Second-Order Separation of Multidimensional Sources with Constrained Mixing System

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Abstract. The case of sources that generate multidimensional signals, filling a subspace of dimensionality K, is considered. Different coordinate axes of the subspace ("subspace channels") correspond to different signal portions generated by each source, e.g., data from different spectral bands or different modalities may be assigned to different subspace channels. The mixing system that generates observed signals from the underlying sources is modeled as superimposing within each subspace channel the contributions of the different sources. This mixing system is constrained as it allows *no* mixing of data that occurs in *different* subspace channels. An algorithm based on second order statistics is given which leads to a solution in closed form for the separating system. Correlations across different subspace channels are utilized by the algorithm, whereas properties such as higher-order statistics or spectral characteristics within subspace channels are not considered. A permutation problem of aligning different sources' subspace channels is solved based on ordering of eigenvalues derived from the separating system. Effectiveness of the algorithm is demonstrated by application to multidimensional temporally i.i.d. Gaussian signals.

1 Introduction

The notion of multi-dimensional or subspace ICA has been developed in [3] and [6] to account for the fact that not all sources may reasonably be modeled as one-dimensional processes with mutual independence. Rather, some sources may generate signals that fill a multi-dimensional subspace that resists decomposition into one-dimensional mutually independent sources. This can occur both in situations where underlying sources are unknown and rather a plausible model of the observed data is sought for, and in situations where analytical reasons dictate a multi-dimensional character of the sources, such as separation of spectral domain speech, which has originally motivated this work.

The present work suggests a second-order approach to the separation of multidimensional sources and considers a constrained version of the general linear mixing system.

2 Multidimensional Sources and Constrained Mixing

The multidimensional signal generated by source i ($i = 1, \ldots, N$) is denoted by $s_i^f(t)$. The source is regarded as stationary and ergodic with respect to parameter t, $t =$

J. Rosca et al. (Eds.): ICA 2006, LNCS 3889, pp. 16–23, 2006.

$t_1, \ldots, t_L$, i.e., expectation values can be estimated as sample means w.r.t. t. E.g., t may denote time or spatial position in an image, provided stationarity may be assumed w.r.t. these variables. Without loss of generality, we limit our treatment to zero mean sources.

Index f spans the subspace of dimensionality K that is filled by the source, with $f = 1, \ldots, K$ denoting the "subspace channels" (or "channels") of the source. Source statistics w.r.t. individual channels $f \neq f'$ may differ, hence, expectations cannot be computed as sample averages across f. Subspace channels, e.g., may correspond to different frequency bands for which data of an audio source or multispectral image data has been collected. More generally, subspace channels may also correspond to different modalities of recorded data, e.g., audio and video data may be stored in different channels. Another example would be data with non-constant mixing, e.g., image data with position-dependent mixing parameters; here, different subspace channels could correspond to different spatial positions.

As they are generated by the same source, data in different subspace channels $f \neq f'$ of a single source may in general covary,

$$E\left\{s_i^f(t)\,(s_i^{f'}(t))^*\right\} \neq 0. \tag{1}$$

Data from different sources $i \neq j$ must be uncorrelated since the sources are assumed to be independent systems. If correlations are computed from source components at two subspace channels, the result is zero for all pairs of channels (f, f'),

$$E\left\{s_i^f(t)\,(s_j^{f'}(t))^*\right\} = 0 \qquad \forall i \neq j,\, \forall f, f'. \tag{2}$$

Mixing of sources is assumed to be separable in the sense that the mixing system only mixes data from corresponding subspace channels of different sources, but does *not* mix data "across" different channels. Gathering data from the f-th subspace channel of all N sources into a single vector $\mathbf{s}^f(t) = [s_1^f(t), \ldots, s_N^f(t)]^T$, mixing is written as

$$x_i^f(t) = \sum_{j=1}^{N} a_{ij}^f\, s_j^f(t) \qquad \Longleftrightarrow \qquad \mathbf{x}^f(t) = \mathbf{A}^f\, \mathbf{s}^f(t) \tag{3}$$

This model is compatible with the mixing scenarios in the examples mentioned above. E.g., multimodal data may plausibly be explained by superposition of basis-patterns within each modality. A-priori, intermingling of data from different subspace channels may be regarded as a less significant process and may be ruled out completely in some applications (e.g., frequency-domain separation of convolutive audio mixtures) on the grounds of known physics.

From knowledge of the mixed signals $\mathbf{x}^f(t)$, only, it is aimed to find an estimate $\hat{\mathbf{A}}^f$ of the mixing matrix so that unmixed signals

$$\mathbf{u}^f(t) = [\hat{\mathbf{A}}^f]^{-1}\, \mathbf{x}^f(t) \tag{4}$$

can be obtained which resemble the source signals.

The simplest approach to solve system (3) would be to perform ICA or second-order source separation separately for each subspace channel f. For two reasons this approach might not be optimal. First, by neglecting information present in the across-channel correlations (Eq.s 1, 2), the obtained separation quality may not be optimal, e.g., because the data in individual channels might be lacking sufficient higher-order or spectral cues. In the evaluation (Sec. 4), we demonstrate that across-channel correlations make it possible to separate data that have no spectral, higher-order or non-stationarity cues.

Second, care has to be taken to reconstruct coherent subspaces, each pertaining to one source process. When treating Eq. 3 as N individual source separation problems, the permutation invariance inherent to blind source separation algorithm applies individually to each one, so that gathering the subspace channels for each source process is a non-trivial issue. A similar problem is frequently encountered in frequency-domain approaches to convolutive blind source separation. It is shown that across-channel correlations as in Eq.s 1, 2 can be exploited to this end.

3 Solution Based on Correlations Across Subspace Channels

Defining the sources' cross-covariance matrix $\mathbf{R}_s^{ff'}$ computed from channels f and f' as

$$\mathbf{R}_s^{ff'} = E\left\{\mathbf{s}^f(t)\,(\mathbf{s}^{f'}(t))^H\right\}, \tag{5}$$

equations (2) and (1) can be restated such that $\mathbf{R}_s^{ff'}$ is diagonal for all (ff'),

$$\left[\mathbf{R}_s^{ff'}\right]_{ij} = \delta_{ij} E\left\{s_i^f(t)\,(s_i^{f'}(t))^*\right\}, \tag{6}$$

where δ_{ij} is the Kronecker symbol.

Since the mixed signals are not independent, their covariance matrix $\mathbf{R}_x^{ff'}$,

$$\mathbf{R}_x^{ff'} = E\left\{\mathbf{x}^f(t)\,(\mathbf{x}^{f'}(t))^H\right\}, \tag{7}$$

is not diagonal. It can be expressed in terms of the sources' covariance matrix as

$$\mathbf{R}_x^{ff'} = \mathbf{A}^f\,\mathbf{R}_s^{ff'}\left(\mathbf{A}^{f'}\right)^H. \tag{8}$$

If the mixing system was identical in both subspace channels, $\mathbf{A}^f = \mathbf{A}^{f'}$, then an eigenvalue equation could be derived in exactly the same manner as presented by [7]. However, since in general $\mathbf{A}^f \neq \mathbf{A}^{f'}$, the analog derivation is not possible.

It is observed that by forming the products

$$\mathbf{Q}_s^{ff'} = \mathbf{R}_s^{ff'}\,[\mathbf{R}_s^{f'f'}]^{-1}\,\mathbf{R}_s^{f'f} \tag{9}$$

$$\mathbf{Q}_x^{ff'} = \mathbf{R}_x^{ff'}\,[\mathbf{R}_x^{f'f'}]^{-1}\,\mathbf{R}_x^{f'f} \tag{10}$$

the algebraic relation between the sources' $\mathbf{Q}_s^{ff'}$ and the mixed signals' $\mathbf{Q}_x^{ff'}$ involves matrix $\mathbf{A}^f$, but not $\mathbf{A}^{f'}$,

$$\mathbf{Q}_s^{ff'} = [\mathbf{A}^f]^{-1}\,\mathbf{Q}_x^{ff'}\,[\mathbf{A}^f]^{-H} \tag{11}$$

Hence, $[\mathbf{A}^f]^{-1}$ diagonalizes $\mathbf{Q}_x^{ff'}$ for all f'.

An eigenvalue equation for $\mathbf{A}^f$ can be derived from (11) by forming the product

$$\mathbf{Q}_x^{ff'}\,[\mathbf{Q}_x^{ff}]^{-1}, \tag{12}$$

yielding

$$\mathbf{A}^f\,\mathbf{\Lambda}^{ff'} = \mathbf{Q}_x^{ff'}\,[\mathbf{Q}_x^{ff}]^{-1}\,\mathbf{A}^f, \tag{13}$$

where

$$\mathbf{\Lambda}^{ff'} = \mathbf{Q}_s^{ff'}\,[\mathbf{Q}_s^{ff}]^{-1} \tag{14}$$

is diagonal and contains the eigenvalues of $\mathbf{Q}_x^{ff'}\,[\mathbf{Q}_x^{ff}]^{-1}$.

Similarly, $\mathbf{A}^{f'}$ is obtained from the Eigenvalue equation

$$\mathbf{A}^{f'}\,\mathbf{\Lambda}^{f'f} = \mathbf{Q}_x^{f'f}\,[\mathbf{Q}_x^{f'f'}]^{-1}\,\mathbf{A}^{f'}. \tag{15}$$

3.1 Conditions for Identifiability

Equation (13) has a unique solution if all eigenvalues on the diagonal of $\mathbf{\Lambda}^{ff'}$ are different. Similarly, for (15) it must hold that the diagonal elements of $\mathbf{\Lambda}^{f'f}$ are different. Since $\mathbf{R}_s^{ff'}$ is diagonal and $\mathbf{R}_s^{ff'} = [\mathbf{R}_s^{f'f}]^H$, we obtain

$$\begin{aligned}\mathbf{\Lambda}^{ff'} \;=\; \mathbf{\Lambda}^{f'f} \;=\; \\ \mathbf{R}_s^{ff'}\,[\mathbf{R}_s^{ff'}]^H\,[\mathbf{R}_s^{ff}]^{-1}\,[\mathbf{R}_s^{f'f'}]^{-1}.\end{aligned} \tag{16}$$

Hence, together with (6) it follows that for $\mathbf{A}^f$ and $\mathbf{A}^{f'}$ to be identifiable it must be fulfilled that $\forall i \neq j$

$$\frac{\left|E\{s_i^f(t)\,(s_i^{f'}(t))^*\}\right|^2}{E\{\left|s_i^f(t)\right|^2\}\,E\{\left|s_i^{f'}(t)\right|^2\}} \neq \frac{\left|E\{s_j^f(t)\,(s_j^{f'}(t))^*\}\right|^2}{E\{\left|s_j^f(t)\right|^2\}\,E\{\left|s_j^{f'}(t)\right|^2\}}. \tag{17}$$

3.2 Solving the Permutation Problem

Since the eigenvectors corresponding to the solution of (13) are unambiguous only upto their order and a scale factor, the mixing matrix $\mathbf{A}^f$ cannot be determined uniquely. Rather, any matrix $\tilde{\mathbf{A}}^f$ which can be expressed as

$$\tilde{\mathbf{A}}^f = \mathbf{A}^f\,\mathbf{D}^f\,\mathbf{P}^f, \tag{18}$$

where $\mathbf{D}^f$ is a diagonal matrix and $\mathbf{P}^f$ a permutation matrix, represents a solution of (13). Hence, it is only possible to determine $\mathbf{A}^f$ upto an unknown rescaling and permutation of its columns by $\mathbf{D}^f$ and $\mathbf{P}^f$, respectively. This corresponds to the well-known invariances inherent to all blind source separation algorithms.

For one-dimensional source signals this is usually not a problem. With multidimensional sources, the components belonging to a single source are reconstructed with disparate (unknown) order and scale in different subspace channels $f \neq f'$ if the corresponding channel-specific permutation and diagonal matrices differ, i.e.,

$$\mathbf{P}^f \neq \mathbf{P}^{f'} \qquad \mathbf{D}^f \neq \mathbf{D}^{f'}. \tag{19}$$

Thus, a coherent picture of each source's activity cannot be obtained.

No solution is given for the invariance with respect to varied scaling in different channels. Instead, each row of the estimated unmixing matrix $[\hat{\mathbf{A}}^f]^{-1}$ is rescaled to have unit norm.

The solution to the permutation problem is based on the observation that transformation (18) results in rearranged eigenvalues $\tilde{\mathbf{\Lambda}}^{ff'}$,

$$\tilde{\mathbf{\Lambda}}^{ff'} = [\mathbf{P}^f]^T \, \mathbf{\Lambda}^{ff'} \, \mathbf{P}^f. \tag{20}$$

That is, the column permutation of $\mathbf{A}^f$ results in a corresponding permutation of the eigenvalues' order on the diagonal of $\tilde{\mathbf{\Lambda}}^{ff'}$.

Denote by $\hat{\mathbf{A}}^f$ and $\hat{\mathbf{A}}^{f'}$ the estimates of the true mixing matrices $\mathbf{A}^f$ and $\mathbf{A}^{f'}$, respectively. Without loss of generality, we assume

$$\hat{\mathbf{A}}^f = \mathbf{A}^f \qquad \hat{\mathbf{A}}^{f'} = \mathbf{A}^{f'} \, \mathbf{P}, \tag{21}$$

so that the estimates $\hat{\mathbf{\Lambda}}^{ff'}$ and $\hat{\mathbf{\Lambda}}^{f'f}$ of the true eigenvalue matrices $\mathbf{\Lambda}^{ff'}$ and $\mathbf{\Lambda}^{f'f}$, respectively, are

$$\hat{\mathbf{\Lambda}}^{ff'} = \mathbf{\Lambda}^{ff'} \tag{22}$$

$$\hat{\mathbf{\Lambda}}^{f'f} = \mathbf{P}^T \mathbf{\Lambda}^{f'f} \mathbf{P} \tag{23}$$

Since, according to (16) we have $\mathbf{\Lambda}^{f'f} = \mathbf{\Lambda}^{ff'}$, it follows

$$\hat{\mathbf{\Lambda}}^{f'f} = \mathbf{P}^T \mathbf{\Lambda}^{ff'} \mathbf{P} = \mathbf{P}^T \hat{\mathbf{\Lambda}}^{ff'} \mathbf{P}. \tag{24}$$

Therefore, the permutation matrix $\mathbf{P}$ can be directly read from the relative ordering of the eigenvalues on the diagonals of $\hat{\mathbf{\Lambda}}^{ff'}$ and $\hat{\mathbf{\Lambda}}^{f'f}$. Permutations are corrected by replacing $\hat{\mathbf{A}}^{f'}$ by $\hat{\mathbf{A}}^{f'} \, \mathbf{P}^T$ whose columns are ordered in accordance with $\hat{\mathbf{A}}^f$.

3.3 More Than Two Subspace Channels

Separation. If channels $f = 1, \ldots, K$, $K \geq 2$, are to be used for separation, the mixing matrix $\mathbf{A}^f$ is obtained as the matrix which simultaneously solves the K diagonalization equations

$$\begin{aligned} \mathbf{Q}_s^{f,1} &= [\mathbf{A}^f]^{-1} \, \mathbf{Q}_x^{f,1} \, [\mathbf{A}^f]^{-H} \\ \mathbf{Q}_s^{f,2} &= [\mathbf{A}^f]^{-1} \, \mathbf{Q}_x^{f,2} \, [\mathbf{A}^f]^{-H} \\ &\vdots \\ \mathbf{Q}_s^{f,K} &= [\mathbf{A}^f]^{-1} \, \mathbf{Q}_x^{f,K} \, [\mathbf{A}^f]^{-H}. \end{aligned} \tag{25}$$

The solution can be obtained by using numerical techniques for simultaneous diagonalization [4].

Identifiability. Equations (25) have a unique solution (up to rescaling and permutation) if, analogous to Eq. (17), for each $f = 1, \ldots, K$ there exists at least one subspace channel f' for which it is fulfilled that $\forall i \neq j$

$$\frac{\left|E\{s_i^f(t)\,(s_i^{f'}(t))^*\}\right|^2}{E\{\left|s_i^f(t)\right|^2\}\,E\{\left|s_i^{f'}(t)\right|^2\}} \neq \frac{\left|E\{s_j^f(t)\,(s_j^{f'}(t))^*\}\right|^2}{E\{\left|s_j^f(t)\right|^2\}\,E\{\left|s_j^{f'}(t)\right|^2\}}. \tag{26}$$

Permutations. The permutations must be sorted for each pair of subspace channels (f, f') by using the method outlined in section 3.2.

4 Evaluation

A synthetic data set of Gaussian i.i.d. noise in two channels is separated. Since the data in each subspace channel is purely Gaussian, these data cannot be separated by looking at a single channel only.

The data consisted of four sources $s_1^f(t), \ldots, s_4^f(t)$, each containing a two-dimensional subspace with channels, $f = 1, 2$, and time-points $t = 1, \ldots, 10000$. Within each subspace channel of each source, the data was chosen to be i.i.d. noise with Gaussian distribution. To enable separation by the proposed algorithm, correlations were introduced between the data in different channels of each source by composing the signals as the sum

$$s_i^f(t) = \xi_i^f(t) + \zeta_i(t) \tag{27}$$

of channel-dependent and channel-independent Gaussian random variables $\xi_i^f(t)$ and $\zeta_i(t)$, respectively.

Since the data within each subspace channel contained neither cues related to higher-order statistics, nor cues related to auto-correlation information or non-stationarity, it is inseparable for any algorithm looking at isolated channels. Only integrating information across different channels makes separation feasible.

The correlations within each source and the independence of the different sources are reflected by the covariance matrices $\mathbf{R}_s^{ff'}$,

$$\mathbf{R}_s^{1,1} = \begin{pmatrix} 1.99 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.89 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.20 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.04 \end{pmatrix} \qquad \mathbf{R}_s^{1,2} = \begin{pmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.64 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.16 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.04 \end{pmatrix} \tag{28}$$

$$\mathbf{R}_s^{2,1} = \begin{pmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.64 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.16 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.04 \end{pmatrix} \qquad \mathbf{R}_s^{2,2} = \begin{pmatrix} 2.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.89 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.20 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.04 \end{pmatrix}. \tag{29}$$

Since the different sources are independent, the off-diagonal terms of all covariance matrices are zero. The diagonals of $\mathbf{R}_s^{1,2}$ and $\mathbf{R}_s^{2,1}$ are non-zero due to the correlations across channels within each source subspace.

The eigenvalues of equation (16) are computed as

$$\operatorname{diag} \mathbf{\Lambda}^{1,2} = (\,\Lambda_1^{1,2},\, \ldots,\, \Lambda_4^{1,2}\,) = (\,0.25,\, 0.51,\, 0.64,\, 1.00\,). \tag{30}$$

Since all eigenvalues are different, the condition for identifiability (17) is fulfilled.

The 4×4 mixing matrices $\mathbf{A}^1$ and $\mathbf{A}^2$ were chosen at random. Covariance matrices of the mixed signals were processed by the proposed algorithm using the eigenvalue method, yielding the combined mixing-unmixing system $[\hat{\mathbf{A}}^f]^{-1}\mathbf{A}^f$

$$[\hat{\mathbf{A}}^1]^{-1}\mathbf{A}^1 = \begin{pmatrix} \mathbf{-3.11} & -0.03 & 0.01 & -0.02 \\ 0.00 & 0.01 & \mathbf{-2.43} & -0.02 \\ -0.01 & \mathbf{2.39} & 0.04 & -0.01 \\ 0.00 & 0.00 & 0.00 & \mathbf{2.23} \end{pmatrix} \quad [\hat{\mathbf{A}}^2]^{-1}\mathbf{A}^2 = \begin{pmatrix} 0.00 & 0.00 & 0.00 & \mathbf{-1.50} \\ \mathbf{1.69} & 0.01 & -0.03 & -0.02 \\ -0.01 & \mathbf{1.49} & 0.02 & 0.02 \\ 0.00 & 0.00 & \mathbf{-1.43} & -0.01 \end{pmatrix}$$

Since each row of the combined system contains only one significant non-zero element, the algorithm has successfully separated the signals. The increase in signal-to-interference ratio from before to after separation amounts to 37.8 dB.

Sources' components are reconstructed in a different order in the two frequency channels, as can be seen from the different positions of the non-zero elements of $(\hat{\mathbf{A}}^{-1}\mathbf{A})(1)$ and $(\hat{\mathbf{A}}^{-1}\mathbf{A})(2)$. Therefore, the method for sorting permutations described in section 3.2 must be employed. To this end, the estimated eigenvalue matrices $\hat{\mathbf{\Lambda}}(1,2)$ and $\hat{\mathbf{\Lambda}}(2,1)$ obtained from solving the eigenvalue problems (13) and (15), respectively, are

$$\hat{\mathbf{\Lambda}}(1,2) = \begin{pmatrix} \mathbf{0.25} & 0.00 & 0.00 & 0.00 \\ 0.00 & \mathbf{0.64} & 0.00 & 0.00 \\ 0.00 & 0.00 & \mathbf{0.51} & 0.00 \\ 0.00 & 0.00 & 0.00 & \mathbf{1.00} \end{pmatrix} \quad \hat{\mathbf{\Lambda}}(2,1) = \begin{pmatrix} \mathbf{1.00} & 0.00 & 0.00 & 0.00 \\ 0.00 & \mathbf{0.25} & 0.00 & 0.00 \\ 0.00 & 0.00 & \mathbf{0.51} & 0.00 \\ 0.00 & 0.00 & 0.00 & \mathbf{0.64} \end{pmatrix}. \tag{31}$$

By permuting the eigenvalues on the diagonals of $\hat{\mathbf{\Lambda}}(1,2)$ and $\hat{\mathbf{\Lambda}}(2,1)$ to occur in the same order in both matrices, and by performing the same permutations for the rows of $\hat{\mathbf{A}}^{-1}(1)$ and $\hat{\mathbf{A}}^{-1}(2)$, respectively, it is ensured that the sources' components are reconstructed in the same order in both frequencies.

5 Discussion

We have proposed a solution to the BSS problem when sources generate subspaces with second-order dependencies within each source subspace. Under the assumption of a constrained mixing system, that can be separated into one linear instantaneous mixing system per subspace channel, an eigenvalue/joint diagonalization based approach has been developed for source identification and correct assignment of subspace dimensions across different sources.

Under additional assumptions, existing second-order separation methods are recovered as special cases of our method. If different subspace channels are derived from underlying one-dimensional sources by temporal shifting, approaches like SOBI [2], Molgedey-Schuster [7] and TDSEP [8] are recovered. In this case, the signal $s_i^f(t)$ would be constructed from a one-dimensional signal $s_i(t)$ as $s_i^f(t) = s_i(t + \tau^f)$, for time-shifts $\tau^1, \ldots, \tau^K$, and constant mixing matrices $\mathbf{A}^f = \mathbf{A}$ would be assumed.

In the two-input-two-output (TITO) case with a whitening preprocessing step, the separation equations of our algorithm boil down to the TITO identification of FIR channels proposed by [5] (while the permutation alignment step of both algorithms remains different).

It is straight-forward to combine the techniques outlined here with standard second-order separation techniques that can employ spectral cues or non-stationarity of variance within each source subspace channel. Such a combination approach yields a large number of equations for simultaneous diagonalization that are expected to lead to decent signal separation.

The developed method may be useful in two applications. For the separation of data with multiple spectral bands, e.g., spectrogram sound data or spectral image data, correlations across different frequency-channels constitute a criterion for source separation that can be used on its own, or in addition to existing methods of decorrelation with respect to time- or spatial shifts. By using this additional source of information, it should be possible to improve on the performance of source separation algorithms in a similar way as, e.g., decorrelation with multiple time-delays can improve over decorrelation with only a single time-delay.

Concerning separation of time-varying mixtures, present approaches average over short time segments to estimate the averaged unmixing system. The presented method may improve the quality of separation since it allows to estimate the unmixing system for time t taking into account data from time $t + \tau$ even though the unmixing system at both times is different, and without necessarily averaging over the entire temporal range $t \dots t + \tau$.

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