)$ and the exact contribution are plotted in fig.1 and 2 with marks, for each iteration. They are averaged on 50 realizations of the sources. It shows good results for the convergence speed and the residual quadratic error. The results can still be improved by adding some constraints. Indeed, four contributions must be computed in this scheme by projecting $y_1(n)$ (respectively $y_2(n)$) on the two sensors: $z_{11}(n)$, $z_{21}(n)$ (respectively $z_{12}(n)$, $z_{22}(n)$). The convergence speed is increasing by adding the mutual information between the projections on the second sensor $I(z_{21}(n), z_{22}(n))$ to $I(z_{11}(n), z_{12}(n))$ (as previously) in the minimization. The results are displayed in figures 1 and 2 in solid line and show the increasing of the convergence. So, the new algorithm is:

Initialization : $\mathbf{y}(n)= \mathbf{x}(n)$
Repeat until convergence :

- Estimate the score function differences: $\beta(z_{11}(n), z_{12}(n))$, $\beta(z_{21}(n), z_{22}(n))$
- Update : $\mathbf{y}(n) \leftarrow \mathbf{y}(n) - \mu\, [\beta(z_{21}(n), z_{22}(n))+\beta(z_{11}(n), z_{12}(n))]$
- Compute the Wiener filters $W_{ij}(z)$, and the contributions : $z_{ij}(n)=W_{ij}(z)\, y_j(n)$
- Replace : $\mathbf{y}(n) \leftarrow [z_{11}(n), z_{12}(n))]$

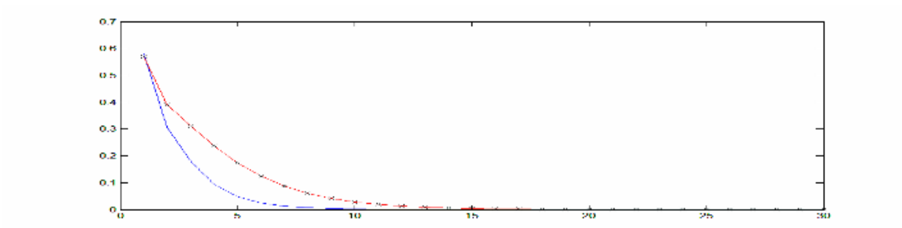


**Fig. 1.** Mutual information versus iterations

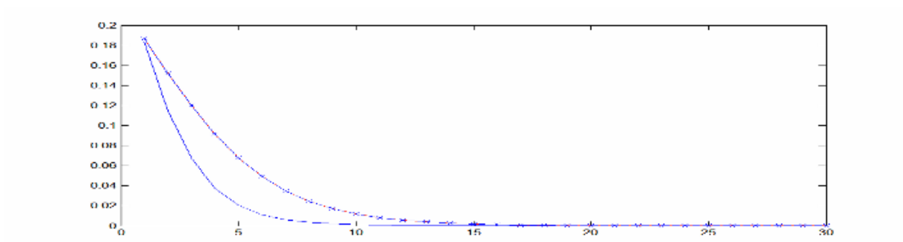


**Fig. 2.** Quadratic error between the contribution of one source on the first sensor and its estimate, versus iterations

## 5 Separation of Piston Slap and Combustion in Diesel Engine

The aim is to characterize the relative noise given out by a diesel engine by quantifying the proportions of mechanical noise coming from piston slap and thermal noise or combustion. Signals are issued of ten accelerometers, placed on a four-stroke and four cylinder diesel engine. They record thermal and mechanical phenomena that are temporally superposed around the TDC, as well as spectrally overlapping. Some sensors respond to vertical moves or horizontal ones, according to their positions. Therefore, some accelerometers are more sensitive to combustion noise whereas the other ones receive more mechanical noise as piston-slap. Nevertheless, all accelerometer signals are convolutive mixtures of thermal and mechanical sources. Signals have been sampled at 25600Hz. In figure 3, we show the power measured by one sensor in the angular window [-40°, 80°] of the crankshaft (in green). This sensor responded to horizontal moves as it was placed on one side of the liner and received more piston-slap. The figure 3 includes the measured pressure and the injection control pulses. The contributions of the two sources on the sensor have been estimated by the algorithm proposed in section 4 and their powers are shown in figures 4 and 5.

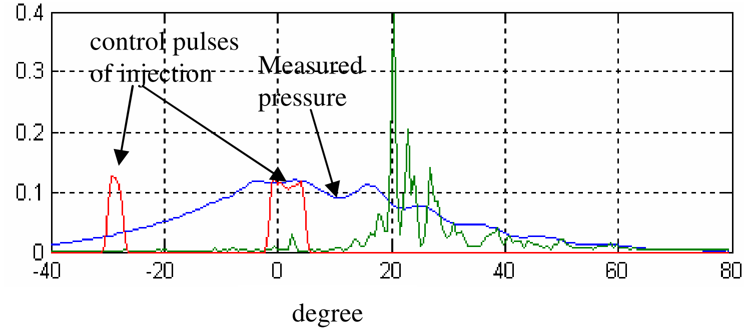


**Fig. 3.** Power of one sensor versus the crankshaft angle in degree

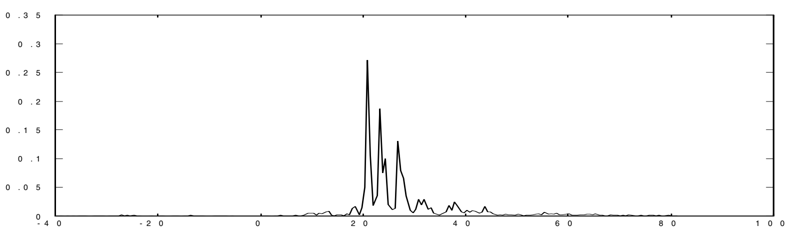


**Fig. 4.** Power of the first separated source versus the crankshaft angle

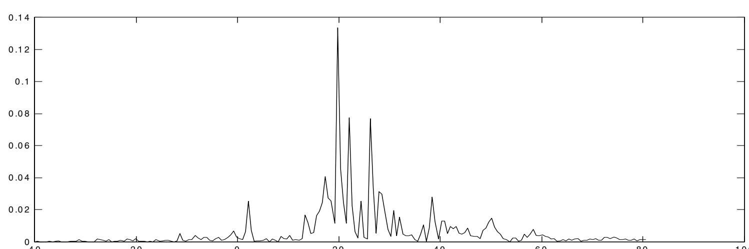


**Fig. 5.** Power of the second separated source versus the crankshaft angle

A first experiment has been done without injection and therefore no combustion noise is present. It helps to know the exact localization of the mechanical and thermal phenomena. This experiment (not presented here) shows that the sensor registers three mechanical shocks, at 20°5, 23° and 27°. By difference, we can conclude that the combustion is present between 10° and 20°, including the position of the main shock and around 0° for the pre-combustion. The two phenomena are well noticeable in figure 3. After separation, we can see that the first contribution is really the most important. It can be correctly attributed to mechanical shocks as they take place at 21°, 23°8 and 28°8. Besides no pre-combustion is seen around 0° and the main combustion (between 10° and 20°) is well separated. The second contribution is a good estimation of the thermal noise as we recover the pre-combustion and the main combustion. Moreover, the position of the pre-combustion is validated by the localization of the control pulses of injection (seen in figure 3).

## 6 Conclusion

We focus on the separability of convolutive mixtures, expressed in time-domain. In the convolutive context, the outputs $y_i(n)$ have to be independent in the sense of stochastic processes which requires the independence of $y_i(n)$ and $y_i(n-m)$ for all discrete times $n$ and $m$. The independence criteria are therefore complicated and computationally expensive. The criterion has been simplified as we recover only the contribution of all sources on all sensors, by using Wiener filtering (or Minimal Distorsion Principal). It has been proved that testing the independence between these contributions on the same sensor also leads to separability, without testing an independence test of delayed outputs. The criterion is easier to test and is implemented here by minimizing the mutual information of the outputs after Wiener filtering. It shows good results on simulation and experimental signals for the separation of piston slap and combustion in diesel engine.

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